Vol 12 No 8, August 2003
1009-1963/2003/12(08)/0866-06
Chinese Physics
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Linear entropy dynamics of the field in the Jaynes–Cummings model with an intensity-dependent coupling in the dispersive approximation*

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(Received 10 February 2003; revised manuscript received 18 March 2003)

We present the linear entropy dynamics of the field state in the dispersive cavity in the Jaynes–Cummings model with an intensity-dependent coupling in the dispersive approximation, and investigate the influence of dissipation on entanglement between the field and the atoms. We show that the coherence properties of the field are also affected by the cavity when the nonlinear process of the field interacting with the atoms with an intensity-dependent coupling is involved, and find that the dissipation constant, the intensity of the field and the atomic distribution angle have different influence on the coherence properties of the field.

Keywords: coherence loss, linear entropy, entanglement
PACC: 4250

1. Introduction

The Jaynes–Cummings model (JCM) with an intensity-dependent coupling is an important model to describe a two-level atom interacting with an electromagnetic field for its exact solvability within the rotating-wave approximation and its periodical and symmetrical dynamics properties. Many authors have studied the squeezing of the field,[1] the phase[2,3] and the entropy of the field,[4] the disentanglement[5,6] in the model. However, all the work was done only on the condition that the influence of the environment is not taken into account. The environment which is represented by a thermal reservoir always exists, and affects the system considered. Because of the dissipation and fluctuation originating from the coupling to the environment, the primary system will collapse from a correlated state into a statistical mixture[7–9] irreversibly (which is usually called decoherence or purity loss). The entanglement between the atom and the field, as well as the decoherence induced by the cavity of the usual dispersive JCM was studied in Ref.[10]. It shows that the cavity has practically no influence on the coherence properties of the field from a qualitative point of view; however, although the atom is not directly coupled to the cavity modes, its coherence properties are strongly influenced by dissipation both qualitatively and quantitatively.

The purpose of this paper is to study the linear entropy dynamics of the field in the JCM with an intensity-dependent coupling in the dispersive approximation and examine whether the coherence properties of the field are influenced by the reservoir during the passage of the atom through the height Q of the cavity. We find that the dissipation suppresses the amplitude of the entanglement between the field and the atom, and makes the field lose its coherence. Finally, we discuss the influence of the dissipation constant, the intensity of the field and the atomic distribution on the coherence properties of the field.

2. The JCM with an intensity-dependent coupling in the dispersive approximation and the linear entropy of the field

First, we consider a single-mode field interacting with an effective two-level atom with an intensity-

*Project supported by the National Natural Science Foundation of China (Grant No 19874020), by the Natural Science Foundation of Hunan Province, China (Grant No 01JJY3030) and by the Ministry of Education, China.
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dependent coupling without considering the influence of the dissipation. The Hamiltonian of the system under the rotating wave approximation is

$$\hat{H} = \omega_0 \hat{S}_z + \omega \hat{a}^+ \hat{a} + \lambda[\hat{S}_+ \hat{R} + \hat{S}_- \hat{R}^+] \quad (\hbar = 1), \quad (1)$$

where $\hat{a}^+$ and $\hat{a}$ are the creation and annihilation operators of the radiation field, $\hat{S}_+$ and $\hat{S}_-$ are pseudopsia operators of the atom, λ is the coupling constant, ω and ω_0 are the frequencies of the field and the atomic transition, respectively, θ is the atomic distribution angle. The operators $\hat{R}$ and $\hat{R}^+$ are defined in the following way[11]

$$\hat{R} = \hat{a}\sqrt{\hat{N}}, \qquad \hat{R}^+ = \sqrt{\hat{N}}\hat{a}^+, \quad (2)$$

where $\hat{N} = \hat{a}^+\hat{a}$. In an invariant subspace spanned by $|+\rangle \otimes |n\rangle$ and $|-\rangle \otimes |n+1\rangle$, the Hamiltonian takes the following form:

$$\hat{H} = \begin{bmatrix} \omega n + \omega_0/2 & \lambda(n+1) \\ \lambda(n+1) & \omega(n+1) - \omega_0/2 \end{bmatrix}. \quad (3)$$

The eigenvalues of H are given by:

$$E_{n\pm} = w(n+1) \pm \frac{1}{2}\sqrt{\delta^2 + 4\lambda^2(n+1)^2}, \quad (4)$$

where $\delta = \omega - \omega_0$ measures how far the two systems are off resonance. In the large detuning approximation, we have

$$|\delta|/\lambda \gg n+1, \quad (5)$$

for any relevant n. In this approximation, the eigenvalues are given by:

$$E_{n\pm} = \omega(n+1) \pm \frac{|\delta|}{2} \pm \frac{\lambda^2(n+1)^2}{|\delta|}. \quad (6)$$

Eq. (5) reflects that δ is related not only to the coupling constant λ but also to the intensity of the field.[14−16] If condition Eq.(5) is fulfilled for all n values, which plays a significant role in the problem, we can work with the effective Hamiltonian:

$$\begin{aligned}\hat{H}_{\rm eff} =& \omega\left(\hat{a}^+\hat{a} + \frac{1}{2}\right) + \omega_0 \hat{S}_z/2 \\ &+ g\Big[(\hat{a}^+\hat{a}+1)^2|+\rangle\langle+| - (\hat{a}^+\hat{a})^2|-\rangle\langle-|\Big], \quad (7)\end{aligned}$$

where for simplicity we have set $g = \lambda^2/|\delta|$.

Now, we assume that there is a reservoir coupling to the field in the usual way. In the dispersive approximation, we assume that the initial state of the system can be described by the ket:

$$|\psi_{a-f}\rangle = \left[\cos\left(\frac{\theta}{2}\right)|+\rangle + {\rm i}\ \sin\left(\frac{\theta}{2}\right)|-\rangle\right] \otimes |\alpha\rangle, \quad (8)$$

where $\alpha = \sqrt{\overline{n}}\exp({\rm i}\beta)$, $|+\rangle$ ($|-\rangle$) denotes the excited (ground) state of the atom, $\overline{n}$ is the initial average photon number, and β is the phase of the coherent field (here for simplicity, we let $\beta = 0$), $0 \le \theta \le \pi$ denotes the atomic distribution. We obtain the equation of motion of the density operator for the system in a standard form

$$\frac{{\rm d}\hat{\rho}(t)}{{\rm d}t} = -{\rm i}[\hat{H}_{\rm eff}, \hat{\rho}] + L\hat{\rho}, \qquad (\hbar = 1). \quad (9)$$

The losses in the cavity are phenomenologically represented by the superoperator L. At zero temperature, this superoperator can be written as

$$L = k(2\hat{a}.\hat{a}^+ - \hat{a}^+\hat{a}. - .\hat{a^+}\hat{a}), \quad (10)$$

where k is the dissipation constant. The action of $\hat{a}.$ and $.\hat{a}^+$ on an operator $\hat{O}$ is defined as

$$(\hat{a}.)\hat{O} = \hat{a}(\hat{O}), \qquad (.\hat{a}^+)\hat{O} = (\hat{O})\hat{a}^+. \quad (11)$$

In the interaction picture, the master equation takes the form

$$\begin{aligned}\frac{{\rm d}\hat{\rho}(t)}{{\rm d}t} =& -{\rm i}g[(\hat{a}^+\hat{a}+1)^2|+\rangle\langle+| - (\hat{a}^+\hat{a})^2|-\rangle\langle-|, \hat{\rho}(t)] \\ &+ k(2\hat{a}\hat{\rho}(t)\hat{a}^+ - \hat{a}^+\hat{a}\hat{\rho}(t) - \hat{\rho}(t)\hat{a}^+\hat{a}). \quad (12)\end{aligned}$$

This process of the field interacting with the atom with an intensity-dependent coupling is a nonlinear process. The reduced field operator is

$$\hat{\rho}_f(t) = {\rm Tr}_A\hat{\rho}(t) = \hat{\rho}_{++}(t) + \hat{\rho}_{--}(t). \quad (13)$$

Liouvillians corresponding to the matrix elements $\hat{\rho}_{--}(t)$ and $\hat{\rho}_{++}(t)$ have the forms

$$\begin{aligned}L_{++} =& -{\rm i}g(M^2 - P^2) - 2g(M-P) \\ &- k(M+P) + 2kF, \\ L_{--} =& -{\rm i}g(P^2 - M^2) - k(M+P) + 2kF, \quad (14)\end{aligned}$$

where the superoperators in the equations are defined as $M = \hat{a}^+\hat{a}.$, $P = .\hat{a}^+\hat{a}$, $F = \hat{a}.\hat{a}^+$. They satisfy the commutation relation[10]

$$[F, M+P] = 2F. \quad (15)$$

The master equation can be solved by applying the dynamical symmetry method proposed in Ref.[12], and we obtain

$$\begin{aligned}\hat{\rho}_{++}(t) =& {\rm e}^{L_{++}t}\hat{\rho}_{++}(0) \\ =& \exp[-{\rm i}g(M^2 - P^2) \\ &- 2{\rm i}g(M-P) - k(M+P)] \\ &\times \exp\Big\{[1 - \exp(-2kt - 2{\rm i}gMt + 2{\rm i}gPt)] \\ &\times \frac{kF}{(k + {\rm i}g(M-P)}\Big\}\rho_{++}(0), \quad (16)\end{aligned}$$

$$\begin{aligned}\hat{\rho}_{--}(t) =& \mathrm{e}^{L_{--}t}\hat{\rho}_{--}(0)\\ =& \exp[-\mathrm{i}g(-M^2+P^2)-k(M+P)]\\ &\times\exp\Big\{[1-\exp(-2kt+2\mathrm{i}gMt-2\mathrm{i}gPt)]\\ &\times\frac{kF}{(k-\mathrm{i}g(M-P))}\Big\}\rho_{--}(0). \end{aligned}\tag{17}$$

From Eq.(8), we can obtain

$$\rho_{++}(0)=\cos^2\left(\frac{\theta}{2}\right)|\alpha\rangle\langle\alpha|,$$

$$\rho_{--}(0)=\sin^2\left(\frac{\theta}{2}\right)|\alpha\rangle\langle\alpha|. \tag{18}$$

By using Eqs.(16)–(18), we can have

$$\begin{aligned}\hat{\rho}_{++}(t) =& \cos^2\left(\frac{\theta}{2}\right)\exp(-|\alpha|^2)\\ &\times\sum_{m,n=0}^{\infty}\frac{\alpha^m\alpha^{*n}}{\sqrt{m!n!}}\exp(\Gamma_{mn}(t)\\ &+\mathrm{i}\Theta_{emn}(t))|m\rangle\langle n|, \end{aligned}\tag{19}$$

$$\begin{aligned}\hat{\rho}_{--}(t) =& \sin^2\left(\frac{\theta}{2}\right)\exp(-|\alpha|^2)\\ &\times\sum_{m,n=0}^{\infty}\frac{\alpha^m\alpha^{*n}}{\sqrt{m!n!}}\exp(\Gamma_{mn}(t)\\ &+\mathrm{i}\Theta_{gmn}(t))|m\rangle\langle n|, \end{aligned}$$

where

$$\begin{aligned}\Gamma_{mn}(t) =& -k(m+n)t+\frac{|\alpha|^2k}{k^2+g^2(m-n)^2}\\ &\times\{k-\mathrm{e}^{-2kt}[k\cos 2(m-n)gt\\ &-g(m-n)\sin 2(m-n)gt]\}, \end{aligned}\tag{20}$$

$$\begin{aligned}\Theta_{emn}(t) =& -(m^2-n^2)gt-2(m-n)gt\\ &+\frac{k|\alpha|^2}{k^2+g^2(m-n)^2}\\ &\times\{-g(m-n)+\mathrm{e}^{-2kt}[k\sin 2(m-n)gt\\ &-g(m-n)\sin 2(m-n)gt]\}, \end{aligned}$$

$$\begin{aligned}\Theta_{gmn}(t) =& -(n^2-m^2)gt+\frac{|\alpha|^2k}{k^2+g^2(m-n)^2}\\ &\times\{-g(m-n)-\mathrm{e}^{-2kt}[g(m-n)\\ &\times\cos 2(m-n)gt+k\sin 2(m-n)gt]\}. \end{aligned}$$

Here, we discuss the coherence loss of the field by means of linear entropy, which is defined as[13]

$$S_f=1-\mathrm{Tr}(\hat{\rho}_f^2). \tag{21}$$

For a pure state S_f is zero, $S_f\simeq 1$ corresponds to a mixture with information and coherence effectively lost. From Eqs.(19)–(21), we obtain

$$\begin{aligned}S_f(t) =& 1-\exp(-2|\alpha|^2)\\ &\times\sum_{m,n=0}^{\infty}\frac{|\alpha|^{2(m+n)}}{m!n!}\exp[2\Gamma_{mn}(t)]\\ &\times\Big\{\cos^4\left(\frac{\theta}{2}\right)+\sin^4\left(\frac{\theta}{2}\right)\\ &+2\cos^2\left(\frac{\theta}{2}\right)\sin^2\left(\frac{\theta}{2}\right)\\ &\times[\cos(\Theta_{emn}(t)-\Theta_{gmn}(t))]\Big\}. \end{aligned}\tag{22}$$

Here, the distinguished roles for dissipation becomes clear. The term of $S_f(t)$ containing the factor $\Gamma_{mn}(t)$ in the form of exponential reflects the presence of the reservoir (for when $k=0$, $\Gamma_{mn}(t)$ vanishes). In the system considered, the field is initially prepared in the coherence state and $S_f(0)=0$, but in a short stage $S_f(t)$ increases and the coherence of the field loses. The function $\Gamma_{mn}(t)$, which is always nonpositive and decreases with time, affects the value of $S_f(t)$ and shows that the cavity has an influence on the coherence properties of the field.

3. Numerical results and conclusions

The function $\Gamma_{mn}(t)$ presenting in the form of exponential in the term of $S_f(t)$ controls the coherence loss of the field and has a similar form to those of dispersive JCM.[10,17] In the usual dispersive JCM,[10] only the form of the linear entropy of the atom $S_a(t)$ contains $\Gamma(t)$, and only the coherence of the atom is influenced by the cavity, though the atom does not couple to the cavity directly. The coherence of the field remains unchanged by the environment and $S_f(t)$ behaves periodically, with a maximum value 0.5 and a minimum value 0, which correspond to the entanglement and disentanglement between the field and the atom, respectively. However, in the system we considered, the field is also affected by the cavity and its coherence will lose due to the dissipation. Next, we will discuss the influence of the dissipation constant k, the intensity of the field $\overline{n}$ and the atomic distribution angle θ on the coherence properties of the field in detail.

Firstly, we study the influence of the dissipation constant on the coherence properties of the field. Figure 1 investigates the linear entropy of the field as a

function of time for several values of k with $\overline{n} = 1$ and $\theta = \pi/2$. The behaviour of $S_f(t)$ is complicated, with the presence of local maxima and minima values. The presence of the local maxima and minimal is due to the entanglement and disentanglement between the field and the atom. Because of the influence of dissipation on the entanglement, the amplitude of the entanglement decreases with the time. During the repeating periods of entanglement and disentanglement, the field loses and gains its coherence. However, the coherence recovered by the atom is always smaller than what was lost. The amplitude of the entanglement tends to zero after some time. One finds that the larger the k is, the more quickly the amplitude is suppressed, and the more rapidly $S_f(t)$ reaches its maximum value and the asymptotic value 0.

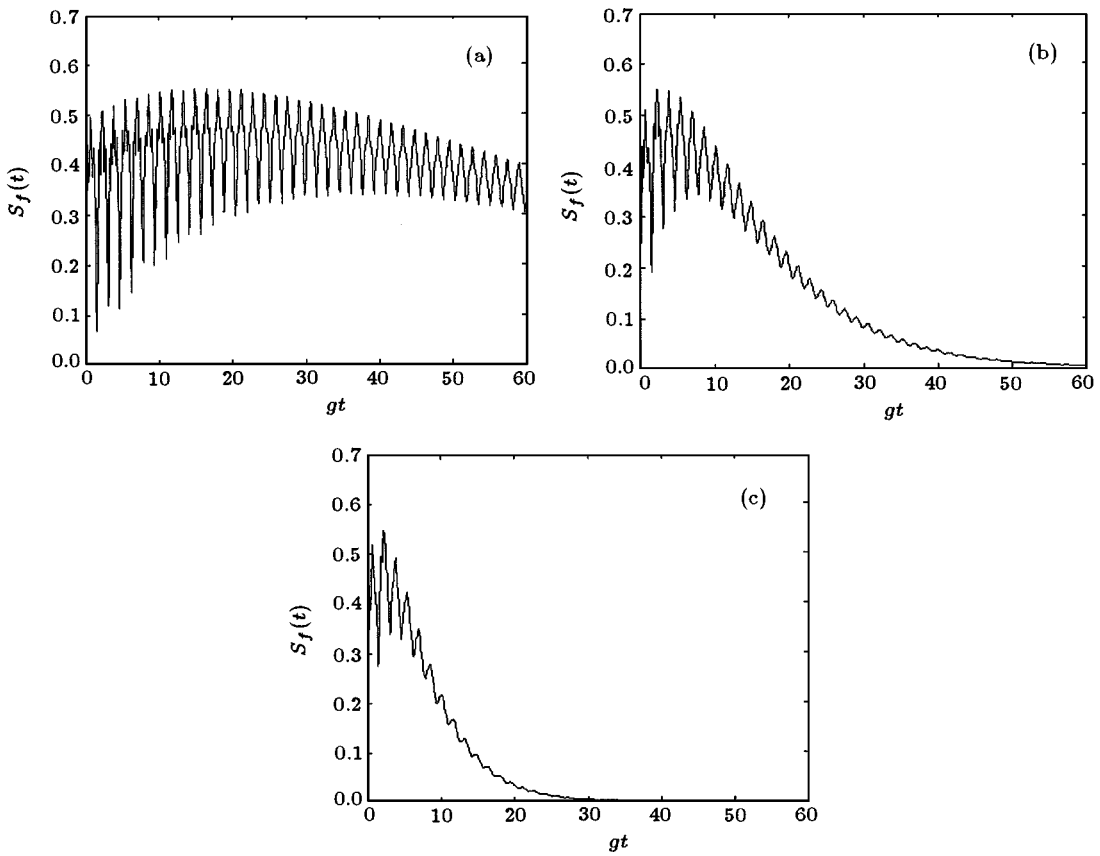


Fig.1. Linear entropy of the field as a function of gt for different values of the dissipation constant. $\overline{n} = 1$, $\theta = \pi/2$. (a) $k/g = 0.01$. (b) $k/g = 0.05$. (c) $k/g = 0.1$.

More examination finds that there is a great difference in the results when k is small enough and large enough, even the other parameters remain unchanged. When $k \leq 0.001g$, the behaviour of $S_f(t)$, which approximately appears in a period of π (plotted in Fig.2(a)), is very similar to that of the same model without dissipation. In this case, the amplitude of the entanglement is suppressed very slowly. But if $k \geq 0.5g$ (plotted in Fig.2(b)), $S_f(t)$ reaches its maximum value (the field reaches its maximum mixture state) and the asymptotic value 0 (the field reaches the vacuum state) in a very short while. Here, we can see that if k is small enough ($\leq 0.001g$), the influence of the cavity on the field can be ignored and the coherence of the field is determined by the interaction between the field and the atom. However, if $k \geq 0.5g$, the coherence of the field is predominantly controlled by the cavity, whose influence is to lead the field into the vacuum state and lose its coherence completely.

Secondly, we examine the influence of the intensity of the field. The evolutions of the linear entropy of the field are shown in Figs.1(c), 3(a), 3(b) for $\overline{n} = 1,\ 2,\ 3$ with $k = 0.05g$ and $\theta = \pi/2$. We see that with the increasing value of $\overline{n}$, the amplitude of the oscillation of $S_f(t)$ is further suppressed, and the entanglement between the field and the atom becomes weaker. It is noticeable that the larger the $\overline{n}$ is, the stronger the mixture of the field is.

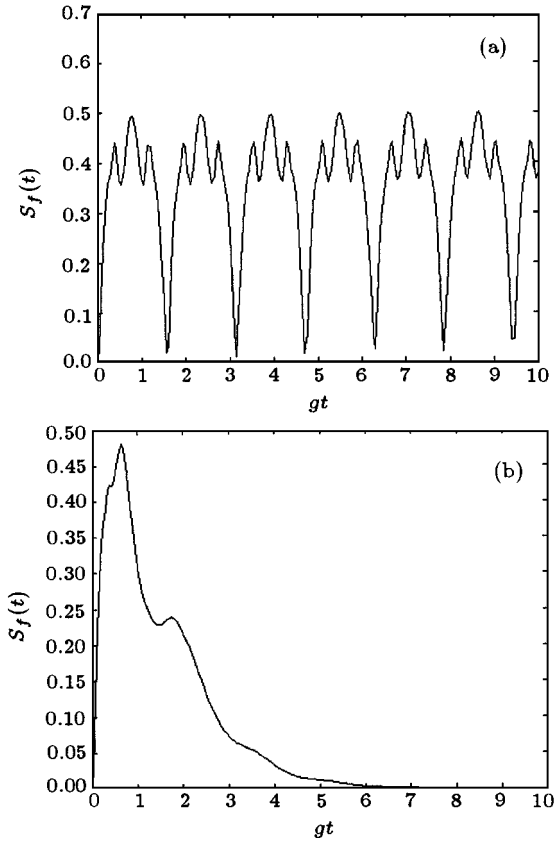


Fig.2. Same as Figs.1. (a) $k/g = 0.0001$. (b) $k/g = 0.5$.

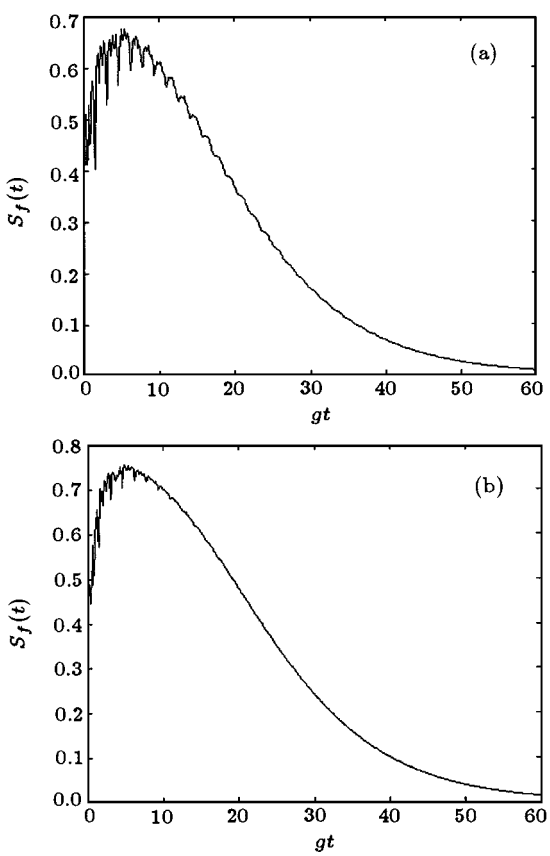


Fig.3. Linear entropy of the field as a function of gt for different values of the intensity of the field. $k/g = 0.05$, $\theta = \pi/2$. (a) $\overline{n} = 2$. (b) $\overline{n} = 3$.

Thirdly, we investigate $S_f(t)$ with different atomic distribution angle θ and plot $S_f(t)$ for $\theta = \pi/2$, $\theta = \pi/4$ and $\theta = 0$ with $k = 0.1g$ and $\overline{n} = 1$ in Figs.1(c), 4(a) and 4(b). Comparing these figures, one can find that the more the θ tends to $\pi/2$, the larger the degree of the maximum mixture state of the field is, and the stronger the entanglement between the two subsystems. We also find that the linear entropy of the field is symmetric to $\theta = \pi$ (on the condition that the other parameters are unchanged). When $\theta \to 0$ (or π), the amplitude of the entanglement always tends to zero.

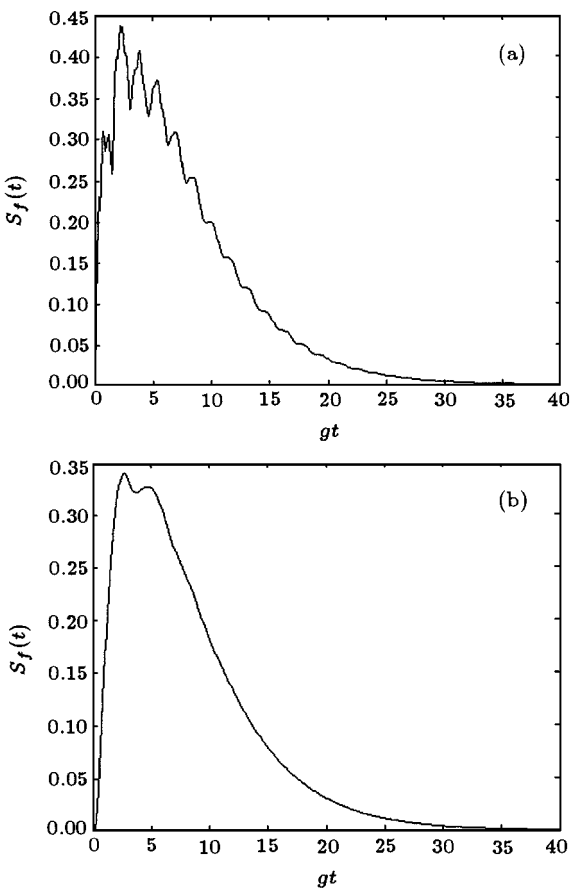


Fig.4. Linear entropy of the field as a function of gt for different values of the atomic distribution angle. $\overline{n} = 1$, $k/g = 0.1$. (a)$\theta = \pi/4$. (b) $\theta = 0$.

In summary, we have studied the linear entropy dynamics of the field in the JCM with an intensity-dependent coupling in the dissipative cavity. Our result shows that the coherence properties of the field are also affected by the cavity both qualitatively and quantitatively when the nonlinear process of the field interacting with the atom with an intensity-dependent coupling is involved. In the dissipation approximation, we obtain: (1) If the dissipation constant k is small

enough, the influence of the cavity on the coherence of the field can be ignored. (2) The larger the intensity of the field is, the weaker the entanglement between the field and the atom is, and the larger the degree of the maximum mixture state of the field is. (3) The more the θ tends to $\pi/2$, the stronger the maximum mixture state and the entanglement between the two subsystems become.

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