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The periodic wave solutions for the generalized Nizhnik–Novikov–Veselov equation*

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The periodic wave solutions expressed by Jacobi elliptic functions for the generalized Nizhnik–Novikov–Veselov equation are obtained using the F-expansion method proposed recently. In the limiting cases, the solitary wave solutions and other types of travelling wave solutions for the system are obtained.

Keywords: generalized Nizhnik–Novikov–Veselov equation, F-expansion method, Jacobi elliptic functions, periodic wave solutions, solitary wave solutions

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1. Introduction

Many phenomena in physics and other fields are often described by nonlinear partial differential equations (PDEs). When we want to understand the physical mechanism of natural phenomenon described by nonlinear PDE, exact solutions for the nonlinear PDE have to be explored, thus the methods for deriving exact solutions for the governing equation have to be developed. Exploring exact solutions of nonlinear PDEs has become one of most important topics in mathematical physics. In the recent years, a number of powerful methods have been proposed, namely the inverse scattering method,[1] Hirota's bilinear method,[2] Bäcklund transformation method,[2] Darboux transformation method,[2] Miura transformation method,[3] Hyperbolic function method,[4,5] truncated expansion method,[6] formally variable separation approach,[7] rank analysis technique,[8] homogeneous balance method,[9–15] and the method based on the cubic nonlinear Klein–Gordon equation.[16]

Recently, the Jacobi elliptic function method[17–22] has been proposed, and has been paid attention by some researchers (see Ref.[23–25]). Fan[24] and Chen[25] extended the Jacobi elliptic function method and used the methods to derive multiple travelling wave solutions for a group of nonlinear evolution equations.

In this paper, we apply the F-expansion method put forward recently to derive the periodic wave solutions expressed by Jacobi elliptic functions for the generalized Nizhnik–Novikov–Veselov equations[26–30]

$$u_t + au_{xxx} + bu_{yyy} + cu_x + du_y - 3a(uv)_x - 3b(uw)_y = 0, \tag{1}$$

$$u_x = v_y, \tag{2}$$

$$u_y = w_x, \tag{3}$$

where a and b are non-zero constants, and c and d are arbitrary constants.

This paper is organized as follows: in the next section, we briefly describe what is the F-expansion method, and how to use it to find the periodic wave solutions of a nonlinear wave equation(s) in mathematical physics. In section 3, we use F-expansion method to find the periodic wave solutions expressed by various Jacobi elliptic functions for the generalized Nizhnik–Novikov–Veselov equation as an example. In the limiting cases, the solitary wave solutions and the

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other type of travelling wave solutions are also obtained. At last, in section 4, our conclusions are summarized.

2. Description of F-expansion method

We consider the nonlinear wave equation for $u(x,t)$

$$\Phi(u, u_x, u_t, u_{xx}, u_{xt}, u_{tt}, u_{xxx}, \cdots) = 0. \tag{4}$$

In general, the left-hand side of Eq.(4) is a polynomial in u and its various derivatives. The F-expansion method for solving Eq.(4) proceeds in the following four steps:

(i) Look for travelling wave solutions of Eq.(4) by taking

$$u(x,t) = u(\xi), \quad \xi = kx + \varpi t + \xi_0, \tag{5}$$

where k, ϖ are nonzero constants to be determined later, ξ_0 is an arbitrary real constant.

Substituting (5) into Eq.(4) yields an ODE for $u(\xi)$,

$$\Phi(u, ku', \varpi u', k^2u'', k\varpi u'', \varpi^2 u'', k^3u''', \cdots) = 0, \tag{6}$$

where

$$u' = \frac{\mathrm{d}u(\xi)}{\mathrm{d}\xi}, \quad u'' = \frac{\mathrm{d}^2u(\xi)}{\mathrm{d}\xi^2}, \quad u''' = \frac{\mathrm{d}^3u(\xi)}{\mathrm{d}\xi^3}, \quad ,\text{etc.}$$

(ii) Suppose that $u(\xi)$ can be expressed by a finite power series of $F(\xi)$:

$$u(\xi) = \sum_{n=0}^{N} a_n F^n(\xi), \quad a_N \neq 0, \tag{7}$$

where $a_0, a_1, \cdots, a_N$ are constants to be determined later, $F(\xi)$ satisfies the first-order nonlinear ODE

$$F'^2(\xi) = PF^4 + QF^2 + R, \tag{8}$$

in which P, Q, R are real constants, integer N in (7) can be determined by considering homogeneous balance between the nonlinear term(s) with the highest-order derivatives of $u(\xi)$ in Eq.(6). Substituting (7) into ODE (6) and using ODE (8) simultaneously, the left-hand side of Eq.(6) becomes a polynomial for $F(\xi)$. Setting coefficients of the polynomial to zero yields a set of algebraic equations for $a_1, a_1, \cdots, a_N, \varpi$, (and possibly k).

(iii) With the aid of Mathematica or Maple, solving the algebraic equations obtained above, $a_0, a_1, \cdots, a_N$ and ϖ (possibly k) can be expressed by P, Q, R. Substituting these results into F-expansion (7), a general form of travelling wave solution of Eq.(4) can be obtained.

(iv) Choosing properly (P, Q, R) in ODE (8), the corresponding solution $F(\xi)$ of ODE (8) is one of Jacobi elliptic functions. Substituting the values of (P, Q, R) and the corresponding Jacobi elliptic function $F(\xi)$ chosen from the Appendix into the general form of travelling wave solution, a periodic wave solution expressed by a Jacobi elliptic function can be obtained.

3. Periodic wave solutions for the generalized Nizhnik–Novikov–Veselov equation

In this section, we apply the F-expansion method to derive the periodic wave solutions for the famous generalized NNV Eqs. (1)–(3).

Suppose the travelling wave solutions for Eqs.(1)–(3) is of the form

$$u = u(\xi), \quad v = v(\xi), \quad w = w(\xi),$$
$$\xi = kx + ly + \gamma t + \xi_0, \tag{9}$$

where k, l, γ are constants to be determined later, and ξ_0 is an arbitrary constant.

Inserting (9) into Eqs.(1)–(3) yields ODEs

$$(\gamma + ck + dl)u' + (ak^3 + bl^3)u''' - 3ak(u'v + uv') - 3bl(u'w + uw') = 0, \tag{10}$$

$$ku' = lv', \tag{11}$$

$$lu' = kw'. \tag{12}$$

Integrating (11), (12) once yields the relations between u and v, w:

$$v = \frac{k}{l}u, \tag{13}$$

$$w = \frac{l}{k}u. \tag{14}$$

Substituting (13),(14) into Eq.(10) yields ODE in $u(\xi)$,

$$(\gamma + ck + dl)u' + (ak^3 + bl^3)u''' - \frac{6(ak^3 + bl^3)}{kl}uu' = 0. \tag{15}$$

Considering the balance between u''' with uu' in Eq.(15), we suppose that $u(\xi)$ can be expressed by

$$u = \alpha_0 + a_1F + \alpha_2F^2, \quad a_2 \neq 0, \tag{16}$$

where $\alpha_0, \alpha_1, \alpha_2$ are constants to be determined later, $F(\xi)$ is a solution of ODE (8).

Inserting (16) into Eq.(15), and considering Eq.(8) simultaneously, the left-hand side of the Eq.(15) becomes a polynomial for $F(\xi)$. Setting the coefficients of the polynomial to zero yields a set of algebraic equations:

$$F^3: \quad 24P(ak^3+bl^3)\alpha_2 - \frac{6(ak^3+bl^3)}{kl}\cdot 2\alpha_2^2 = 0, \tag{17}$$

$$F^2: \quad 6P(ak^3+bl^3)\alpha_1 - \frac{6(ak^3+bl^3)}{kl}\cdot 3\alpha_1\alpha_2 = 0, \tag{18}$$

$$F: \quad 8Q(ak^3+bl^3)\alpha_2 - \frac{6(ak^3+bl^3)}{kl}\cdot(\alpha_1^2+2\alpha_0\alpha_2) + 2(\gamma+ck+dl)\alpha_2 = 0, \tag{19}$$

$$F^0: \quad Q(ak^3+bl^3)\alpha_1 - \frac{6(ak^3+bl^3)}{kl} \times \alpha_0\alpha_1 + (\gamma+ck+dl)\alpha_1 = 0. \tag{20}$$

Solving the algebraic Eqs. (17)–(20), we have the solutions

$$\alpha_1 = 0, \tag{21}$$

$$\alpha_2 = 2Pkl, \tag{22}$$

$$\gamma = -4Q(ak^3+bl^3) - ck - dl + \frac{6\alpha_0(ak^3+bl^3)}{kl}, \tag{23}$$

where k, l, α_0 are arbitrary constants.

Substituting (21)–(23) into (13), (14) and (16), we obtain the general travelling wave solutions of Eqs.(1)–(3):

$$u = \alpha_0 + 2PklF^2(\xi), \tag{24}$$

$$v = \frac{k}{l}[\alpha_0 + 2PklF^2(\xi)], \tag{25}$$

$$w = \frac{l}{k}[\alpha_0 + 2PklF^2(\xi)], \tag{26}$$

where

$$\xi = kx + ly + \left[-4Q(ak^3+bl^3) - ck - dl + \frac{6\alpha_0(ak^3+bl^3)}{kl}\right]t + \xi_0, \tag{27}$$

and k, l, α_0, ξ_0, are arbitrary constants.

From the first line of the Appendix, it is easy to see that the Jacobi sn-function

$$F(\xi) = \mathrm{sn}\xi \tag{28}$$

is a solution for Eq.(8) with

$$P = m^2, \quad Q = -(1+m^2), \quad R = 1. \tag{29}$$

Inserting (28), (29) into (24)–(27), the periodic wave solutions of Eqs.(1)–(3) are derived as follows:

$$u_1 = \alpha_0 + 2m^2kl\,\mathrm{sn}^2\xi_1, \tag{30}$$

$$v_1 = \frac{k}{l}[\alpha_0 + 2m^2kl\,\mathrm{sn}^2\xi_1], \tag{31}$$

$$w_1 = \frac{l}{k}[\alpha_0 + 2m^2kl\,\mathrm{sn}^2\xi_1], \tag{32}$$

where

$$\xi_1 = kx + ly + \left[4(1+m^2)(ak^3+bl^3) - ck - dl + \frac{6\alpha_0(ak^3+bl^3)}{kl}\right]t + \xi_0, \tag{33}$$

and k, l, α_0, ξ_0, are arbitrary constants.

In the limiting case when $m \to 1$, $\mathrm{sn}\xi \to \tanh\xi$; thus (30)–(33) become the solitary wave solutions of Eqs.(1)–(3):

$$u_{11} = \alpha_0 + 2kl - 2kl\,\mathrm{sech}^2\xi_{11}, \tag{34}$$

$$v_{11} = \frac{k}{l}[\alpha_0 + 2kl - 2kl\,\mathrm{sech}^2\xi_{11}], \tag{35}$$

$$w_{11} = \frac{l}{k}[\alpha_0 + 2kl - 2kl\,\mathrm{sech}^2\xi_{11}], \tag{36}$$

where

$$\xi_{11} = kx + ly + \left[8(ak^3+bl^3) - ck - dl + \frac{6\alpha_0(ak^3+bl^3)}{kl}\right]t + \xi_0, \tag{37}$$

and k, l, α_1, ξ_0, are arbitrary constants.

From the second line of the Appendix, it is obvious that the Jacobi cn-function

$$F(\xi) = \mathrm{cn}\xi \tag{38}$$

is a solution of Eq.(8) with

$$P = -m^2, \quad Q = 2m^2 - 1, \quad R = 1 - m^2. \tag{39}$$

Inserting (28),(29) into (24)–(27), the periodic wave solutions to Eqs.(1)–(3) are derived as follows:

$$u_2 = \alpha_0 - 2m^2kl\,\mathrm{cn}^2\xi_2, \tag{40}$$

$$v_2 = \frac{k}{l}[\alpha_0 - 2m^2kl\,\mathrm{cn}^2\xi_2], \tag{41}$$

$$w_2 = \frac{l}{k}[\alpha_0 - 2m^2kl\,\mathrm{cn}^2\xi_2], \tag{42}$$

where

$$\xi_2 = kx + ly + \Big[4(1-2m^2)(ak^3+bl^3) - ck - dl + \frac{6\alpha_0(ak^3+bl^3)}{kl}\Big]t + \xi_0, \quad (43)$$

and k, l, α_0, ξ_0, are arbitrary constants.

In the limiting case when $m \to 1$, $\mathrm{nc}\xi \to \mathrm{sech}\xi$, thus (40)–(43) become the solitary wave solutions of Eqs.(1)–(3),

$$u_{21} = \alpha_0 - 2kl\mathrm{sech}^2\xi_{21}, \quad (44)$$

$$v_{21} = \frac{k}{l}[\alpha_0 - 2kl\mathrm{sech}^2\xi_{21}], \quad (45)$$

$$w_{21} = \frac{l}{k}[\alpha_0 - 2kl\mathrm{sech}^2\xi_{21}], \quad (46)$$

where

$$\xi_{21} = kx + ly + \Big[-4(ak^3+bl^3) - ck - dl + \frac{6\alpha_0(ak^3+bl^3)}{kl}\Big]t + \xi_0, \quad (47)$$

and k, l, α_0, ξ_0 are arbitrary constants.

In terms of the relationships listed in the Appendix, the other periodic wave solutions expressed by the Jacobi elliptic functions for Eqs.(1)–(3) can be obtained as follows:

$$\begin{cases} u_3 = \alpha_0 - 2kl\mathrm{dn}^2\xi_3, \\ v_3 = \dfrac{k}{l}[\alpha_0 - 2kl\mathrm{dn}^2\xi_3], \\ w_3 = \dfrac{l}{k}[\alpha_0 - 2kl\mathrm{dn}^2\xi_3], \end{cases}$$

where

$$\xi_3 = kx + ly + \Big[4(m^2-2)(ak^3+bl^3) - ck - dl + \frac{6\alpha_0(ak^3+bl^3)}{kl}\Big]t + \xi_0.$$

$$\begin{cases} u_4 = \alpha_0 + 2lk\mathrm{ns}^2\xi_4, \\ v_4 = \dfrac{k}{l}[\alpha_0 + 2lk\mathrm{ns}^2\xi_4], \\ w_4 = \dfrac{l}{k}[\alpha_0 + 2lk\mathrm{ns}^2\xi_4], \end{cases}$$

where

$$\xi_4 = kx + ly + \Big[4(m^2+1)(ak^3+bl^3) - ck - dl + \frac{6\alpha_0(ak^3+bl^3)}{kl}\Big]t + \xi_0.$$

$$\begin{cases} u_5 = \alpha_0 + 2kl(1-m^2)\mathrm{nc}^2\xi_5, \\ v_5 = \dfrac{k}{l}[\alpha_0 + 2kl(1-m^2)\mathrm{nc}^2\xi_5], \\ w_5 = \dfrac{l}{k}[\alpha_0 + 2lk(1-m^2)\mathrm{nc}^2\xi_5], \end{cases}$$

where

$$\xi_5 = kx + ly + \Big[4(1-m^2)(ak^3+bl^3) - ck - dl + \frac{6\alpha_0(ak^3+bl^3)}{kl}\Big]t + \xi_0.$$

$$\begin{cases} u_6 = \alpha_0 + 2kl(m^2-1)\mathrm{nd}^2\xi_6, \\ v_6 = \dfrac{k}{l}[\alpha_0 + 2kl(m^2-1)\mathrm{nd}^2\xi_6], \\ w_6 = \dfrac{l}{k}[\alpha_0 + 2lk(m^2-1)\mathrm{nd}^2\xi_6], \end{cases}$$

where

$$\xi_6 = kx + ly + \Big[4(m^2-2)(ak^3+bl^3) - ck - dl + \frac{6\alpha_0(ak^3+bl^3)}{kl}\Big]t + \xi_0.$$

$$\begin{cases} u_7 = \alpha_0 + 2kl(1-m^2)\mathrm{sc}^2\xi_7, \\ v_7 = \dfrac{k}{l}[\alpha_0 + 2kl(1-m^2)\mathrm{sc}^2\xi_7], \\ w_7 = \dfrac{l}{k}[\alpha_0 + 2lk(1-m^2)\mathrm{sc}^2\xi_7], \end{cases}$$

where

$$\xi_7 = kx + ly + \Big[4(m^2-2)(ak^3+bl^3) - ck - dl + \frac{6\alpha_0(ak^3+bl^3)}{kl}\Big]t + \xi_0.$$

$$\begin{cases} u_8 = \alpha_0 - 2klm^2(1-m^2)\mathrm{sd}^2\xi_8, \\ v_8 = \dfrac{k}{l}[\alpha_0 - 2klm^2(1-m^2)\mathrm{sd}^2\xi_8], \\ w_8 = \dfrac{l}{k}[\alpha_0 - 2klm^2(1-m^2)\mathrm{sd}^2\xi_8], \end{cases}$$

where

$$\xi_8 = kx + ly + \Big[4(1-2m^2)(ak^3+bl^3) - ck - dl + \frac{6\alpha_0(ak^3+bl^3)}{kl}\Big]t + \xi_0.$$

$$\begin{cases} u_9 = \alpha_0 + 2lk\mathrm{cs}^2\xi_9, \\ v_9 = \dfrac{k}{l}[\alpha_0 + 2lk\mathrm{cs}^2\xi_9], \\ w_9 = \dfrac{l}{k}[\alpha_0 + 2lk\mathrm{cs}^2\xi_9], \end{cases}$$

where

$$\xi_9 = kx + ly + \Big[4(m^2-2)(ak^3+bl^3) - ck - dl + \frac{6\alpha_0(ak^3+bl^3)}{kl}\Big]t + \xi_0.$$

$$\begin{cases} u_{10} = \alpha_0 + 2lk\mathrm{ds}^2\xi_{10}, \\ v_{10} = \dfrac{k}{l}[\alpha_0 + 2lk\mathrm{ds}^2\xi_{10}], \\ w_{10} = \dfrac{l}{k}[\alpha_0 + 2lk\mathrm{ds}^2\xi_{10}], \end{cases}$$

where

$$\xi_{10} = kx + ly + \left[4(1-2m^2)(ak^3+bl^3) - ck - dl + \frac{6\alpha_0(ak^3+bl^3)}{kl}\right]t + \xi_0,$$

and k, l, α_0, ξ_0, are arbitrary constants.

In the limiting case when $m \to 1$,

$$\begin{cases} u_{31} = u_{21} = \alpha_0 - 2kl\mathrm{sech}^2\xi_{21}, \\ v_{31} = v_{21} = \dfrac{k}{l}[\alpha_0 - 2kl\mathrm{sech}^2\xi_{21}], \\ w_{31} = w_{21} = \dfrac{l}{k}[\alpha_0 - 2kl\mathrm{sech}^2\xi_{21}], \end{cases}$$

$$\begin{cases} u_{41} = \alpha_0 + 2lk\coth^2\xi_{41}, \\ v_{41} = \dfrac{k}{l}[\alpha_0 + 2lk\coth^2\xi_{41}], \\ w_{41} = \dfrac{l}{k}[\alpha_0 + 2lk\coth^2\xi_{41}], \end{cases}$$

where

$$\xi_{41} = kx + ly + \left[8(ak^3+bl^3) - ck - dl + \frac{6\alpha_0(ak^3+bl^3)}{kl}\right]t + \xi_0.$$

$$\begin{cases} u_{91} = u_{101} = \alpha_0 + 2lk\mathrm{csch}^2\xi_{91}, \\ v_{91} = v_{101} = \dfrac{k}{l}[\alpha_0 + 2lk\mathrm{csch}^2\xi_{91}], \\ w_{91} = w_{101} = \dfrac{l}{k}[\alpha_0 + 2lk\mathrm{csch}^2\xi_{91}], \end{cases}$$

where

$$\xi_{91} = kx + ly + \left[-4(ak^3+bl^3) - ck - dl + \frac{6\alpha_0(ak^3+bl^3)}{kl}\right]t + \xi_0.$$

In the limiting case when $m \to 0$,

$$\begin{cases} u_{42} = u_{102} = \alpha_0 + 2lk\csc^2\xi_{42}, \\ v_{42} = v_{102} = \dfrac{k}{l}[\alpha_0 + 2lk\csc^2\xi_{42}], \\ w_{42} = w_{102} = \dfrac{l}{k}[\alpha_0 + 2lk\csc^2\xi_{42}], \end{cases}$$

where

$$\xi_{42} = kx + ly + \left[4(ak^3+bl^3) - ck - dl + \frac{6\alpha_0(ak^3+bl^3)}{kl}\right]t + \xi_0.$$

$$\begin{cases} u_{52} = \alpha_0 + 2lk\sec^2\xi_{52}, \\ v_{52} = \dfrac{k}{l}[\alpha_0 + 2lk\sec^2\xi_{52}], \\ w_{52} = \dfrac{l}{k}[\alpha_0 + 2lk\sec^2\xi_{52}], \end{cases}$$

where

$$\xi_{52} = kx + ly + \left[4(ak^3+bl^3) - ck - dl + \frac{6\alpha_0(ak^3+bl^3)}{kl}\right]t + \xi_0.$$

$$\begin{cases} u_{72} = \alpha_0 + 2lk\tan^2\xi_{72}, \\ v_{72} = \dfrac{k}{l}[\alpha_0 + 2lk\tan^2\xi_{72}], \\ w_{72} = \dfrac{l}{k}[\alpha_0 + 2lk\tan^2\xi_{72}], \end{cases}$$

where

$$\xi_{72} = kx + ly + \left[-8(ak^3+bl^3) - ck - dl + \frac{6\alpha_0(ak^3+bl^3)}{kl}\right]t + \xi_0.$$

$$\begin{cases} u_{92} = \alpha_0 + 2lk\cot^2\xi_{92}, \\ v_{92} = \dfrac{k}{l}[\alpha_0 + 2lk\cot^2\xi_{92}], \\ w_{92} = \dfrac{l}{k}[\alpha_0 + 2lk\cot^2\xi_{92}], \end{cases}$$

where

$$\xi_{92} = kx + ly + \left[-8(ak^3+bl^3) - ck - dl + \frac{6\alpha_0(ak^3+bl^3)}{kl}\right]t + \xi_0.$$

4. Conclusion

In this paper, we apply F-expansion method to derive the periodic wave solutions of the generalized Nizhnik–Novikov–Veselov equation, and in the limiting case, the solitary wave solutions and the other type of travelling wave solutions are obtained. From the applications of F-expansion method, it is obvious that F-expansion method is one of the most powerful tools for finding the periodic wave solutions (especially expressed by the Jacobi elliptic functions) for nonlinear PDEs, and is one of the most useful extension of the Jacobi elliptic function expansion method. Application of F-expansion method to other PDE is to be introduced later.

Appendix

Relations between values of (R, Q, R) and corresponding $F(\xi)$ in ODE $F'^2 = PF^4 + QF^2 + R$.

P	Q	R	$F'^2 = PF^4 + QF + R$	$F(\xi)$
m^2	$-(1+m^2)$	1	$F'^2 = (1-F^2)(1-m^2F^2)$	$\mathrm{sn}\xi$
$-m^2$	$2m^2-1$	$1-m^2$	$F'^2 = (1-F^2)(m^2F^2+1-m^2)$	$\mathrm{cn}\xi$
-1	$2-m^2$	m^2-1	$F'^2 = (1-F^2)(F^2+m^2-1)$	$\mathrm{dn}\xi$
1	$-(1+m^2)$	m^2	$F'^2 = (1-F^2)(m^2-F^2)$	$\mathrm{ns}\xi=(\mathrm{sn}\xi)^{-1}$
$1-m^2$	$2m^2-1$	$-m^2$	$F'^2 = (1-F^2)[(m^2-1)F^2-m^2]$	$\mathrm{nc}\xi=(\mathrm{cn}\xi)^{-1}$
m^2-1	$2-m^2$	-1	$F'^2 = (1-F^2)[(1-m^2)F^2-1]$	$\mathrm{nd}\xi=(\mathrm{dn}\xi)^{-1}$
$1-m^2$	$2-m^2$	1	$F'^2 = (1+F^2)[(1-m^2)F^2+1]$	$\mathrm{sc}\xi=\dfrac{\mathrm{sn}\xi}{\mathrm{cn}\xi}$
$-m^2(1-m^2)$	$2m^2-1$	1	$F'^2 = (1+m^2F^2)[(m^2-1)F^2+1]$	$\mathrm{sd}\xi=\dfrac{\mathrm{sn}\xi}{\mathrm{dn}\xi}$
1	$2-m^2$	$1-m^2$	$F'^2 = (1+F^2)[F^2+(1-m^2)]$	$\mathrm{cs}\xi=\dfrac{\mathrm{cn}\xi}{\mathrm{sn}\xi}$
1	$2m^2-1$	$-m^2(1-m^2)$	$F'^2 = (m^2+F^2)[F^2+(m^2-1)]$	$\mathrm{ds}\xi=\dfrac{\mathrm{dn}\xi}{\mathrm{sn}\xi}$

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