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Nonadiabatic effects on population transfer of two Bose–Einstein condensates induced by atomic interaction*

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We investigate the stimulated Raman adiabatic passage for Bose–Einstein condensate (BEC) states which are trapped in different potential wells or two ground states of BEC in the same trap. We consider that lasers are nearly resonant with the atomic transitions. The difference of population transfer processes between BEC atoms and usual atoms is that the atomic interaction of the BEC atoms can cause some nonadiabatic effects, which may degrade the process. But with suitable detunings of laser pulses, the effects can be remedied to some extent according to different atomic interactions.

Keywords: quantum condensate phenomena, photon-atom interaction, Bose systems
PACC: 0375, 0330, 3280

1. Introduction

Stimulated Raman adiabatic passage (STIRAP), which usually uses two delayed laser pulses to couple two ground states with a common excited state of an atom, has been studied extensively.[1−3] The successful population transfer into a desired state is also important for wide applications, including atomic and molecular spectroscopy, atomic interferometer, and atomic collision. Counterintuitive stimulated Raman adiabatic passage has been studied for the generation of photon number states in cavity quantum electrodynamics (QED)[4] and for cavity QED quantum computing,[5] as well as the molecular creation in BEC.[6−8]

It is well known that for a usual atom when counterintuitive pulses are applied and the adiabatic condition is met, little population can occupy the excited state. If in the process the adiabatic condition is not satisfied, population will occupy the excited state. It may decay into an original state and other states. So the adiabatic passage will be failure. For the usual atom case, nonadiabatic effects can be induced by unmatched pulse parameters, such as the Rabi frequencies, the delay and the width. All the above-mentioned applications rely on the existence of stimulated Raman adiabatic passage. BEC is very different from a usual atom.[9−11] In a condensate all atoms are condensed into a single mode which can be approximately described by the mean-field theory. In the mean-field theory, the Gross–Pitaevskii equation clearly includes the atomic two-body interaction. According to the recent BEC experiment,[12] when BEC is taken into consideration, spontaneous emission is enhanced. Usually, spontaneous emission is regarded as a decoherence effect. At present, most experiments produce atom laser used far-off detuning coupling field. In this way, the excited state can be adiabatically eliminated. So one need not consider the excited state spontaneous emission. From Ref.[13], we see clearly that the population has visited the excited state. In Ref.[14], the authors discussed the influence of atomic interaction on the time-dependent tunnelling process. In this paper, we will check the adiabatic passage for BECs in

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which two ground states are coupled to a common internal excited state by two Gaussian laser pulses, which are used near resonance to the transitions. In our treatment the atomic two-body collision is clearly included. We mainly focus on the nonadiabatic effects induced by the atomic interaction in the transfer process. At the same time, we will check the condition for dark state existence analytically, and do some numerical calculations to show the process of population transfer. We find that by considering different atomic interactions of two ground-state atoms, complete population transfer can always be realized when pulses are applied resonantly, and spontaneous emission does not have much influence under the linear approximation of interaction.

2. Theory

We consider a three-level system labelled as $|1\rangle, |2\rangle, |3\rangle$. The $|1\rangle$ and $|3\rangle$ are ground states and $|2\rangle$ is the excited state. As a counterintuitive way, we firstly apply a laser pulse between the states $|3\rangle$ and $|2\rangle$(if initial population is in state $|1\rangle$), then another laser pulse is used between states $|1\rangle$ and $|2\rangle$. The direct transition between the two ground states $|1\rangle$ and $|3\rangle$ is forbidden. Here we use delayed Gaussian pulses for transitions $|1\rangle \leftrightarrow |2\rangle$ and $|2\rangle \leftrightarrow |3\rangle$. Laser pulses can be written in the forms $E_1(t-\xi_1)\mathrm{e}^{-i\omega_{12}t}$ and $E_2(t-\xi_2)\mathrm{e}^{-i\omega_{32}t}$, where $\xi_{1(2)}$ and $\omega_{12(32)}$ are the pulse delay and laser frequency respectively. The intermediate state $|2\rangle$ is detuned by δ_1 and there is a detuning δ_3 in $|3\rangle$ as shown in Fig.1.

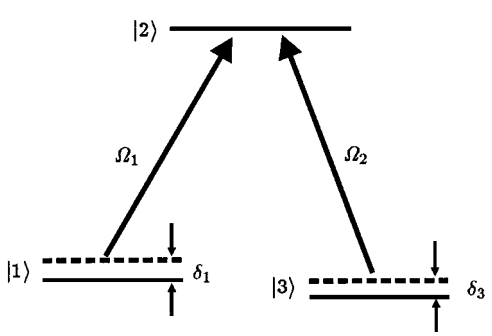


Fig.1. Schematic of three-level system. $|1\rangle$ and $|3\rangle$ are ground states and $|2\rangle$ is excited state. Ω_1 and Ω_2 are Rabi frequencies. δ_1 and δ_3 are the detunings.

The Hamiltonian of this system can be expressed as

$$H = H_{\mathrm{atom}} + H_{\mathrm{atom-atom}} + H_{\mathrm{atom-laser}}, \qquad (1)$$

where H_{atom} is the free part of Hamiltonian which governs the free evolution of the atomic field. $H_{\mathrm{atom-atom}}$ is the interaction between the ground-state atoms which, under the mean-field theory approximation, is the two-body s-wave scattering collisions. $H_{\mathrm{atom-laser}}$ is the interaction between the atom and lasers which drives the population of BEC atoms to transfer from one BEC to another BEC. Here we do not consider the interaction of the excited-state atoms and the interaction between the excited-state atom and other atoms because of the short time for the atom staying in the excited state. Here we use the single mode approximation for the condensate and assume the same spatial distribution among different components. Also atomic spontaneous emission is not included in our treatment. For an ideal population transfer process the atomic spontaneous emission does not have much effects on the process. In the rotating frame of laser pulses, we can obtain the Hamiltonian for the single mode approximation,

$$\begin{aligned} H = &\sum_j \hbar U_j (N_j^2 - N_j) + \sum_{i \neq j} \hbar U_{ij} N_i N_j + \hbar\delta_1 N_1 \\ &+ \hbar\delta_3 N_3 + \left[\frac{\hbar\Omega_1(t)}{2} a_2^\dagger a_1 \right. \\ &\left. + \frac{\hbar\Omega_2(t)}{2} a_2^\dagger a_3 + \mathrm{h.c.} \right], \end{aligned} \qquad (2)$$

where

$$\delta_1 = \omega_1 - \omega_2 + \omega_{12}$$

and

$$\delta_3 = \omega_3 - \omega_2 + \omega_{32}.$$

We define

$$U_j = \frac{2\pi\hbar s_j}{m} \int \mathrm{d}^3\boldsymbol{r} |\phi_j|^4,$$

$$U_{ij} = \frac{2\pi\hbar s_{ij}}{m} \int \mathrm{d}^3\boldsymbol{r} |\phi_i|^2 |\phi_j|^2,$$

where s_j and s_{ij} are the two-body scattering lengths. The total atomic number is conserved, that is $N = N_1 + N_2 + N_3$.

$$\begin{aligned} \hbar\omega_{1,2,3} = &\int \mathrm{d}^3\boldsymbol{r}\, \phi^*_{1,2,3}(\boldsymbol{r}) \\ &\times \left(-\frac{\hbar^2}{2m}\nabla^2 + V_{1,2,3}(\boldsymbol{r}) \right) \phi_{1,2,3}(\boldsymbol{r}) \end{aligned}$$

are the energies corresponding to the levels $|1\rangle$, $|2\rangle$ and $|3\rangle$. Let

$$\Omega_1(t) = \mu_{12} E_1(t)/\hbar \int \mathrm{d}^3\boldsymbol{r}\, \phi_2^*(\boldsymbol{r}) \phi_1(\boldsymbol{r})$$

and

$$\Omega_2(t) = \mu_{32}E_2(t)/\hbar \int \mathrm{d}^3\boldsymbol{r}\, \phi_2^*(\boldsymbol{r})\phi_3(\boldsymbol{r}).$$

Since the number of particles is conserved, we have

$$a_1^\dagger a_1 + a_2^\dagger a_2 + a_3^\dagger a_3 = N, \tag{3}$$

by defining scaled boson operators $a_j' = a_j/\sqrt{N}$ and dropping the primes, it follows

$$a_1^\dagger a_1 + a_2^\dagger a_2 + a_3^\dagger a_3 = 1. \tag{4}$$

Considering $N_i^2 \gg N_i$, by setting $u_1 = 2U_1N$, $u_2 = 2U_2N$ and $u_3 = 2U_3N$, $u_{13} = 2U_{13}N$, we can obtain the following Hamiltonian

$$\begin{aligned} H/N = &\sum_j \hbar u_j N_j^2 + \sum_{i\neq j} \hbar u_{ij} N_i N_j + \hbar\delta_1 N_1 \\ &+ \hbar\delta_3 N_3 + \left[\frac{\hbar\Omega_1(t)}{2} a_2^\dagger a_1 \right. \\ &\left. + \frac{\hbar\Omega_2(t)}{2} a_2^\dagger a_3 + \text{h.c.}\right]. \end{aligned} \tag{5}$$

We suppose that the time for an atom staying at the excited state is very short, so the atomic interaction at the excited state can be neglected. Because the spatial part has been integrated, under the basis $|1\rangle, |2\rangle, |3\rangle$, we can have the eigenvalue equation

$$\begin{vmatrix} u_1N_1 + u_{13}N_3 + \delta_1 - \lambda & \frac{\hbar\Omega_1}{2} & 0 \\ \frac{\hbar\Omega_1}{2} & -\lambda & \frac{\hbar\Omega_2}{2} \\ 0 & \frac{\hbar\Omega_2}{2} & u_3N_3 + u_{13}N_1 + \delta_3 - \lambda \end{vmatrix} = 0. \tag{6}$$

From Eq.(6) it can be easily checked that when the following conditions are satisfied, one of the eigenvalues is zero. It corresponds to the dark state eigenvalue.

$$u_1N_1 + u_{13}N_3 + \delta_1 = 0, \tag{7a}$$

$$u_3N_3 + u_{13}N_1 + \delta_3 = 0. \tag{7b}$$

When the above conditions are satisfied, for wavefunction $\psi = c_1|1\rangle + c_2|2\rangle + c_3|3\rangle$, the dark state is

$$\psi = \frac{\Omega_2(t)}{\sqrt{\Omega_1^2(t) + \Omega_2^2(t)}}|1\rangle - \frac{\Omega_1(t)}{\sqrt{\Omega_1^2(t) + \Omega_2^2(t)}}|3\rangle.$$

Under the mean-field approximation[13] or by deriving c-number equations in the positive-P representation of quantum optics,[15] an atomic field can be expressed as c-numbers.[6,16] Here we use the mean-field theory approximation. From now on, the symbols a_1, a_2, a_3 in the equations refer to c-numbers, rather than operators. According to the Hamiltonian and mean-field approximation, we obtain the following equations of motion

$$\mathrm{i}\dot{a}_1 = u_1a_1^*a_1^2 + u_{13}a_3^*a_3a_1 + \delta_1a_1 + \frac{\Omega_1(t)}{2}a_2, \tag{8a}$$

$$\mathrm{i}\dot{a}_2 = u_2a_2^*a_2^2 + \frac{\Omega_1(t)}{2}a_1 + \frac{\Omega_2(t)}{2}a_3, \tag{8b}$$

$$\mathrm{i}\dot{a}_3 = u_3a_3^*a_3^2 + u_{13}a_1^*a_1a_3 + \delta_3a_3 + \frac{\Omega_2(t)}{2}a_2. \tag{8c}$$

There are no analytical solutions for the above operator equations. But during the process of population transfer, a_1 and a_3 are not constants and always changed with the process. That means: when BEC is considered, the strict dark state cannot exist if δ_1 and δ_3 are constants.

3. Numerical results

In order to verify our statement, we perform some numerical calculations. As stated above, we use Gaussian laser pulse $\Omega_{1,2} = \Omega_0 \exp\left[-\left(\frac{t-\xi_{1,2}}{\sigma}\right)^2\right]$ and choose time $t = \tau\sigma$. We assume that the atomic interaction in the excited state is zero. The Rabi frequencies (after multiplying the pulse width) are plotted in Fig.2.

With these pulse parameters and for comparison, we calculate the population transfer for $\delta_1 = \delta_3 = 0, u_1 = u_2 = u_3 = 0$. This corresponds to the ideal case. For the counterintuitive pulse sequence, the dark state can exist. Clearly, for $t \to -\infty$, $\Omega_2 \gg \Omega_1$, population stays in the initial ground state $|1\rangle$; and for $t \to +\infty$, $\Omega_1 \gg \Omega_2$, population transfers to the ground state $|3\rangle$. In this process, there is little population in the excited state $|2\rangle$. If the laser pulses are allowed of adiabatic evolution, population can be completely transferred to the desired state without suffering the loss of spontaneous emission. In the case of outcoupling atom laser, the outcoupling rate is the highest. For this case the population evolution is plotted in Fig.3.

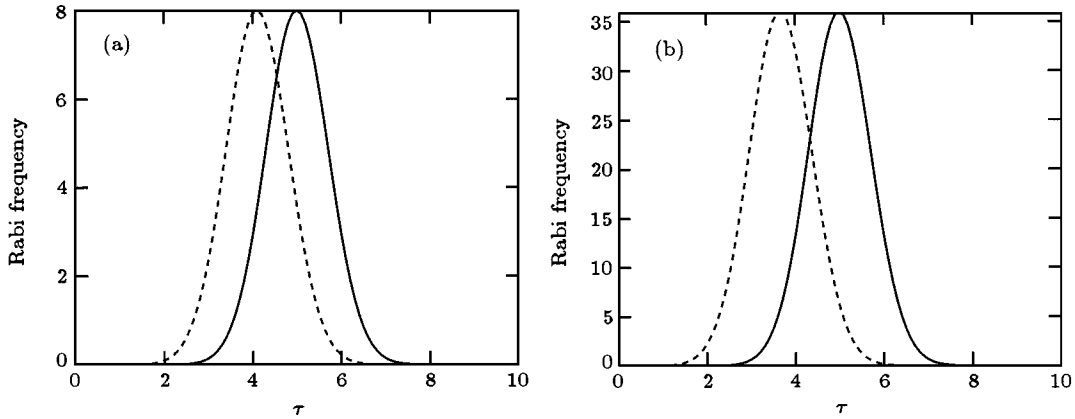


Fig.2. The pulse sequence. Solid and dashed lines represent pulse 1 and pulse 2, respectively. (a) $\Omega\sigma = 8, \sigma = 20, \xi_1 = 100, \xi_2 = 82$. (b) $\Omega\sigma = 36, \sigma = 6, \xi_1 = 30, \xi_2 = 22$.

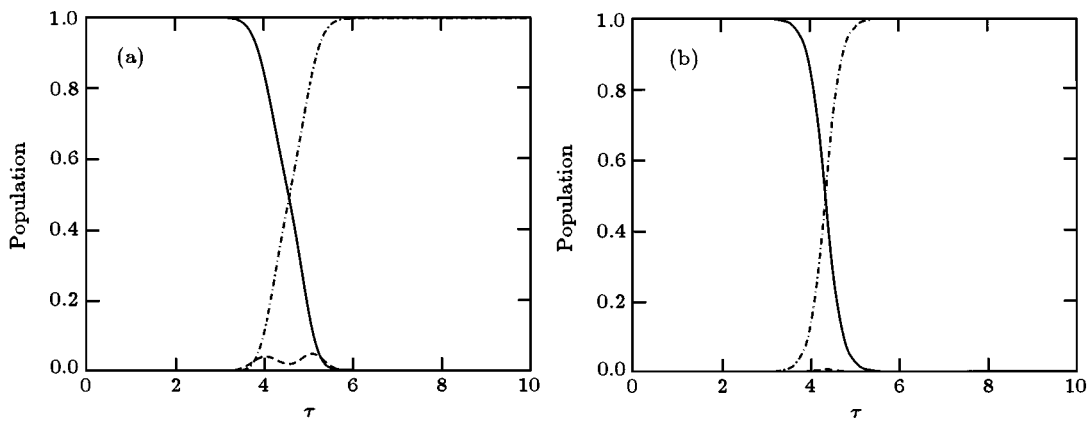


Fig.3. The population transfer in an ideal case. All the interatomic interactions and detunings are zeros. The curves show that the population transfer is complete. There is little population which can visit the excited state. (a) $\Omega\sigma = 8, \sigma = 20, \xi_1 = 100, \xi_2 = 82, u_1 = u_3 = u_{13} = 0$. (b) $\Omega\sigma = 36, \sigma = 6, \xi_1 = 30, \xi_2 = 22, u_1 = u_3 = u_{13} = 0$.

We compute the populations of three states and plot them in Fig.4 for various interaction strengths without detunings. For the strong and short laser pulses, the effect of atomic interaction is not evident. An almost ideal atomic population transfer is held. This can be seen from Figs.4(a) and (b). Although the population transfer is complete, when the interaction strength is increase, there are some oscillations in Fig.4(b). These oscillations is caused by atomic interactions. But we see clearly in the Figs.4(c) and (d) that the population transfer is not an ideal one. In these two figures, we have chosen weak and longer laser pulses and relative small atomic interaction compared with Figs.4(a) and (b). During the process some population will visit the excited state. The stronger the atomic interaction, the worse the population transfer process is. In this situation the adiabatic population transfer process has been destroyed because of the atomic interaction. The atomic interaction causes some nonadiabatic effects in the process. This is the difference between the processes undergone by BEC atoms and usual atoms. But the nonadiabatic effects can be eliminated when we use ground states detuning to compensate the effects of atomic interaction. The result of this kind of calculations has been plotted in Fig.5.

We can see from the graphs, when the atomic interaction strength is smaller (see $u\sigma = 1$ in Fig.5(a)), the atomic interaction effects are not obvious. But with increasing interaction, the improvement of transfer process is significant. For $u_1\sigma = u_3\sigma = 4, \delta_1\sigma = \delta_3\sigma = -4$, we can obtain an almost ideal population transfer. During this process, there is little population visiting the excited state. For other cases with proper detunings, higher population transfer can also be obtained. But at the same time, due to the large

nonlinear interaction, to completely remove the effects of atomic interaction is impossible. With too huge an atomic interaction, some population will still visit the excited state. This will result in a nonideal population transfer finally.

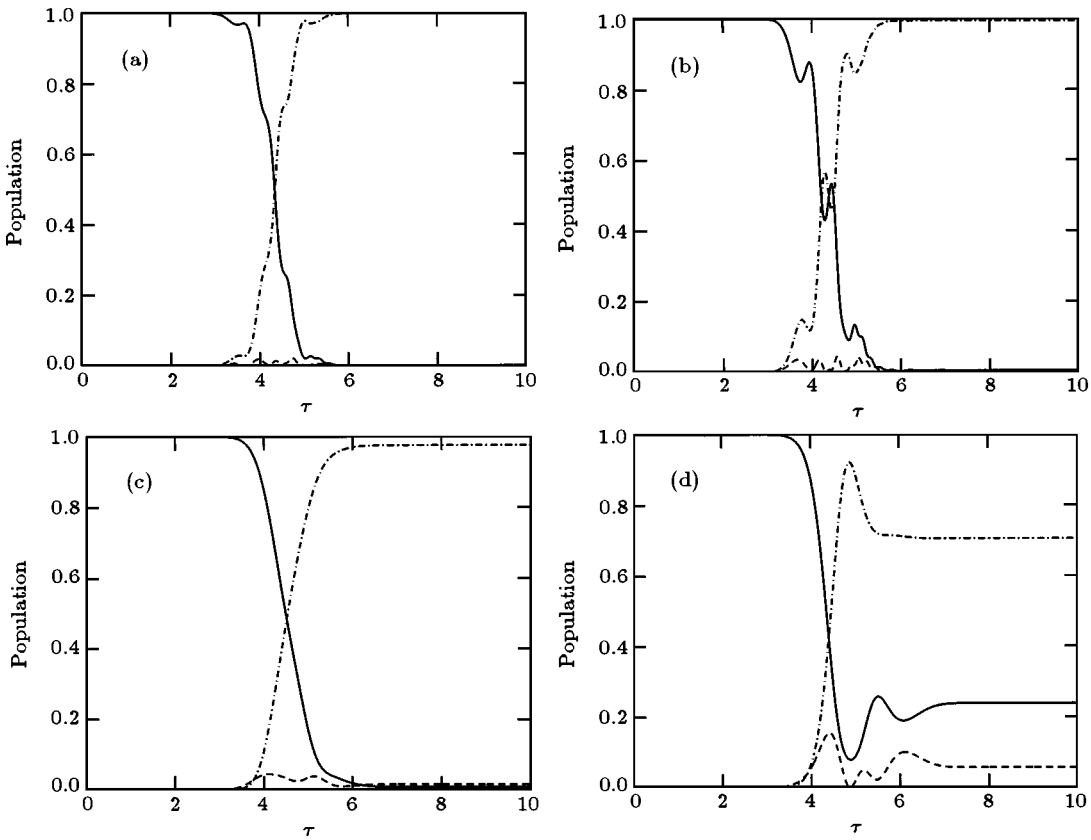


Fig.4. The population transfer for different atomic interactions. The pulse parameters in (a), (b) and (c), (d) are the same as those in Figs.2(a) and (b) respectively. All the detunings are zero. (a) $u_1\sigma = u_3\sigma = 12, u_{13}\sigma = 6$. (b) $u_1\sigma = u_3\sigma = 18, u_{13}\sigma = 9$. (c) $u_1\sigma = u_3\sigma = 1, u_{13}\sigma = 0.5$. (d) $u_1\sigma = u_3\sigma = 4, u_{13}\sigma = 2$.

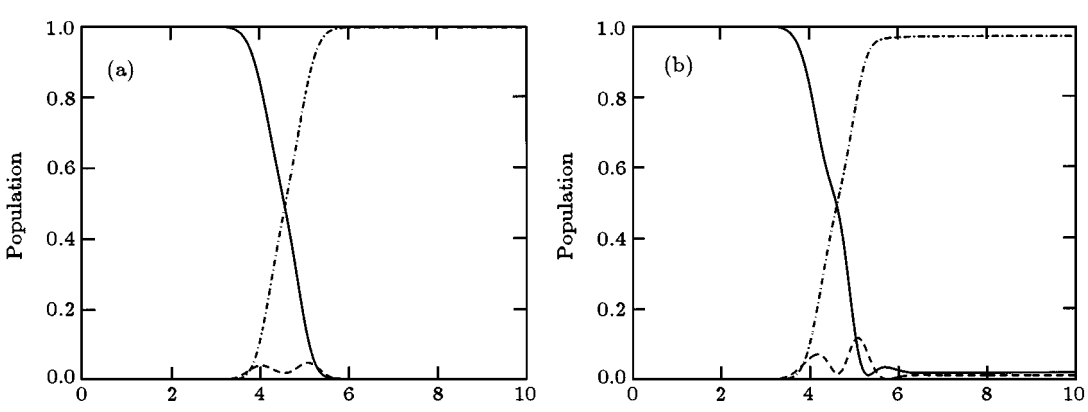


Fig.5. Solid, dashed and dot-dashed lines represent the population variations of states $|1\rangle, |2\rangle, |3\rangle$ respectively. The pulse parameters are same as Fig.2(a). (a) $u_1\sigma = u_3\sigma = 1, u_{13} = 0.5, \delta_1 = \delta_3 = -1$. (b) $u_1\sigma = u_3\sigma = 4, u_{13} = 2, \delta_1 = \delta_3 = -4$.

4. Conclusions

In conclusion, we have considered the population transfer of two Bose–Einstein condensates by using counterintuitive laser pulses. Due to the atomic interaction in the ground state, the situation is different from the usual atomic adiabatic population transfer. For relative weak and long laser pulses, atomic interactions will cause some nonadiabatic effects. This kind of nonadiabatic effects is evident when the atomic interaction is stronger. Nonadiabatic effects can be eliminated by a suitable choice of ground-state detuning. This means that if we properly choose the pulse detuning, we can have an approximate dark state and thus the population transfer is almost completed to some extent. When this is used to outcouple BEC atoms, higher outcouple rate can be obtained without suffering too much spontaneous emission. These situations have been confirmed by numerical calculations.

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