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Collapse and revival of a single-Cooper-pair box in a single-mode quantized field

Yao Yan-Sun(姚延荪), Zou Jian(邹 健), and Shao Bin(邵 彬)

Department of Physics, Beijing Institute of Technology, Beijing 100081, *China*

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We study the quantum dynamics of a single-Cooper-pair box biased by a classical voltage and also irradiated by a single-mode quantized field. We demonstrate that under weak damping of the quantized field, the collapse-revival phenomena can exist in this system, and the oscillations of the collapse and revival depend sensitively on the initial state of the single-mode quantized field and the damping rate κ. We also demonstrate that this system can show the beats phenomena.

Keywords: Josephson junction, quantized field, collapse and revival, damping rate
PACC: 7450, 7215R

1. Introduction

In recent years considerable attention has been focused on the quantum two-level system potential applications in quantum computers.[1−5] It has been shown that quantum coherence of charge states in a single-Cooper-pair box and flux states in a rf superconducting quantum interference device (SQUID) can be used to create charge and flux qubits for quantum computation. Furthermore, another important aspect of research activities in the Cooper-pair box is that it unveils fundamental problems related to the quantum mechanical behaviour of the macroscopic systems. And the single-Cooper-pair box is an ideal macroscopic quantum system in that, although there is a large number of electrons in the metal box electrode, at very low temperatures they all form Cooper-pairs and condense into a single macroscopic ground state, separated by a superconductivity gap Δ from the excited states of quasi-particles. In order to exploit the fully quantum mechanical nature of mesoscopic Josephson junctions, an ultra small capacitance weak link biased by a classical voltage source and also coupled to a quantized electromagnetic field was considered. Some interesting results have been obtained,[6−8] which prove to be reliable in view of recent development in quantum optics.

In this paper we investigate the quantum dynamics of a single-Cooper-pair box biased by a classical voltage and also irradiated by a single-mode quantized field. We will continue the discussion of a former paper.[9] In that paper we demonstrated that the probability difference $W(t)$ between two states $\{|\downarrow\rangle, |\uparrow\rangle\}$ can behave in collapse-revival phenomena when the system is under a weak damping. In this paper we will make a thorough discussion of the dynamics of the number of Cooper-pairs. We demonstrate that the probability difference between two states $\{|0\rangle, |1\rangle\}$ can also show similar collapse-revival phenomena. In our previous paper we only needed to calculate the diagonal matrix elements of the reduced density operator ρ, however, in this paper we must take all of the matrix elements including the off-diagonal ones into account.

2.Model and method

In what follows we consider a superconducting box with a low-capacitance Josephson junction (with capacitance $C_{\rm J}$ and Josephson energy $E_{\rm J}$), biased by a classical voltage source $V_{\rm g}$ through a gate capacitance $C_{\rm g}$, and placed inside a single-mode microwave cavity. Suppose that the gate capacitance is screened from the quantized radiation field, then the junction-field Hamiltonian can be written as[10]

$$H = \frac{(Q - C_{\rm g}V_{\rm g} - C_{\rm J}V)^2}{2(C_{\rm g} + C_{\rm J})} - E_{\rm J}\cos\phi + \hbar\omega\left(a^+a + \frac{1}{2}\right), \tag{1}$$

where the relevant conjugate variables are the charge $Q = 2Ne$ on the island (where N is the number

of Cooper-pairs) and ϕ the phase difference across the junction; a^+ and a are respectively the creation and annihilation operators of the microwave with frequency ω. The radiation field is to produce an alternating electric field of the same frequency across the junction, and V is the effective voltage difference produced by the microwave across the junction. We also assume that the field is linearly polarized, and is taken perpendicular to the plane of electrodes, then V may be written as[11]

$$V = \mathrm{i}\left(\frac{\hbar\omega}{2C_{\mathrm{F}}}\right)^{1/2}(a - a^+), \tag{2}$$

where C_{F}, the capacitance parameter, depends on the thickness of the junction, the relative dielectric constant of the thin insulating barrier, and the dimension of the cavity.

In this paper we consider the case that the charging energy with scale $E_C = e^2/2(C_{\mathrm{g}} + C_{\mathrm{J}})$ dominates over the Josephson coupling energy E_{J}, and we concentrate on the value of $V_{\mathrm{g}} = e/C_{\mathrm{g}}$, so that only the low-energy charge states N=0 and N=1 are relevant.[2,3] In this case the Hamiltonian in a basis of the charge states $|0\rangle$ and $|1\rangle$ reduces to a two-state form in a spin-1/2 language[2,3,11]

$$\begin{aligned} H =& E_C\left[1 + \frac{C_{\mathrm{J}}^2}{e^2}V^2\right] - \frac{1}{2}E_{\mathrm{J}}\tau_X \\ & - 2E_C\frac{C_{\mathrm{J}}}{e}V\tau_Z + \hbar\omega\left(a^+a + \frac{1}{2}\right), \end{aligned} \tag{3}$$

where τ_Z and τ_X are the Pauli matrices in the pseudospin basis $\{|0\rangle, |1\rangle\}$.

Even in the absence of quantized radiation field, i.e., V=0, the charge states are not the eigenstates of the Hamiltonian (3), and for simplicity we diagonalize H in the two-charge-state subspace and denote the corresponding two eigenstates as

$$|\downarrow\rangle = \frac{\sqrt{2}}{2}(|1\rangle - |0\rangle), \quad |\uparrow\rangle = \frac{\sqrt{2}}{2}(|1\rangle + |0\rangle). \tag{4}$$

Now we consider an example of a realistic system achievable by the present technology: $C_{\mathrm{J}} \sim 10^{-15}$F, $\omega \sim 10^{10}$Hz, $C_{\mathrm{F}} \sim 10^{-11}$F, and $kT \ll E_{\mathrm{J}} \sim h\omega \ll E_C \ll \Delta$, where Δ is the superconductivity gap. In this case we can neglect the term containing V^2 in Eq.(3), and from Eq.(2) in the rotating wave approximation we can rewrite the Hamiltonian in the new basis as

$$H = \frac{1}{2}E_{\mathrm{J}}\sigma_Z - \mathrm{i}\,\hbar g(a\sigma_+ - a^+\sigma_-) + \hbar\omega a^+a, \tag{5}$$

where

$$g = \frac{eC_{\mathrm{J}}}{2(C_{\mathrm{g}} + C_{\mathrm{J}})}\left(\frac{\omega}{2\hbar C_{\mathrm{F}}}\right)^{1/2}. \tag{6}$$

In the new basis $(|\downarrow\rangle, |\uparrow\rangle)$, σ_Z is the Pauli matrix, σ_+ and σ_- are the raising and lowering operators respectively. To derive Eq.(5), the constant term has been omitted. For the parameters we have chosen, $g/\omega \sim 10^{-2}$. It is noted that the Hamiltonian (5) is just like the simplest form of the cavity-field system interaction, which is known as Jaynes–Cummings model.[12,13]

This system naturally involves a dissipative process characterized by a cavity damping rate $\kappa = \omega/Q$, where Q is the quality factor of the cavity. In the single-mode quantized field characterized by a damping rate κ, we use the Markovian approximation to include dissipation into the master equation. The resulting master equation of the reduced density operator ρ is[12,14]

$$\frac{\partial}{\partial t}\rho = \frac{1}{\mathrm{i}}[H, \rho] + \kappa(2a\rho a^+ - a^+a\rho - \rho a^+a), \tag{7}$$

where κ is the damping rate of the single-mode quantized field. Besides, in this paper we redefined $\hbar$=1 for simplicity. Meanwhile, we suppose the system is in the resonant case, i.e., $E_{\mathrm{J}} = \hbar\omega$.

To solve Eq.(7) we work in the dressed-states representation, i.e. the representation consisting of the complete set of eigenstates of H which are given by

$$\begin{aligned} & H|0, \downarrow\rangle = -\frac{\omega}{2}|0, \downarrow\rangle, \\ & H|\psi_n^{\pm}\rangle = \lambda_n^{\pm}|\psi_n^{\pm}\rangle, \quad n = 0, 1, 2, \cdots, \infty; \end{aligned} \tag{8}$$

$$\begin{aligned} & |\psi_n^{\pm}\rangle = \frac{1}{\sqrt{2}}(|n, \uparrow\rangle \pm \mathrm{i}|n+1, \downarrow\rangle), \\ & \lambda_n^{\pm} = \omega\left(n + \frac{1}{2}\right) \pm g\sqrt{n+1}, \quad n = 0, 1, 2, \cdots, \infty, \end{aligned} \tag{9}$$

where $|n, \uparrow\rangle$ or $|n, \downarrow\rangle$ refers to a state with n photons in the cavity mode and the Cooper-pair box in the excited state $|\uparrow\rangle$ or in the ground state $|\downarrow\rangle$.

In what follows, we use the dressed-state representation to solve the equations for the density matrix. We first write the field operators appearing in Eq.(7) in terms of the dressed states. Note that on the basis of the states $|n, i\rangle$ $(i = \uparrow, \downarrow)$ we have,[15]

$$a = \sum_{n=0}^{\infty}\sqrt{n}(|n-1, \uparrow\rangle\langle n, \uparrow| + |n-1, \downarrow\rangle\langle n, \downarrow|). \tag{10}$$

Here we rewrite Eq.(10) by substituting Eq.(9),

$$\begin{aligned} a =&\frac{1}{\sqrt{2}}(|0,\downarrow\rangle\langle\psi_0^+| - |0,\downarrow\rangle\langle\psi_0^-|) \\ &+\frac{1}{2}\sum_{n=1}^{\infty}[(\sqrt{n+1}+\sqrt{n})(|\psi_{n-1}^+\rangle\langle\psi_n^+|+|\psi_{n-1}^-\rangle\langle\psi_n^-|) \\ &-(\sqrt{n+1}-\sqrt{n})(|\psi_{n-1}^+\rangle\langle\psi_n^-|+|\psi_{n-1}^-\rangle\langle\psi_n^+|)]. \end{aligned} \tag{11}$$

The number operator can be similarly expressed as

$$\begin{aligned} a^+a =&\frac{1}{2}\sum_{n=0}^{\infty}[(2n+1)(|\psi_n^+\rangle\langle\psi_n^+| + |\psi_n^-\rangle\langle\psi_n^-|) \\ &-(|\psi_n^+\rangle\langle\psi_n^-| + |\psi_n^-\rangle\langle\psi_n^+|)]. \end{aligned} \tag{12}$$

Next, we turn to the Heisenberg picture by defining

$$W(t) = \exp(\mathrm{i}\,Ht)\rho(t)\exp(-\mathrm{i}\,Ht), \tag{13}$$

and obtain an equation for $W(t)$ by substituting Eqs.(11) and (12) in Eq.(7). The exact equation for $W(t)$ is found to be a sum of time-independent and time-dependent terms. The time-dependent terms in this equation oscillate at the frequency proportional to g. It can be shown that the contribution of the oscillatory terms is of the order of κ^2/g^2. Hence, if we make the secular approximation, i.e., if we neglect the oscillatory terms by assuming $\kappa \ll g$, then the equation for $W(t)$ is found to be

$$\begin{aligned} \dot{W}(t) =&\kappa[|0,\downarrow\rangle\langle\psi_0^+|W(t)|\psi_0^+\rangle\langle 0,\downarrow| \\ &+|0,\downarrow\rangle\langle\psi_0^-|W(t)|\psi_0^-\rangle\langle 0,\downarrow|] \\ &+\kappa/2\sum_{n=1}^{\infty}\{(\sqrt{n+1}+\sqrt{n})^2 \\ &\times[|\psi_{n-1}^+\rangle\langle\psi_n^+|W(t)|\psi_n^+\rangle\langle\psi_{n-1}^+| \\ &+|\psi_{n-1}^-\rangle\langle\psi_n^-|W(t)|\psi_n^-\rangle\langle\psi_{n-1}^-|] \\ &+(\sqrt{n+1}-\sqrt{n})^2 \\ &\times[|\psi_{n-1}^+\rangle\langle\psi_n^-|W(t)|\psi_n^-\rangle\langle\psi_{n-1}^+| \\ &+|\psi_{n-1}^-\rangle\langle\psi_n^+|W(t)|\psi_n^+\rangle\langle\psi_{n-1}^-|]\} \\ &-\kappa/2\sum_{n=0}^{\infty}(2n+1)[(|\psi_n^+\rangle\langle\psi_n^+| \\ &+|\psi_n^-\rangle\langle\psi_n^-|)W(t) \\ &+W(t)(|\psi_n^+\rangle\langle\psi_n^+| + |\psi_n^-\rangle\langle\psi_n^-|)]. \end{aligned} \tag{14}$$

We now solve Eq.(13) to find various matrix elements of $W(t)$. From Eq.(13) it easily follows that:

$$\begin{aligned} \langle\psi_m^+|W(t)|\psi_n^+\rangle =&\exp[-\kappa t(m+n+1)]\langle\psi_m^+|W(0)|\psi_n^+\rangle, \\ \langle\psi_m^-|W(t)|\psi_n^-\rangle =&\exp[-\kappa t(m+n+1)]\langle\psi_m^-|W(0)|\psi_n^-\rangle, \\ \langle\psi_m^+|W(t)|\psi_n^-\rangle =&\exp[-\kappa t(m+n+1)]\langle\psi_m^+|W(0)|\psi_n^-\rangle, \\ \langle\psi_m^-|W(t)|\psi_n^+\rangle =&\exp[-\kappa t(m+n+1)]\langle\psi_m^-|W(0)|\psi_n^+\rangle; \end{aligned} \tag{15}$$

and from Eq.(13) we know that

$$\begin{aligned} \rho(0) =&W(0), \\ \rho(t) =&\exp(-\mathrm{i}\,Ht)W(t)\exp(\mathrm{i}\,Ht). \end{aligned} \tag{16}$$

Using Eqs.(15) and (16), we can obtain the results

$$\begin{aligned} &\langle\psi_m^+|\rho(t)|\psi_m^+\rangle \\ =&\exp\{\mathrm{i}\,t[\omega(n-m)+g(\sqrt{n+1}+\sqrt{m+1})]\} \\ &\times\exp[-\kappa t(m+n+1)]\langle\psi_m^+|\rho(0)|\psi_m^+\rangle, \\ &\langle\psi_m^-|\rho(t)|\psi_m^-\rangle \\ =&\exp\{\mathrm{i}\,t[\omega(n-m)-g(\sqrt{n+1}+\sqrt{m+1})]\} \\ &\times\exp[-\kappa t(m+n+1)]\langle\psi_m^-|\rho(0)|\psi_m^-\rangle. \end{aligned} \tag{17}$$

It is known that the probability difference between two states $\{|0\rangle, |1\rangle\}$ of the number of Cooper-pairs can be written as

$$P(t) = \mathrm{Tr_F}(\rho_{11} - \rho_{00}), \tag{18}$$

where $\mathrm{Tr_F}$ is the trace of the single-mode quantized field.

In the basis $(|\downarrow\rangle, |\uparrow\rangle)$ we know that

$$\rho_{11} - \rho_{00} = \sigma_+ + \sigma_-. \tag{19}$$

Thus we can rewrite Eq.(18) by substituting Eq.(19),

$$P(t) = \sum_{\varepsilon,\varepsilon'=+,-}\sum_{m,n=0}^{\infty}\langle\psi_m^{\varepsilon}|\rho(t)|\psi_n^{\varepsilon'}\rangle\langle\psi_n^{\varepsilon'}|\sigma_+ + \sigma_-|\psi_m^{\varepsilon}\rangle, \quad (m \neq n). \tag{20}$$

Using Eq.(17), we solve Eq.(20) and obtain the result of $P(t)$,

$$\begin{aligned} P(t) =&\frac{\mathrm{i}}{2}\sum_{m=0}^{\infty}\{\exp[\mathrm{i}\,t(\omega+g\sqrt{m+1}+g\sqrt{m+2})] \\ &\times\exp[-2\kappa t(m+1)]\langle\psi_m^+|\rho(0)|\psi_{m+1}^+\rangle \\ &+\exp[\mathrm{i}\,t(-\omega+g\sqrt{m}+g\sqrt{m+1})] \\ &\times\exp[-2\kappa mt]\langle\psi_m^+|\rho(0)|\psi_{m-1}^+\rangle \\ &+\exp[\mathrm{i}\,t(\omega+g\sqrt{m+1}-g\sqrt{m+2})] \\ &\times\exp[-2\kappa t(m+1)]\langle\psi_m^+|\rho(0)|\psi_{m+1}^-\rangle \\ &-\exp[\mathrm{i}\,t(-\omega-g\sqrt{m}+g\sqrt{m+1})] \\ &\times\exp[-2\kappa mt]\langle\psi_m^+|\rho(0)|\psi_{m-1}^-\rangle \\ &-\exp[\mathrm{i}\,t(\omega-g\sqrt{m+1}+g\sqrt{m+2})] \\ &\times\exp[-2\kappa t(m+1)]\langle\psi_m^-|\rho(0)|\psi_{m+1}^+\rangle \\ &+\exp[\mathrm{i}\,t(-\omega+g\sqrt{m}-g\sqrt{m+1})] \\ &\times\exp[-2\kappa mt]\langle\psi_m^-|\rho(0)|\psi_{m-1}^+\rangle \\ &-\exp[\mathrm{i}\,t(\omega-g\sqrt{m+1}-g\sqrt{m+2})] \\ &\times\exp[-2\kappa t(m+1)]\langle\psi_m^-|\rho(0)|\psi_{m+1}^-\rangle \\ &-\exp[\mathrm{i}\,t(-\omega-g\sqrt{m}-g\sqrt{m+1})] \\ &\times\exp[-2\kappa mt]\langle\psi_m^-|\rho(0)|\psi_{m-1}^-\rangle\}. \end{aligned} \tag{21}$$

In this case we assume that the system is initially in the ground state $|\downarrow\rangle$, and the field is in the coherent state $|\alpha\rangle$, then we can write

$$\rho(0) = |\alpha\rangle\langle\alpha| \otimes |\downarrow\rangle\langle\downarrow|, \tag{22}$$

where

$$|\alpha\rangle\langle\alpha| = \exp(-|\alpha|^2) \sum_{m,n=0}^{\infty} \frac{\alpha^{m+n}}{\sqrt{m!n!}} |n\rangle\langle m|, \quad (m \neq n). \tag{23}$$

Using Eqs.(22) and (23) we can have the result from Eq.(21),

$$\begin{aligned} P(t) =& \exp(-|\alpha|^2) \sum_{m=0}^{\infty} \{ \mathrm{i} \exp[-2\kappa t(m+1)] \\ & \times \frac{\alpha^{2m+3}}{\sqrt{(m+1)!(m+2)!}} \\ & \times \sin\omega t \cos gt\sqrt{m+1} \cdot \sin gt\sqrt{m+2} \\ & - \mathrm{i} \exp(-2\kappa m t) \cdot \frac{\alpha^{2m+1}}{\sqrt{(m+1)!m!}} \\ & \times \sin\omega t \cos gt\sqrt{m} \cdot \sin gt\sqrt{m+1} \\ & + \exp[-2\kappa t(m+1)] \cdot \frac{\alpha^{2m+3}}{\sqrt{(m+1)!(m+2)!}} \\ & \times \cos\omega t \cos gt\sqrt{m+1} \cdot \sin gt\sqrt{m+2} \\ & + \exp(-2\kappa m t) \cdot \frac{\alpha^{2m+1}}{\sqrt{(m+1)!m!}} \\ & \times \cos\omega t \cos gt\sqrt{m} \cdot \sin gt\sqrt{m+1} \}. \end{aligned} \tag{24}$$

3. Results and discussion

We now use Eq.(24) to study the quantum dynamics of this system. Using computer simulation we draw a series of figures (Figs.1–4) to discuss various phenomena under different conditions. We have also checked up the accuracy of the calculation by drawing the figure of the sum of the imaginary part, and find that the result is keeping zero.

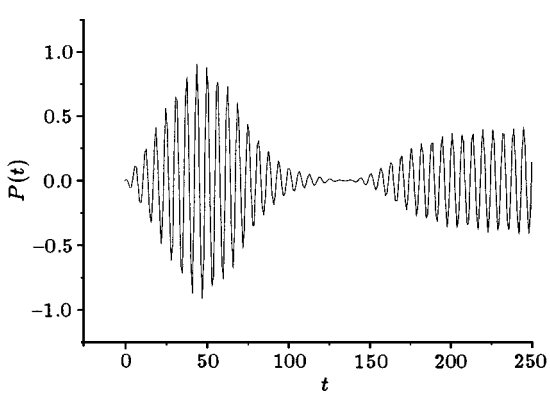


Fig.1. The time evolution of $P(t)$ for the system initially in the ground state, and the single-mode quantized field being in a coherent state with an intensity $\overline{n}$ equal to 3.1 for κ=0, ω=1, g=0.01.

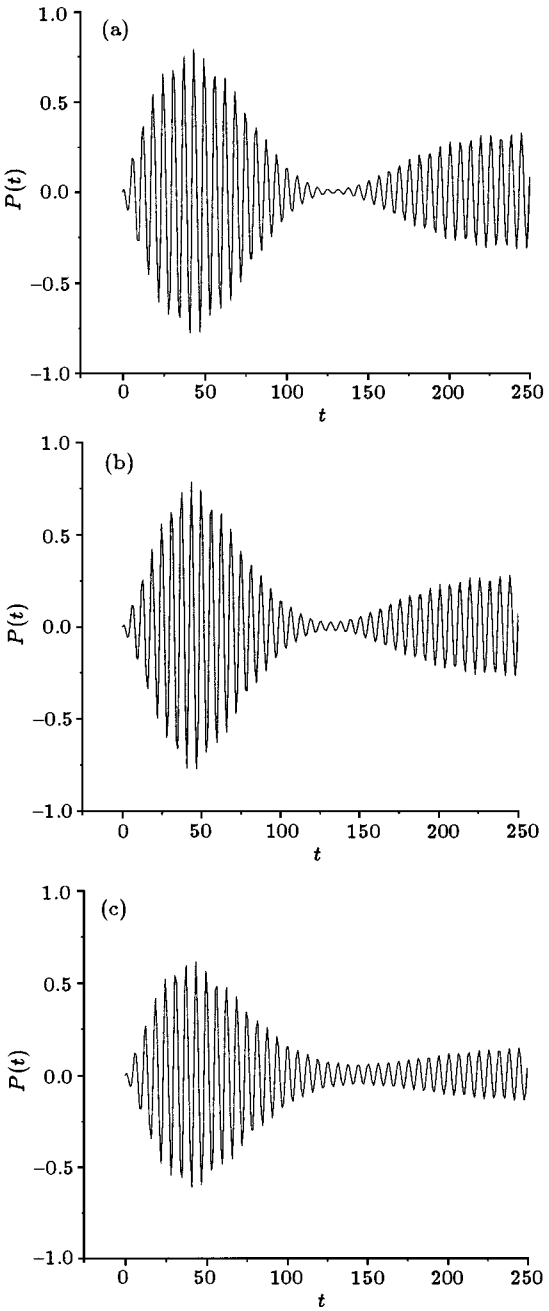


Fig.2. The time evolution of $P(t)$ for the system initially in the ground state, and the single-mode quantized field being in a coherent state with an intensity $\overline{n}$ equal to 3.1, ω=1, g=0.01 for (a) κ=0.0007; (b) κ=0.001; (c) κ=0.003.

In the figures we plot $P(t)$ versus time t, and in all the figures we define ω=1, g=0.01, and $\overline{n}$ is the intensity of the single-mode quantized field. First, we discuss the possibility of the collapse-revival phenomena and the general influence induced by the damping rate κ. In Fig.1 we consider the situation with κ=0 and $\overline{n}$=3.1. We can easily see from the curve that the system exhibits collapse-revival phenomena, and we can also find a long period for $P(t)$=0. In Fig.2 we keep the other condition unchanged, compared with Fig.1,

and choose three different damping rates κ. Through various curves in Fig.2 we can see clearly that, under the condition $\kappa \neq 0$, at first there is a small oscillation in the quiescent period, as shown in Fig.2(a); if the value of κ becomes bigger, the amplitude of oscillation will become more obvious. Secondly, we can also see that the higher the damping rate κ, the smaller the amplitude of the oscillations in the revivals.

In Figs.3 and 4 we go into a deep discussion about the relation between intensity $\overline{n}$ and the damping rate κ. In Fig.3 we discuss the cases with $\overline{n}$ near 3.1, and choose $\overline{n}$ equal to 3.2. After comparing the two cases in the Fig.2 (a) and Fig.3, which differ only in the damping rate κ, we find that in the case with a cavity damping (κ=0), collapse-revival phenomena can also be observed if the $\overline{n}$ changes according to κ. Furthermore, through many cases we have simulated, we find that in order to show the collapse-revival phenomena, increasing κ results in an increase of $\overline{n}$. But, when the value of κ is too large and goes beyond some limit, no collapse-revival phenomena can be observed. In Fig.4 we discuss the condition that $\overline{n}$ is large enough. We can find the continuous beats phenomena, and also find that the higher the κ, the smaller the amplitude of the beats. It is obvious that if κ is too large, no such beat can be detected.

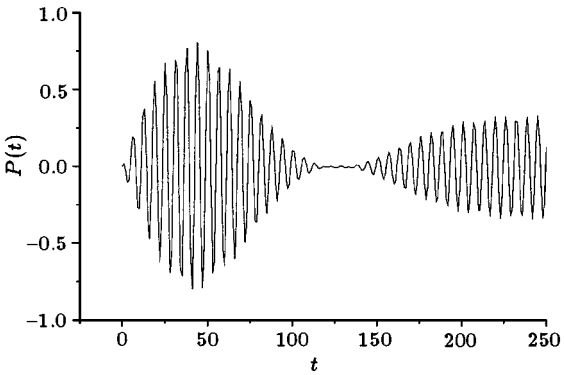


Fig.3. The time evolution of $P(t)$ for the system initially in the ground state, and the single-mode quantized field being in a coherent state for κ=0.0007, $\overline{n}$=3.2, ω=1, g=0.01.

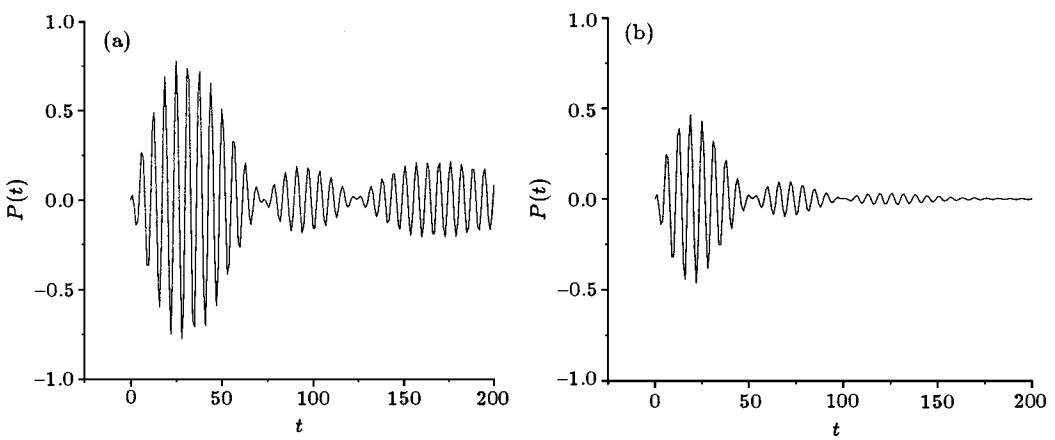


Fig.4. The time evolution of $P(t)$ for the system initially in the ground state, and the single-mode quantized field being in a coherent state; (a) κ=0.0006, $\overline{n}$=6, ω=1, g=0.01; (b) κ=0.0012, $\overline{n}$=12, ω=1, g=0.01.

In view of the recent progress towards field state generation in micromaser,[16] and the progress in the study of interaction between junctions and microwave,[17,18] the eventual experimental verification of collapse and revival phenomena discussed in this paper does not appear to be impossible, although it would certainly not be easy to achieve. In conclusion, from this model we could learn about the influence of a quantized radiation field on dynamical behaviour at Josephson junction. We hope that our work may stimulate further experimental and theoretical work in this direction.

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