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The periodic magnetic cylinder tube for atom guidance: transport efficiency analysis*

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We have proposed a model of periodic axial magnetized hollow tube for atom guidance (1999 *Opt. Commun.* **160** 72–4). In the present paper, an analytic solution of the magnetic field inside the tube is obtained and the transport efficiency of the cold atoms in the magnetic tube at different tube intensities of magnetization is discussed. The effect of the curvature of magnetic tube on the atom transport efficiency is also considered. We have found that the tube has a cooling effect on the transverse temperature of the transported atoms.

Keywords: atomic waveguide, magnetic tube, atom transport efficiency, transverse temperature
PACC: 3280P, 3260V

1. Introduction

In recent years, with the development of atom cooling technology,[1] a new field of atom optics has come into being. In this technology, it would be useful and interesting to transport the cooled atoms with a waveguide, such as optical fibre guiding laser radiation. At present, there have been many ways to guide cold atoms. Ito *et al*[2] and Renn *et al*[3] reported respectively their experiments on atom waveguide made of a hollow optical fibre with red detuned Gauss laser inside the fibre or blue detuned evanescent laser near the inside wall of the fibre. Key *et al*[4] guided the cold atoms with a quadrupole magnetic field supplied by four current lines. Folman *et al*[5] used micro magnetic trap by "atom chip" for atom guide. Yin *et al* proposed arrays of microscopic magnetic traps for cold atom.[6]

Our group has proposed an atomic waveguide model before.[7] The hollow magnetic tube was constructed with a magnetized material (Fig.1). The thickness of the magnetic material coating was t and the inner radius of the tube was a, and the tube was magnetized periodically with a period λ along the axial direction of the tube. In our previous paper,[7] we calculated the magnetic field inside the tube numerically for a sinusoidal magnetization. Now we have obtained the analytic solution of the magnetic field inside the tube for the cases of general periodic magnetization. In this paper we calculate the magnetic field distribution inside the hollow magnetic tube in section 2, and discuss the transport efficiency of the atoms in section 3, including the case of straight tube and the case of the curved tube. In section 4, we consider the cooling effect of the waveguide on the transverse temperature of the atoms.

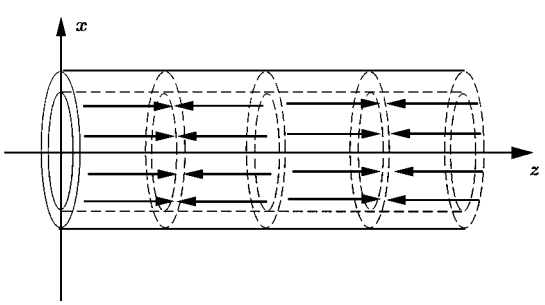


Fig.1. The structure of the magnetic tube with alternating magnetism along the axial direction.

2.The analytic solution of the magnetic field

The static field B inside the magnetic tube can be described by a scalar potential φ related to the mag-

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netization $\boldsymbol{M}(\boldsymbol{r}')$ within the material of the tube wall by Eq.(5.100) of Jackson's book[8]

$$\varphi(\boldsymbol{r}) = -\frac{\mu_0}{4\pi}\int_V \frac{\nabla'\cdot\boldsymbol{M}(\boldsymbol{r}')}{|\boldsymbol{r}-\boldsymbol{r}'|}\mathrm{d}^3\boldsymbol{r}' + \frac{\mu_0}{4\pi}\oint_S \frac{\boldsymbol{n}'\cdot\boldsymbol{M}(\boldsymbol{r}')}{|\boldsymbol{r}-\boldsymbol{r}'|}\mathrm{d}^2\boldsymbol{r}', \quad (1)$$

where V and S are the volume and surface of the material and $\boldsymbol{n}'$ is the normal to the surface. In our magnetic tube model, $\boldsymbol{M}(\boldsymbol{r}) = M(z)\boldsymbol{e}_z$, where $M(z)$is a periodic function satisfying $M(z) = M(z+\lambda)$. Since $\boldsymbol{M}$ is parallel to the surface of the tube wall, obviously we have $\boldsymbol{n}'\cdot\boldsymbol{M} = 0$. As the magnetization of the tube is periodic with a period of λ, it is convenient to expand the magnetization in a Fourier series

$$M(z) = \frac{1}{2}\sum_{n=0}^{\infty} M_n \mathrm{e}^{\mathrm{i}nkz} + \text{c.c.}, \quad (2)$$

in which $k = 2\pi/\lambda$ and c.c. is the complex conjugate. Then the scalar potential becomes

$$\varphi(\boldsymbol{r}) = -\frac{\mu_0}{8\pi}\sum_n \mathrm{i}nk\int_V \frac{M_n \mathrm{e}^{\mathrm{i}nkz'}}{|\boldsymbol{r}-\boldsymbol{r}'|}\mathrm{d}^3\boldsymbol{r}' + \text{c.c.}, \quad (3)$$

and the magnetic field $\boldsymbol{B}(r)$ can be obtained by

$$\boldsymbol{B}(\boldsymbol{r}) = -\nabla\varphi(\boldsymbol{r}). \quad (4)$$

Considering the cylindrical symmetry of the magnetization, we use cylindrical coordinates(ρ,ϕ,z) to calculate the magnetic scale potential. In cylindrical coordinates, we have

$$|\boldsymbol{r}-\boldsymbol{r}'| = \sqrt{\rho^2+\rho'^2-2\rho\rho'\cos(\phi-\phi')+(z-z')^2}. \quad (5)$$

Substituting Eq.(5) into Eq.(3) and integrating it, we can obtain the scale potential as

$$\varphi(\rho,\phi,z) = -\frac{\mu_0}{2}\sum_n \mathrm{i}M_n \mathrm{e}^{\mathrm{i}nkz} I_0(nk\rho)[aK_1(nka) - (a+t)K_1(nk(a+t))] + \text{c.c.}, \quad (6)$$

where $I_0(x)$ is the zeroth-order modified Bessel function of the first kind, $K_1(x)$ is the first-order modified Bessel functions of the second kind. Therefore, the components of the field are

$$B_\rho = -\sum_n B_n I_1(nk\rho)\sin(nkz+\delta_n), \quad (7)$$

$$B_z = -\sum_n B_n I_0(nk\rho)\cos(nkz+\delta_n), \quad (8)$$

in which δ_n is the phase of the magnetization defined by $M_n = |M_n|\mathrm{e}^{\mathrm{i}\delta_n}$, and when the magnetization is constant throughout the thickness, B_n is determined by

$$B_n = \mu_0|M_0|[nkaK_1(nka) - nk(a+t)K_1(nk(a+t))]. \quad (9)$$

Thus, we have obtained the analytic solution of the magnetic field for the general case. The most useful case is that the magnetization contains only one Fourier component $\boldsymbol{M}(\boldsymbol{r}) = M_0\cos(kz)\boldsymbol{e}_z$, i.e., the magnetic tube is magnetized sinusoidally. Then the magnetic field inside the tube is

$$B_\rho = -B_1 I_1(k\rho)\sin(kz), \quad (10)$$

$$B_z = -B_1 I_0(k\rho)\cos(kz), \quad (11)$$

where

$$B_1 = \mu_0 M_0[kaK_1(ka) - k(a+t)K_1(k(a+t))]. \quad (12)$$

From Eqs.(10) and (11), the amplitudes of the two magnetic fields are not equal, which means that the magnitude of the magnetic field along the axial direction varies periodically according to the period of the magnetization. This is an unfavourable factor for the atom to propagate along the axial direction. Fortunately, when $k\rho \gg 1$,the functions $I_0(k\rho)$ and $I_1(k\rho)$ tend to have a same value. So we should make the period of magnetization λ satisfy $\lambda \ll a$, in which, a is the inner radius of the magnetic tube. Figure 2 is the magnitude distributions of the magnetic field inside the tube along the radial and axial directions. Figures 2(a) and 2(b) show the case of $\lambda/a = 0.1$, and Figs.2(c) and 2(d) show the case of $\lambda/a = 1.0$. From the figures, it is easy to see that when $a \gg \lambda$, the profile of the radial magnetic field distribution is steeper in Fig.2(b) compared with that in Fig.2(d); and the magnitude of the field along the axial direction is almost constant [Fig.2(a)]. However, when the period of magnetization have the same order of the magnetic tube's inner radius, the profile of the radial magnetic field distribution varies not so rapidly [Fig.2(d)] and the field along the axial direction becomes fluctuating [Fig.2(c)]. So for the purpose of guiding atoms, the period of the magnetization should be kept much shorter than the inner radius of the magnetic tube.

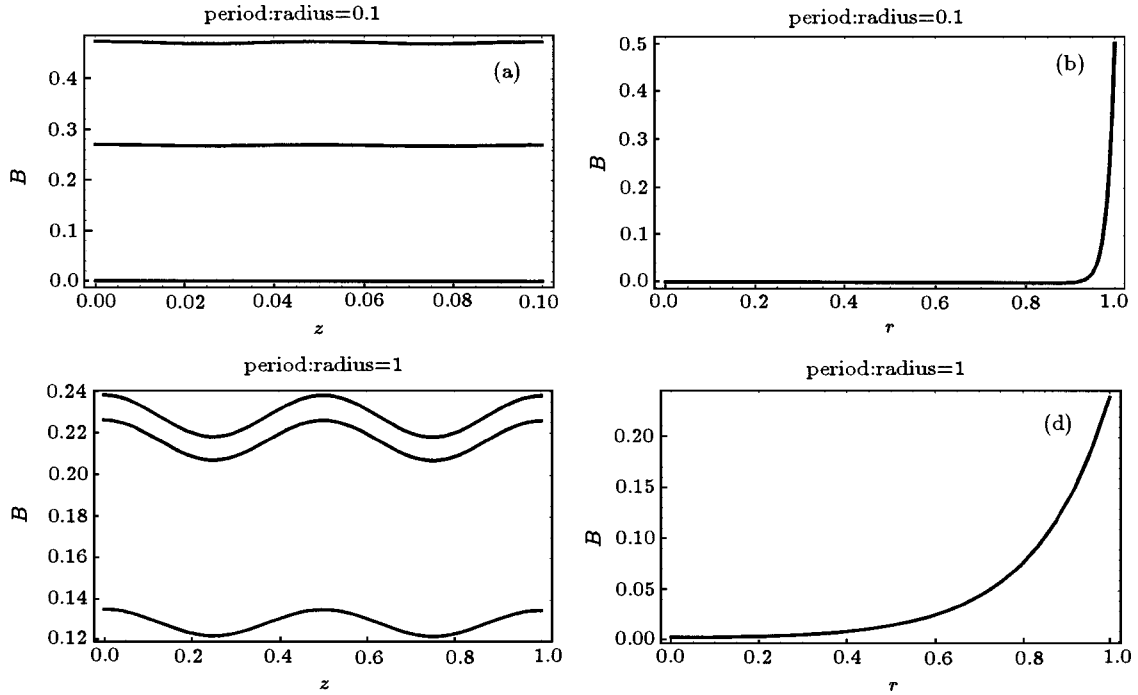


Fig.2. The magnitude distributions of the magnetic field inside the magnetic tube along the radial and axial directions. (a)$\lambda/a = 0.1$,axial (b)$\lambda/a = 0.1$,radial (c)$\lambda/a = 1$,axial (d)$\lambda/a = 1$, radial. (Here, the unit of B is $\mu_0 M_0$).

When $ka \gg 1$, we can have an approximate formula for the magnetic field near the inner wall of the magnetic tube by an asymptotic expansion of the Bessel function of the imaginary argument when $x \to \infty$

$$I_\nu(x) = \frac{\mathrm{e}^x}{\sqrt{2\pi x}}[1 + O(x^{-1})], \tag{13}$$

$$K_\nu(x) = \sqrt{\frac{\pi}{2x}}\mathrm{e}^{-x}[1 + O(x^{-1})]. \tag{14}$$

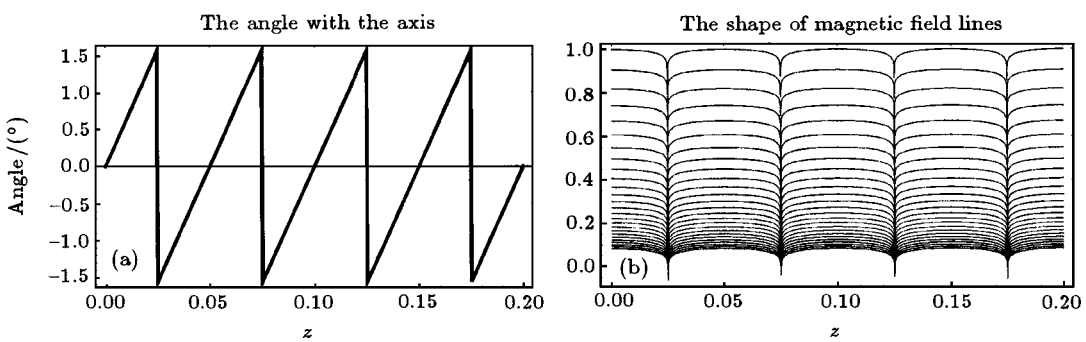


Fig.3. (a) The angle between the magnetic field and the axis of the tube. (b) the magnetic field line shape near the magnetic tube wall.

By using Eqs.(13) and (14), the approximate formula of the magnetic field can be easily written out

$$B(\rho,\phi,z) \approx \frac{\mu_0 M_0}{2}\sqrt{\frac{a}{\rho}}\left[1 - \sqrt{\frac{a+t}{a}}\mathrm{e}^{-kt}\right]\mathrm{e}^{-k(a-\rho)}; \tag{15}$$

Eq. (15) is valid when $\rho \approx a$, and the field is attenuated fast inside the tube along the radial direction so that

$$B(\rho,\phi,z) \approx \frac{\mu_0 M_0}{2}\left[1 - \sqrt{\frac{a+t}{a}}\mathrm{e}^{-kt}\right]\mathrm{e}^{-k(a-\rho)}. \tag{16}$$

When $a \gg t$, Eq.(16) becomes

$$B(\rho,\phi,z) \approx \frac{\mu_0 M_0}{2}\left[1 - \mathrm{e}^{-kt}\right]\mathrm{e}^{-k(a-\rho)}, \tag{17}$$

which is just the same as the magnetic field near a magnetic mirror.[9]

The direction of the magnetic field can be described by angle θ between the direction of the field and the axis of the tube, which satisfies

$$\theta = \arctan\left[\frac{I_1(k\rho)}{I_0(k\rho)}\tan(kz)\right]. \tag{18}$$

Figure 3(a) shows the variation of the magnetic field direction along the axis of the tube, and Fig.3(b) illustrated the shape of the magnetic field lines near the magnetic tube wall.

3. The transport efficiency analysis

The interaction energy between the alkali metals and the magnetic field inside the tube can be given by Breit–Rabi formula,[10]

$$E_m(\rho, F = I \pm 1/2) = \pm\frac{\triangle E_0}{2}\sqrt{1 + \frac{4M_{\rm F}}{2I+1}x + x^2}, \tag{19}$$

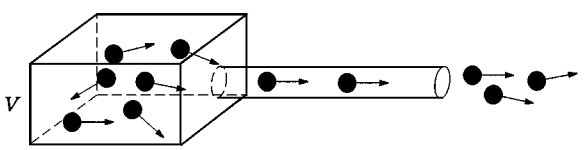


Fig.4. The transport of cold atoms through the magnetic tube.

in which $\triangle E_0$ is the hyperfine structure energy gap when the magnetic field is zero and x is a parameter of the magnetic field defined by

$$x = g_J\frac{\mu_{\rm B}}{\triangle E_0}B(\rho, z), \tag{20}$$

where g_J is the Landé factor and $\mu_{\rm B}$ is the Bohr magneton. We think that the motion of the cold atoms inside the magnetic tube is adiabatic so that the magnetic quantum number related to the local magnetic field would be constant. For $ka \gg 1$ and weak magnetic field $x \ll 1$, the interaction energy between the atoms and the magnetic field is

$$E_m(\rho, z) = g_{\rm F}M_{\rm F}\mu_{\rm B}\frac{\mu_0 M_0}{2}\left[1 - \sqrt{\frac{a+t}{a}}{\rm e}^{-kt}\right]{\rm e}^{-k(a-\rho)}. \tag{21}$$

The adiabatic condition is invalid when the magnetic field is near zero inside the tube, and the atoms will be "flipped out" by transiting from the bound magnetic states to the unbound magnetic states. But we can simply add an additional bias magnetic field to keep the adiabatic condition valid. This bias is much smaller near the wall of the tube, but is very important near the centre of the magnetic tube.

In order to evaluate the transport efficiency of the atoms through the magnetic tube (Fig.4), we calculate the atom flux probability. Several assumptions were made which simplify the flux calculation.[11] First, we assume that the distribution of atoms at the magnetic tube input face is spatially uniform and has a Maxwell–Boltzmann velocity distribution. Second, the atoms is not too cold so that they are classical particles. Third, the total energy of the atoms in the magnetic field, i.e. kinetic energy plus potential energy, is conservative. Fourth, if an atom hits the wall of the tube, we consider it to be lost from the waveguide.

The distributions of the position and velocity of the atoms before they enter the magnetic tube are

$$f_p(x, y, z) = \frac{1}{V}, \tag{22}$$

$$f_v(v_x, v_y, v_z) = \left(\frac{m}{2\pi k_{\rm B}T}\right)^{3/2}{\rm e}^{-\frac{m}{2k_{\rm B}T}(v_x^2+v_y^2+v_z^2)}. \tag{23}$$

Then the input flux can be calculated by the following formula

$$\begin{aligned} J_i &= \int\limits_{x^2+y^2<a^2} f_p(x, y, z){\rm d}x{\rm d}y \\ &\quad\times \int\limits_{v_z>0} v_z f_v(v_x, v_y, v_z){\rm d}v_x{\rm d}v_y{\rm d}v_z \\ &= \sqrt{\frac{\pi}{2}}\sqrt{\frac{k_{\rm B}T}{m}}\frac{a^2}{V}. \end{aligned} \tag{24}$$

We consider first the atom flux in the absence of magnetization. In this case, only the atoms that do not strike the wall emerge at the output end. So it is simply to calculate the output flux $J_{\rm o} = \sqrt{\frac{\pi}{2}}\sqrt{\frac{k_{\rm B}T}{m}}\frac{a^2}{V}\frac{a^2}{l^2} = J_i a^2/l^2$. As the length of the tube is much larger than the radius of the tube, the efficiency of the guide is trivial when the tube is not magnetized. In the presence of the magnetization of the tube, the transverse atomic motion is bound by the magnetic potential. In order to calculate the output flux, we should determine the velocity region S in which the atoms can pass through the magnetic atom waveguide. Since the potential is cylindrically symmetric, the angular momentum is conserved, which can be expressed as $v_\phi\rho$=constant. The atoms that can be bounded by the magnetic potential must satisfy the following condition

$$S(\rho): \frac{1}{2}mv_\rho^2 + \frac{1}{2}mv_\phi^2 + E_m(\rho) < \frac{1}{2}m\frac{\rho^2}{a^2}v_\phi^2 + E_m(a), \tag{25}$$

where $E_m(\rho)$ is the magnetic potential energy defined by Eq.(21). The output flux is calculated by the following integration

$$J_{\rm o}(M_0) = \iint\limits_{\rho<a} \frac{1}{V}\rho {\rm d}\rho {\rm d}\phi \times \iiint\limits_{S(\rho),v_z>0} v_z f_v(v_\rho, v_\phi, v_z) {\rm d}v_\rho {\rm d}v_\phi {\rm d}v_z. \quad (26)$$

In order to calculate the integration of Eq.(26), we make the following transformations

$$r = \frac{\rho}{a}, \quad (27)$$

$$u_\rho = v_\rho \sqrt{\frac{m}{2k_{\rm B}T}}, \quad (28)$$

$$u_\phi = v_\phi \sqrt{\frac{m}{2k_{\rm B}T}}. \quad (29)$$

Then the integration for the output flux becomes

$$J_{\rm o}(M_0) = \frac{a^2}{V}\sqrt{\frac{2k_{\rm B}T}{\pi m}} \int_0^1 rG(r){\rm d}r, \quad (30)$$

in which $G(r)$ is defined by the following integration

$$G(r) = \iint\limits_{S'(r)} {\rm e}^{-(u_\rho^2+u_\phi^2)} {\rm d}u_\rho {\rm d}u_\phi, \quad (31)$$

$$S'(r): \frac{u_\rho^2}{\frac{E_m(a)-E_m(ar)}{4k_{\rm B}T}} + \frac{u_\phi^2}{\frac{E_m(a)-E_m(ar)}{4k_{\rm B}T}(1-r^2)^{-1}} < 1.$$

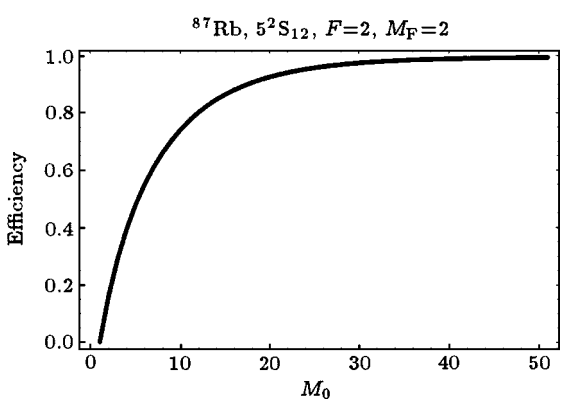


Fig.5. The ^{87}Rb atom transport efficiency versus the magnetization of the magnetic tube. The unit of M_0 is $k_{\rm B}T/\mu_0\mu_{\rm B}$.

Defining the transport efficiency as $\eta = J_{\rm o}/J_i$, from Eqs.(24) and (30), we have

$$\eta = \frac{2}{\pi}\int_0^1 rG(r){\rm d}r. \quad (32)$$

We calculate the transport efficiency of the magnetic tube in the case of guiding cold ^{87}Rb in ground state F=2, $M_{\rm F}$=2. Figure 5 shows the result of the transport efficiency versus the magnetization. It illustrates that the efficiency rises with increasing the magnetization of the tube wall. This is different from that of hollow optical fibre atomic waveguide, in which the laser is used for atom guidance. When the power of the laser increases, the "heating effect" will be significant so that the efficiency of the atom transport will be reduced. Here with the magnetic tube atomic waveguide, the efficiency can reach 100% with increasing the magnetization. From this point of view, the magnetic atomic waveguide has a great advantage over the light guidance atomic waveguide.

Now we consider the effect of the curvature of the magnetic tube on the atom transport efficiency. Figure 6 illustrates a magnetic tube with a radius of curvature R. When the atoms move inside the curved magnetic tube, they will experience a centrifugal force which presses them toward the outer wall of the magnetic tube and they will be lost if the magnetic field inside cannot exert a sufficiently large force to overcome the centrifugal potential.

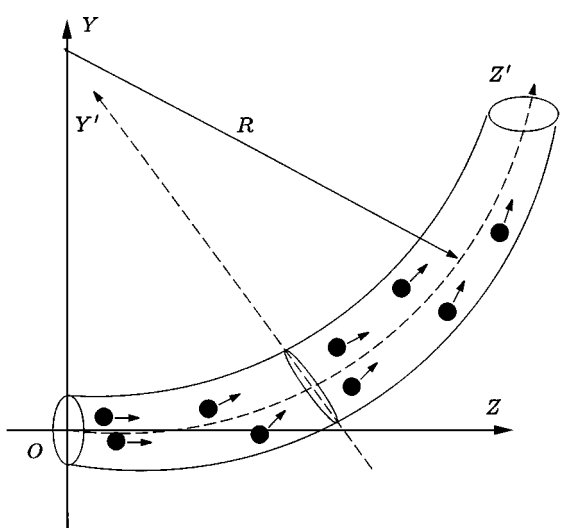


Fig.6. Illustration of a bent magnetic tube atomic waveguide. The radius of curvature is R and the new coordinate system (X', Y', Z') is used.

In the following discussion, we assume that $R \gg a$. For the bent magnetic tube, we transform the coordinates from (X, Y, Z) to(X', Y', Z'). In Fig.6, the axis Z' follows the curvature of the bent magnetic tube and we have the following transformation formula

$$X = X', \quad (33)$$

$$Y = R[1-\cos(Z'/R)] + Y'\cos(Z'/R), \quad (34)$$

$$Z = (R - Y')\sin(Z'/R). \tag{35}$$

The unit vectors $(\boldsymbol{e}_X, \boldsymbol{e}_Y, \boldsymbol{e}_Z)$ for the old coordinate system are related to the new ones $(\boldsymbol{e}_{X'}, \boldsymbol{e}_{Y'}, \boldsymbol{e}_{Z'})$ by the expressions

$$\boldsymbol{e}_X = \boldsymbol{e}_{X'}, \tag{36}$$

$$\boldsymbol{e}_Y = \boldsymbol{e}_{Y'}\cos(Z'/R) + \boldsymbol{e}_{Z'}\sin(Z'/R), \tag{37}$$

$$\boldsymbol{e}_Z = -\boldsymbol{e}_{Y'}\sin(Z'/R) + \boldsymbol{e}_{Z'}\cos(Z'/R). \tag{38}$$

When $R \gg a$, we can define $a_{X'} = \ddot{X}'$, $a_{Y'} = \ddot{Y}'$, $a_{Z'} = \ddot{Z}$. We can obtain the transformation of the acceleration between these two coordinate systems by applying Eqs. (33)–(38)

$$\begin{aligned}\boldsymbol{a} =& \ddot{X}\boldsymbol{e}_X + \ddot{Y}\boldsymbol{e}_Y + \ddot{Z}\boldsymbol{e}_Z \\ =& a_{X'}\boldsymbol{e}_{X'} + \left(a_{Y'} + \frac{v_{Z'}^2}{R}\right)\boldsymbol{e}_{Y'} \\ &+ \left(a_{Z'} - \frac{2v_{Y'}v_{Z'}}{R}\right)\boldsymbol{e}_{Z'}.\end{aligned} \tag{39}$$

So the expressions of the Newton's equations of motion for the atoms inside the bent tube are

$$ma_{X'} = F_{X'}, \tag{40}$$

$$m\left(a_{Y'} + \frac{v_{Z'}^2}{R}\right) = F_{Y'}, \tag{41}$$

$$m\left(a_{Z'} - \frac{2v_{Y'}v_{Z'}}{R}\right) = 0. \tag{42}$$

The region of the input velocities and the coordinates corresponding to the trapped atoms can be determined by integrating Eqs.(40)–(42) as follows

$$v_{Z'}R = \text{constant}, \tag{43}$$

$$\frac{1}{2}mv_{X'}^2 + \frac{1}{2}mv_{Y'}^2 + m\frac{v_{Z'}^2}{R}Y' + E_m(X', Y') = \text{constant}. \tag{44}$$

Equation (43) implies that the angular momentum is related to the centre of curvature of the magnetic tube. Since $R \gg a$, the velocity $v_{Z'}$ is approximately constant. Equation (44) presents the energy conservation when the atoms move inside the tube. For simplicity, we assume that the cross section of the tube is a square $-a < X' < a$,$-a < Y' < a$; and we consider that the square magnetic potential is near the magnetic tube wall

$$E_m(X', Y') = \begin{cases} 0, & \text{if } |X'| < a \text{ and } |Y'| < a, \\ U_0, & \text{if } |X'| = a \text{ or } |Y'| = a, \end{cases} \tag{45}$$

in which U_0 is the potential on the wall that can be taken as $\frac{\mu_0 M_0}{2}\left(1 - \mathrm{e}^{-kt}\right)$. Equation(44) demonstrates that the motion of the atoms is analogous to the motion of atoms in a gravity field with the acceleration constant $v_{Z'}^2/R$. Similar to the discussion for a straight magnetic tube, we can obtain the transport efficiency of the cold atoms through the magnetic tube with a square cross section,

$$\begin{aligned}\eta =& \left[\mathrm{erf}\left(\sqrt{\frac{U_0}{k_\mathrm{B}T}}\right)\right]^2 \\ &- \frac{1}{2}\int_{-1}^{1}\mathrm{d}y \frac{\mathrm{e}^{-\frac{U_0}{k_\mathrm{B}T}\frac{R}{2a(1+y)}}}{\sqrt{\frac{R}{2a(1+y)} - 1}} \\ &\times \mathrm{erfi}\left(\sqrt{\frac{U_0}{k_\mathrm{B}T}\left(\frac{R}{2a(1+y)} - 1\right)}\right),\end{aligned} \tag{46}$$

where $y = Y'/a$ and $\mathrm{erf}(x) = \frac{2}{\sqrt{\pi}}\int_0^x \mathrm{e}^{-x^2}\mathrm{d}x$ is the error function and $\mathrm{erfi}(x)$=$\mathrm{erf}(\mathrm{i}x)/\mathrm{i}$ is the error function with an imaginary argument. Equation (46) has a lucid physical picture: the first term of the right-hand side of the equation is the transport efficiency when $R \to \infty$ and the second term is the effect of the curvature to the transport efficiency. The transport efficiencies versus magnetic potential at different curvatures are illustrated in Fig.7. From the figure, we can find that there is a threshold magnetic potential in the bent magnetic tube and the threshold increases as the radius of curvature of the tube decreases. The explanation is that a smaller radius of the magnetic tube needs a larger centripetal force supplied by the magnetic potential.

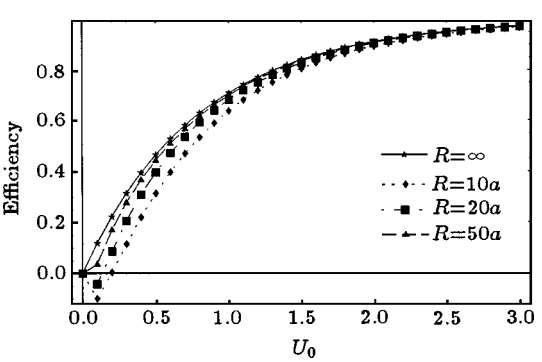


Fig.7. The transport efficiency at different radii $R/a = 10, 20, 50, \infty$, where the unit of U_0 is $k_\mathrm{B}T$.

4. The transverse cooling effect

In this section we discuss the transverse cooling effect of the magnetic tube on the atoms. We only discuss the straight magnetic tube. The velocity of atoms which can emerge at the output end of the magnetic tube must satisfy Eq.(25), which means that

only the atoms with slow transverse velocity can pass through the magnetic tube. So the magnetic tube atomic waveguide can be used as a transverse velocity filter for the input atoms by adjust the magnetization of the tube. In other words, the magnetic tube can be used to decrease the transverse temperature of the guided atoms, which is defined as

$$T_{\rm trans} = \frac{m\left\langle v_\perp^2 \right\rangle}{2k_{\rm B}}. \tag{47}$$

As the potential between the magnetic tube and the atoms is conservative, the transverse kinetic energy does not change when the atoms move from the input end to the output end. When the magnetization of the tube is zero, only those atoms that do not strike the wall can pass through the tube. Under this condition we can calculate the transverse temperature of the output atoms

$$T_{\rm trans} \approx \frac{3}{2}\left(\frac{a}{l}\right)^2 T, \tag{48}$$

where l is the length of the magnetic tube. Since $l \gg a$, the transverse temperature of the output atoms is far lower than that of the input atoms. But unfortunately, the transport efficiency is trivial in this case. When the tube is magnetized, the transverse temperature of the output atoms will increase. Figure 8 illustrates increasing the transverse temperature of the output atoms with increasing the magnetic field. The relation between the transverse temperature and the transport efficiency is plotted in Fig.9. One can find that the higher the transport efficiency, the higher the transverse temperature of the output atoms.

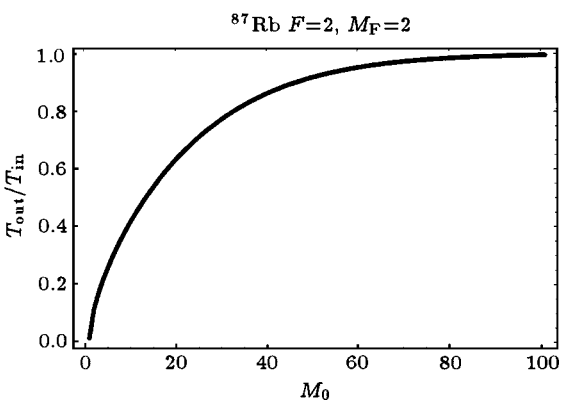


Fig.8. The transverse temperature of the atoms of ^{87}Rb in ground state $F = 2, M_{\rm F} = 2$ at the output end.

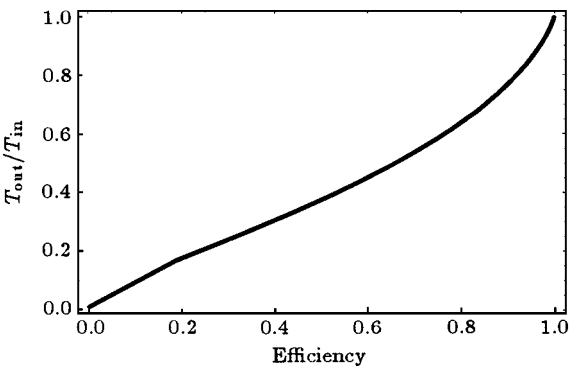


Fig.9. The relation of the transverse temperature of the atoms of ^{87}Rb in the ground state $F = 2, M_{\rm F} = 2$ at the output end with the atom transport efficiency.

5. Conclusion

In this paper, we have discussed the periodic magnetization magnetic tube atom waveguide model. We calculated the analytic solution of the magnetic field and discussed the transport efficiency of the magnetic field atom waveguide, including the straight and curved atom waveguides. We also calculated the transverse atom cooling effect of the magnetic tube atom waveguide. In all of our discussion, we have considered the cold atoms under the condition that the adiabatic motion is valid, and the atoms are not too cold so that they can be considered as classical particles. The quantum motion in the magnetic tube will be discussed in future papers.

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