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Directed transport of coupled systems in symmetric periodic potentials*

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In this paper, we discuss the damped unidirectional motions of a coupled lattice in a periodic potential. Each particle in the lattice is subject to a time-periodic ac force. Our studies reveal that a directed transport process can be observed when the ac forces acting on the coupled lattice have a phase shift (mismatch). This directed motion is a collaboration of the coupling, the substrate potential, and the periodic force, which are all symmetric. The absence of any one of these three factors will not give rise to a directed current. We discuss the complex relations between the directed current and parameters in the system. Results in this paper can be accomplished in experiments. Moreover, our results can be generalized to the studies of directed transport processes in more complicated spatially extended systems.

Keywords: Frenkel–Kontorova model, molecular motor, directed transport, broken symmetry
PACC: 0545, 0540

1. Introduction

Coupled oscillator systems may give rise to various collective phenomena such as synchronization, clustering, nonlinear waves and collective transport.[1−9] These behaviours have been observed in a lot of practical systems in physics, chemistry, biology and geology. Although different systems possess different dynamics, these behaviours are closely related to each other. Many common features have been revealed in various systems on the basis of some universal mathematical models.

In recent years, studies on molecular motors and directed transport have revealed a very important non-equilibrium transport behaviour in physical and biological systems.[10,11] The essential physics behind all these problems can be described by the directional motion of Brownian particles in asymmetric (ratchet) potentials. They are related to a fundamental issue—whether the energy of thermal fluctuations can be transferred to the work for directed motion. If a system is in the equilibrium state (detailed balance), one may judge in terms of the second thermodynamical law that a directional current cannot be observed.[12] However, when a system is immersed in a nonequilibrium environment, a possible directional transport can be observed, even if the average force on the system may be zero. This phenomenon has been observed and further realized in a series of physical experiments,[13−17] such as the vortex dynamics of superconductors, Josephson junctions, quantum dots, nano materials, optics and biology.[18]

Directional transport of coupled systems is a very important and promising issue. In many cases, one has to consider the effect of mutual interactions of units on directed motion. It has been shown that cumulative interactions among units may significantly change the transport properties of the system, as compared with the single-unit case. In studies of biomolecular motors, it was revealed that internal degrees of freedom are essential for a net directional motion to occur.[19,20] For example, the molecular motors in muscles possess a linear structure that consists of many parts. Moreover, the proteins of kinesin superfamily are composed

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of two globular heads, and the kinesin direction of motion along microtubes can be reversed by adjusting the architecture of a small domain of the protein called neck region. Therefore, it is of fundamental importance to study the coupling effect on directed transport processes. On the other hand, studies of coupled molecular motors can also help us gain a deeper understanding of the effect of internal structure of biomolecular motors on ratchet motion. It has been shown that a directional motion can take place in the absence of nonequilibrium fluctuation through mutual couplings (so-called deterministic motor).[21,22]

Until now, most studies on coupled ratchets have been concentrated on the transport dynamics in ratchet potentials.[5] As a matter of fact, the symmetry breaking of the coupling or external forces may also induce a net current. We have studied the transport of asymmetrically coupled oscillators in symmetric periodic potentials. It was found that the directional transport can be realized via the breaking of the symmetry in the coupling, where an external driving force is unnecessary.[23] Flach *et al*[24,25] found that an asymmetric ac force may also lead to a non-zero net current. We recently found that a directional current can be induced[26] by breaking the spatiotemporal symmetry for symmetrically coupled particles in symmetric periodic potentials, which is subject to symmetric external drivings. In this paper, we report the studies on directed transports in coupled oscillators. It is found that a collective motor transport can be induced by a proper mismatch among periodic forces on different oscillators (consequently, leading to a broken spatiotemporal symmetry).

2. Model

We start our discussion by considering the motion of an underdamped single particle in a periodic potential $V(x) = d(1 - \cos x)$ under the driving of a time-periodic force $F(t) = A\cos\omega t$. The equation of motion can be described by the following damped single-pendulum equation:

$$\ddot{x} + \gamma\dot{x} + d\sin x = A\cos\omega t, \tag{1}$$

where $x, \dot{x}$ represent the position and the corresponding instantaneous velocity, respectively. γ, d, A and ω are the damping coefficient, the potential height, the amplitude and frequency of the ac force. Obviously, the periodic force and the potential field are all symmetric, i.e., $F(-t) = F(t), V(x) = V(-x)$. Therefore, no directed motion can be observed for the damped case. Now we further consider the case that many particles are coupled to each other in a diffusive way. When the ac forces acting on all particles are in phase, we cannot find the unidirectional motion of each particle. Let us assume that there exist mismatches in phases of these forces. The simplest case for the mismatch is that there is a uniform phase difference ϕ between two adjacent forces. A realistic example is the case when the coupled lattice is subject to a discrete plane wave $A\exp[\mathrm{j}(\omega t + \mathrm{i}\phi)]$, where $\mathrm{j} = \sqrt{-1}$. Here the phase difference corresponds to the wavevector, the equation of motion for this case can be written as

$$\begin{aligned}&\ddot{x}_i + \gamma\dot{x}_i + d\sin x_i \\ &= K(x_{i+1} - 2x_i + x_{i-1}) + A\cos(\omega t + \mathrm{i}\phi).\end{aligned} \tag{2}$$

Here K denotes the coupling strength. A parameter one cannot find from Eq.(2) is the average distance of adjacent particles in the absence of the substrate potential a, which implicitly appears in the potential function $U(x_{i+1} - x_i) = \frac{1}{2}(x_{i+1} - x_i - a)^2$. Equation (2) in fact is the ac-driven damped Frenkel–Kontorova (FK) model, which has been extensively studied in relation to dislocations in solids, coupled pendula, charge-density wave, Josephson-junction arrays, and nano-tribology.[27–29] However, all the previous studies of ac-driven FK model were focused on the case of in-phase ac forces. Here, for the first time we propose the above phase-mismatch ac-driven FK model. The FK model adopts the simplest forms in the coupling and potentials, and this may reveal the essential features of the wavevector competition. Moreover, the FK model has been constructed or realized in many physical experiments, therefore our results can be experimentally verified.

In numerical simulations we require that the total force on the system is zero, i.e., $F(t) = A\sum_i \cos(\omega t + \mathrm{i}\phi) = 0$. This can be easily achieved by choosing $\phi = 2\pi M/N$, where M is an integer and N is the number of particles. We further introduce the average current:

$$J = \lim_{T\to\infty} \frac{1}{NT} \sum_{i=1}^{N} \int \dot{x}_i(t)\mathrm{d}t. \tag{3}$$

Unless specifically stated, we adopt the following parameters: $A = 100, \omega/2\pi = 0.1, \gamma = 0.1, K = 1.0$. The periodic boundary condition is used: $x_{i+N}(t) = x_i(t) + Na$.

3. Main results and discussions

For the case of in-phase ac forces ($\phi = 0$), we obviously have $J(a, \phi = 0) = 0$; when $a = 0, J(a = 0, \phi) = 0$. We are interested in the possibility of non-zero unidirectional currents for $a \neq 0$ and $\phi \neq 0$. In Fig.1, the evolution of one of the particles (the middle one, for convenience) is plotted for different amplitudes of the periodic forces. During the simulation we use $d = 1.5$, $a/2\pi$=0.34, $\phi/2\pi$=0.15 and A=1.0, 2.0, 6.0, 8.0. It can be found that a net current exists when a and ϕ are not zero (non-zero current means increasing $x_i(t)$ with time t). This effect is due to the mismatch in periodic forces. Moreover, this mismatch-induced directional transport should be a collaboration between the coupling and periodic substrate potential. Obviously, if there is no interaction among particles (K=0), the motion for each particle can be described by Eq.(1), hence J=0. If there is no periodic potential (d=0), the problem is reduced to the motion of a harmonically coupled chain under the ac force, and the system does not possess a modulated structure. This also leads to a null current, J=0. Therefore, the existence of collective directional transport is the consequence of the organization of the periodic potential, the coupling and the ac forces. The absence of any of these factors may lead to a zero current.

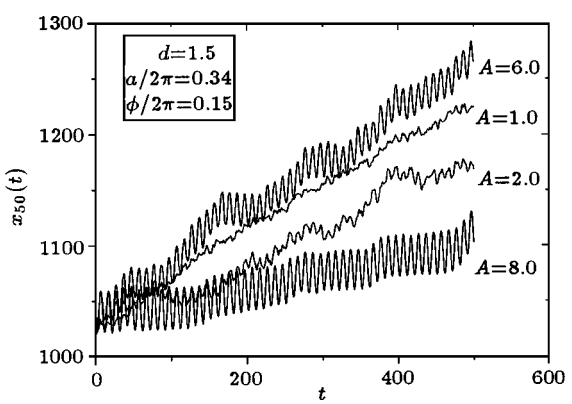


Fig.1. The evolution of the 50th oscillator for different amplitudes of the ac force.

One may investigate the motion of a single particle in the periodic potential under the drive of an effective force. In this case, Eq.(1) is still valid, while the ac force is replaced by the effective force $F_i = K(x_{i+1} - 2x_i + x_{i-1}) + A\cos(\omega t + \mathrm{i}\phi)$. The evolution of this effective force could be periodic or irregular. For the periodic case, it can be found that F_i is asymmetric within one period, which has been discussed. A chaotic effective force can be considered as a superposition of a periodic effective force and an effective noise: $F_i(t) = f_i(t) + \xi_i(t)$, and the periodic part is also temporally asymmetric. Directed transport induced by the broken symmetry in the periodic force has recently been extensively investigated. Our study reveals that the interplay between the mismatched ac force and the symmetric periodic potential gives rise to a time symmetry breaking of the effective force in the presence of coupling, which leads to a spatiotemporal symmetry breaking and directed transport. This is a type of mechanism for the collective transport in coupled molecular motors.

It can be seen from Fig.1 that the relation between the current and the ac force is not monotonic. For example, the currents at A=1.0 and 6.0 are larger than that at A=2.0 and 8.0. This indicates a complicated relation between the current and system parameters. In Fig.2 (a), the current varying with the force amplitude A is given (other parameters are the same as those in Fig.1). It can be found that J varies with A in an oscillatory way. J can be much larger at $A \approx$1.2, 4.2, 6.0, 7.0, and 8.5. At $A \approx$5.0, 6.4, 7.8, and 9.2, Js are very low and even less than zero (inverse current). The oscillating way of J is very significant for large A.

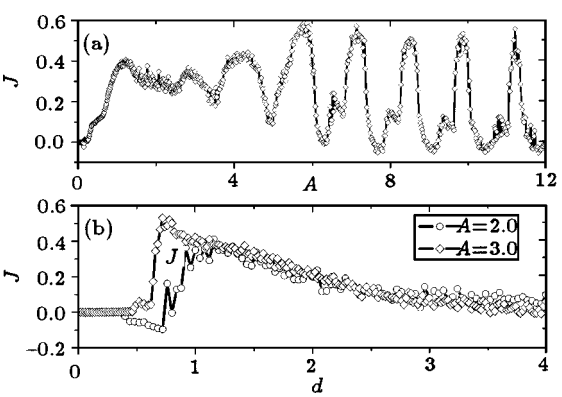


Fig.2. (a) The relation between the current J and the force amplitude A; (b) The relation between the current J and the barrier height d.

In Fig.2 (b), we compute the current J by changing the barrier height d. All parameters are the same as before except A=2.0 and 3.0. Two important results can be found. First, there exists a critical barrier height d_c. When $d \geq d_c$, $J \neq 0$. The critical barrier d_c changes along with system parameters. For example, when A=2.0, $d_c \approx$0.4; when A=3.0, $d_c \approx$0.5. This threshold feature is much different from the dissipative case (d_c is always 0). Second, an optimal barrier

height does exist for the system to possess an optimized current, i.e., one can find a "resonance" for the J-d relation. This can be well predicted by the following analysis. For d=0, we have J=0. For very large d, a finite ac force is not adequate for the particle to jump over the potential barrier, thus J=0; for an intermediate d, a "peak" is expected if there exists a directed motion. Of course, one may find more complex results (e.g., current reversal) except for the "peak" behaviour, which can be found in Fig.2.

The two parameters a and ϕ play very important roles in governing the directional transports. In Figs.3 (a) and 3(b), the current J against a and ϕ are computed for d=2.0 and A=1.5. We change $a/2\pi$ by fixing $\phi/2\pi$=0.09 in Fig.3 (a), while change $\phi/2\pi$ for $a/2\pi$=0.09. Complicated dependence of J on a and ϕ can be seen. An important result is the current reversal. It can be found that $J(a) = -J(2\pi - a)$ and $J(\phi) = -J(2\pi - \phi)$, which originate from the translational symmetry of time and space.[23] Hence one may expect J=0 when $a = \pi$ or $\phi = \pi$.

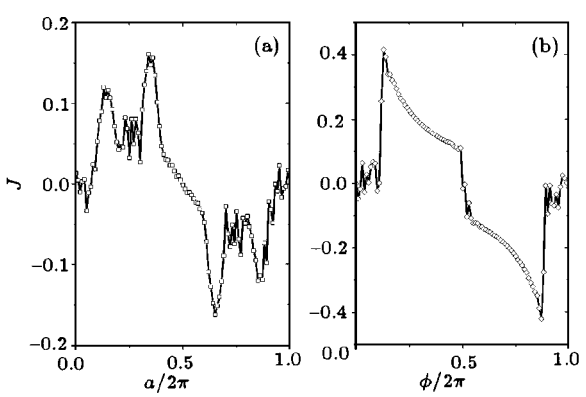


Fig.3. (a) The relation between the current J and the spring constant a; (b) The relation between the current J and the phase shift ϕ.

4. Conclusions

To summarize, in this paper we have discussed the motion of coupled chains in symmetric periodic potentials under the action of ac forces with phase mismatches. We find that directed transport can take place with a collaboration of the coupling, periodic potential and ac force. We investigate the complicated relation between the current and various parameters, and explore the dynamical mechanism of the directed transport. We adopt the Frenkel–Kontorova system, one of the simplest models that describe spatial and temporal competitions, as our working model. Hence the results obtained in this paper should be general, and they can be implemented in different physical experiments, such as coupled Josephson junctions, coupled quantum dots and coupled electronic circuits. Moreover, the present discussion can be generalized to more complicated systems (e.g., general interactions, general periodic potentials).

There still remain some detailed problems that need further investigations. First, it is necessary to gain a deeper understanding of the relation between macroscopic directional transport and microscopic dynamics. In Fig.1, we have observed chaotic motions of particles. It is important to study the effects of spatiotemporal chaos, periodic oscillations, kinks, phase locking and other dynamics on transport. Second, we did not discuss the noise effect. In many cases noise plays a negative role, but it may have a positive influence on the directed transport (e.g., spatiotemporal stochastic resonance). These issues are under study now.

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