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Stochastic perturbation in quasi-ideal dispersion-managed soliton system*

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The model of stochastic perturbation is built up systematically in quasi-ideal dispersion-managed soliton system, its influence on soliton propagation is investigated by both the variational approach and the numerical simulation, and it is found that the stochastic perturbation leads to disintegration of soliton and enhances the interaction between solitons. The nonlinear gain and filter are introduced to suppress effectively the influence on both soliton propagation and interaction.

Keywords: stochastic perturbation, disintegration of soliton, interaction, nonlinear gain
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1. Introduction

Optical solitons in fibres are formed by a balance of group dispersion and Kerr non-linearity. Numerical simulation and experiments have demonstrated that solitons can propagate an extended distance without distortion, so they may become the ideal message carrier in long distance communication.[1] Periodical optical amplifiers are needed to compensate the loss in transmission line in which optical soliton pulse is transmitting. The mean times of the appearance of fluctuation in amplitude leads to instability of coherently amplified solitons in optical fibre transmission, and limits the capacity of long distance communication.[2]

It has been proved that the stochastic perturbations cannot be neglected in soliton communication, and they have some bad influences on soliton transmission system, such as increasing the time jitters in arrival and reducing the communication capacity of soliton system.[3−5]

Recently, soliton propagation in dispersion-managed system is considered most attractive for long-haul optical communication, where dispersion-managed soliton is a new type of optical soliton. For dispersion-managed solitons, such effects as Gorden–Haus jitter, emission of radiation, and phase-match four-wave mixing are strongly reduced.[6] The main advantage of dispersion management is that it enables a transmission line to be easily constructed even conventional fibres are already installed, the use of dispersion-managed solitons has resulted in significant transmission advantages relative to standard solitons or non-return-to-zero formats in single-channel high-data-rate optical communication system, and for dispersion-managed systems distance up to 14000 km at 20 Gb/s and 24500 km at 10 Gb/s have been achieved.[7]

But stochastic perturbation in dispersion-managed system does harm to soliton propagation for long-haul optical communication. For example, stochastic perturbation caused by stochastic dispersion, lump gain or distributing loss leads to disintegration of soliton, and degrades the stability of soliton propagation.[8,9] In this work, the model of stochastic perturbation is built up in quasi-ideal dispersion-managed soliton system, its influence is investigated by the variational approach and the numerical simulation, the control for soliton propagation is introduced, some novel conclusions are obtained. Our results may be used in designing system parameters of the long-haul dispersion-managed soliton system, such as period length and strength of dispersion map and amplification compensating fibre loss.

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2. Model of stochastic perturbation

In a real transmission system, the envelope of field becomes inhomogeneous because of possible variations in the fibre dispersion, the fibre loss and amplification, so the amplitude q of the light-wave envelope in a fibre with periodic dispersion map $d(Z)$, loss Γ compensated by lumped amplifiers placed at $Z = nZ_{\rm a}(n = 0, 1, 2, \cdots)$ satisfies[10]

$$\begin{aligned} &\mathrm{i}\frac{\partial q}{\partial Z} + \frac{d(Z)}{2}\frac{\partial^2 q}{\partial t^2} + |q|^2 q \\ &= -\mathrm{i}\Gamma q + \mathrm{i}\,\alpha \sum_{n=0}^{N} \delta(Z - nZ_{\rm a}) q(t, nZ_{\rm a}), \end{aligned} \tag{1}$$

where $Z_{\rm a}$ is the amplifier period, α is gain coefficient of amplifiers, $\alpha = \exp(\Gamma Z_{\rm a}) - 1$, and Eq.(1) can be transformed to a Hamiltonian form by introducing a new amplitude u through

$$u = \frac{q}{a(Z)}. \tag{2}$$

Then we can obtain

$$\mathrm{i}\frac{\partial u}{\partial \xi} + \frac{1}{2}\frac{\partial^2 u}{\partial t^2} + \frac{a^2(Z)}{d(Z)}|u|^2 u = 0, \tag{3}$$

where the new coordination (propagation distance) ξ is defined as $\xi = \int_0^{\xi} d(Z)\mathrm{d}Z$, and $a(Z)$ is given by

$$\begin{aligned} a(Z) = [2\Gamma Z_{\rm a}/(1 - \mathrm{e}^{-2\Gamma Z_{\rm a}})]^{1/2} \exp[-\Gamma(Z - nZ_{\rm a})], \\ nZ_{\rm a} < Z < (n+1)Z_{\rm a}. \end{aligned} \tag{4}$$

Equation (3) can be rewritten as

$$\mathrm{i}\frac{\partial u}{\partial \xi} + \frac{1}{2}\frac{\partial^2 u}{\partial t^2} + |u|^2 u = \left[1 - \frac{a^2(Z)}{d(Z)}\right]|u|^2 u. \tag{5}$$

When $a^2(Z)/d(Z)$ equals to 1 (so-called ideal dispersion-managed soliton system, we always hope to design the ideal system by optimizing system parameters, such as distributed loss, gain of lumped amplifiers and the periodic dispersion map), all ideal soliton properties are recovered even in a system with lumped amplifiers and distributed loss. But, in fact, $a^2(Z)/d(Z)$ relates to distributed loss, gain of lumped amplifiers and the periodic dispersion map, and becomes the function of propagation distance ξ. For example $a^2(Z)/d(Z)$ may be the stepwise or exponential function of propagation distance in many dispersion-managed schemes.

When $a^2(Z)/d(Z)$ equals to 1, it is obvious that we can obtain exact soliton solution to Eq.(5). In fact, $a^2(Z)/d(Z)$ changes around 1 in a quasi-ideal dispersion-managed soliton system, so $[1 - a^2(Z)/d(Z)]$ is modelled reasonably as randomly varying fluctuation along propagation line. Under the model of stochastic perturbation, Eq.(3) can be rewritten as

$$\begin{aligned} &\mathrm{i}\frac{\partial u}{\partial \xi} + \frac{1}{2}\frac{\partial^2 u}{\partial t^2} + |u|^2 u = \mathrm{i}\,R(\xi). \\ &R(\xi) = -\mathrm{i}\,\sigma(\xi)|u|^2 u, \end{aligned} \tag{6}$$

where $R(\xi)$ is weak perturbation, and stochastic variable $\sigma(\xi)$ satisfies

$$\begin{aligned} &< \sigma(\xi) >= 0, \\ &< \sigma(\xi)\sigma(\xi') >= D\delta(\xi - \xi'), \end{aligned} \tag{7}$$

where D is the stochastic strength.

3. Effects of stochastic perturbation on soliton propagation

Lagrangian density of Eq.(5) can be defined as usual variational approach

$$\begin{aligned} L(\xi, t) =& \frac{\mathrm{i}}{2}u^*\frac{\partial u}{\partial \xi} + \frac{\mathrm{i}}{2}u^*\frac{\partial u}{\partial t} \\ &+ \frac{[1 - \sigma(\xi)]}{4}|u|^4 - \frac{1}{4}\left|\frac{\partial u}{\partial t}\right|^2. \end{aligned} \tag{8}$$

Then we adopt as trial function

$$u(\xi, t) = \sqrt{A}\exp\left[-(1 + \mathrm{i}b)\frac{t^2}{2W^2} + \mathrm{i}\phi\right], \tag{9}$$

where A, W, b and ϕ are the intensity, width, chirp coefficient and phase of pulse, respectively, which are all functions of propagation distance ξ.

Averaged Lagrangian is defined as

$$\overline{L} = \int_{-\infty}^{\infty} L(\xi, \tau)\mathrm{d}\tau. \tag{10}$$

Substitution of Eqs.(8) and (9) into Eq.(10) gives

$$\begin{aligned} \overline{L} =& E\Bigg\{ -\left(-\frac{1}{4}\frac{\mathrm{d}b}{\mathrm{d}Z} + \frac{b}{2W}\frac{\mathrm{d}W}{\mathrm{d}Z}\right) \\ &- \frac{1}{2}\left(\frac{1 + b^2}{2W^2}\right) + \frac{[1 - \sigma(\xi)]E}{2\sqrt{2\pi}W}\Bigg\}, \end{aligned} \tag{11}$$

where $E = \int_{-\infty}^{\infty} |u|^2 \mathrm{d}t = \sqrt{\pi}AW$ is constant.

Varying Eq.(11) leads to the equations

$$\begin{aligned} W\frac{\mathrm{d}W}{\mathrm{d}\xi} &= -b, \\ W\frac{\mathrm{d}W}{\mathrm{d}\xi} &= -(1 + b^2) + \frac{[1 - \sigma(\xi)]E}{\sqrt{2\pi}}W. \end{aligned} \tag{12}$$

A rough estimation can be made below. When the stochastic perturbation is zero, there are stable pulse-width W_s in propagation line. So we assume $W = W_s + \Delta W$, and

$$W_s \frac{d(\Delta W)}{d\xi} = -b,$$

$$\frac{db}{d\xi} = -\frac{b^2}{W_s^2} - \frac{\sigma(\xi)E}{\sqrt{2W}W_s}. \tag{13}$$

When the chirp is small, with the initial condition we can obtain

$$\Delta W(\xi) = \int_0^{\xi} \frac{\sigma(x)E}{\sqrt{2\pi}W_s^2} dx. \tag{14}$$

The fluctuation of pulse-width is given by

$$[\Delta W(\xi)]^2 = \frac{1}{2\pi}\frac{DE^2}{W_s^4}\xi. \tag{15}$$

We can see the even square fluctuation of pulse-width is approximately proportional to the propagation distance when stochastic perturbation or the chirp is small, and it may lead to disintegration of soliton.

As usual, an average of a few different random sequences of stochastic perturbation has been used in the following numerical simulations. Fig.1 shows the even square fluctuation of pulse-width versus the propagation distance ξ. The smooth line is the result from Eq.(15), and the zigzag line is the result of numerical simulation. Initially, the input soliton-like pulse is $\exp\left(-\frac{t^2}{2}\right)$, and the stochastic strength is D=0.5. In quasi-ideal dispersion-managed soliton system, we can see that the fluctuation of pulse-width from numerical simulation is close to the results calculated by Eq.(15), so we can make a rough estimation about soliton pulse-width changing with the propagation distance. Large fluctuation in soliton pulse-width will

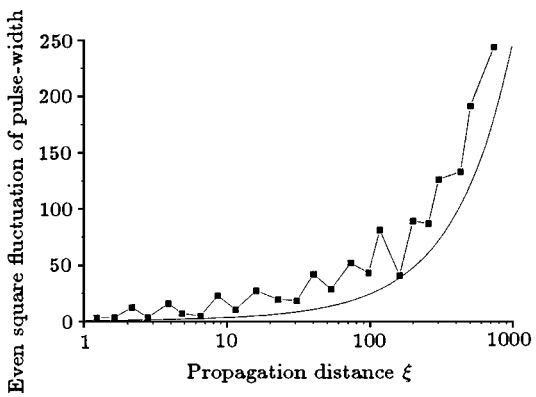


Fig.1. Even square fluctuation of pulse-width versus the propagation distance ξ. The smooth line is the result from Eq.(15), and the zigzag line is the result of numerical simulation.

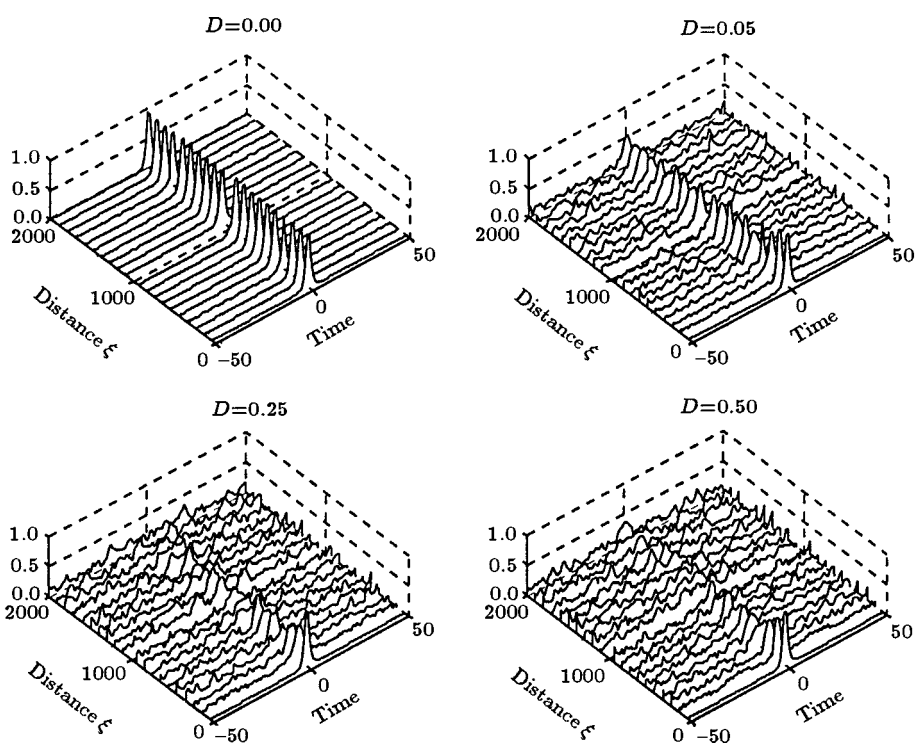


Fig.2. Normalized soliton intensity versus the propagation distance ξ. The stochastic strengths are D=0, 0.05, 0.25 and 0.5, respectively.

lead to the disintegration of soliton. Fig.2 shows the normalized soliton intensity versus the propagation distance ξ, the stochastic strengths being D=0, 0.05, 0.25 and 0.5, respectively. Figure 3 demonstrates the length of disintegration versus the stochastic strength, the initially input soliton pulse is sech(t). We can see stochastic perturbation leads to disintegration of soliton, and the length of disintegration relates to the stochastic strength. The larger the stochastic strength is, the shorter the length of disintegration is.

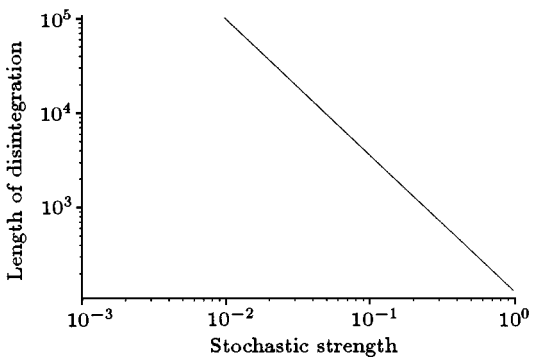


Fig.3. Length of disintegration versus the stochastic strength.

4. Analysis of soliton interaction with stochastic perturbation

Interaction between solitons may induce the crosstalk between two communication channels and the time jitter, thus increasing the bit-error-rate of the transmission system. When initial separation between solitons is larger than five times of soliton width in normal soliton transmission system, interaction between solitons can be suppressed effectively.[1] But in quasi-ideal dispersion-managed soliton system with stochastic perturbation, the situation is very distinguished.

Figure 4 shows the normalized intensity of two solitons versus the propagation distance ξ, the stochastic strengths being D=0, 0.05, 0.25 and 0.5, respectively. Figure 5 demonstrates the length of interaction versus the stochastic strength. Initially, input soliton pulse is sech($t - \Delta/2$)+sech($t + \Delta/2$), and Δ=20 (about 12 times of soliton width). The length of interaction is defined as the distance within which we can identify two solitons. In quasi-ideal dispersion-managed soliton system, we can see that stochastic perturbation enhances the interaction between solitons even if the separation between solitons is large enough (for example, when the separation is far greater than five times of pulse-width), and the length of interaction relates to both the strength of stochastic perturbation and the separation between solitons.

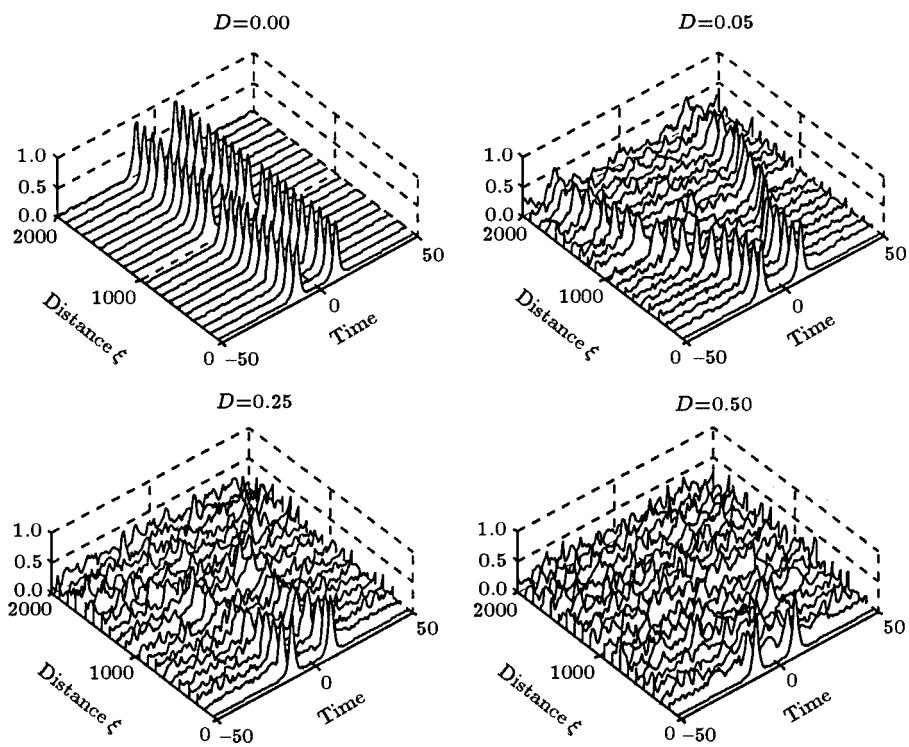


Fig.4. Normalized intensity of two solitons versus the propagation distance ξ. The separation is Δ=20, and the stochastic strengths are D=0, 0.05, 0.25 and 0.5, respectively.

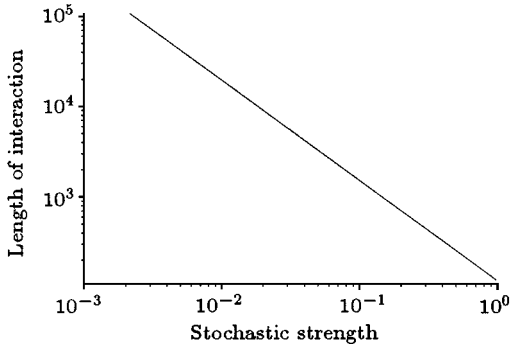


Fig.5. Length of interaction versus the stochastic strength. The separation is Δ=20.

5. Control of stochastic perturbation

In order to stabilize the soliton propagation, a nonlinear gain combining with filter is introduced to suppress the influence of stochastic perturbation. When propagating in the fibre communication system exhibiting the stochastic perturbation, with filter and the nonlinear gain, the soliton pulses can be described by the modified nonlinear Schrödinger equation

$$\mathrm{i}\frac{\partial u}{\partial \xi}+\frac{1}{2}\frac{\partial^2 u}{\partial t^2}+|u|^2u$$
$$=\sigma(\xi)|u|^2u+\mathrm{i}\gamma_1 u+\mathrm{i}\gamma_2|u|^2u+\mathrm{i}\beta\frac{\partial^2 u}{\partial t^2}+\mathrm{i}\gamma_3|u|^4u, \quad (16)$$

where γ_1, γ_2 and γ_3 are the coefficients of the linear and the nonlinear gain, respectively, β is the coefficient of filter. These parameters are chosen based on the condition[11,12]

$$\gamma_1<0, \quad \gamma_3<0, \quad \beta<2\gamma_2,$$
$$15\gamma_1-5(\beta-2\gamma_2)+8\gamma_3=0. \quad (17)$$

Figure 6 demonstrates the normalized soliton intensity and the normalized intensity of two solitons versus the propagation distance ξ, and with controllable nonlinear gain and filter. The coefficients of the linear gain, the nonlinear gain and the filter are $\gamma_1=-0.05$, γ_2=0.5, γ_3=–0.3432, respectively, and β=0.30, the stochastic strengths are D=0.25 and 0.5, the separation of two solitons is Δ=20. We can see that all ideal soliton properties are recovered as if the stochastic perturbation does not exist when there is control of nonlinear gain and filter in the transmission line. In other words, it shows that the use of the nonlinear gain combining with filter can effectively suppress the effect of stochastic perturbation on soliton propagation and soliton interaction between solitons even if the propagation distance is long enough.

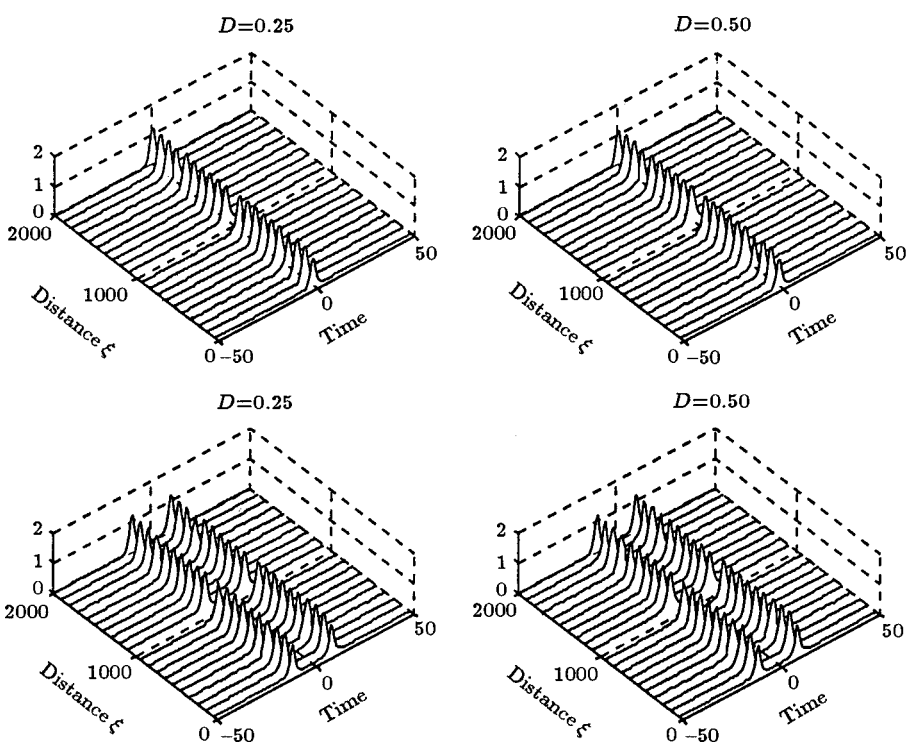


Fig.6. Normalized soliton intensity and normalized intensity of two solitons versus the propagation distance ξ, and with controllable nonlinear gain and filter. The stochastic strengths are D=0.25 and 0.5, respectively, and the separation of two solitons is Δ=20.

6. Conclusion

The model of stochastic perturbation in quasi-ideal dispersion-managed soliton system is built up, the influence of stochastic perturbation on soliton propagation is investigated by the variational approach and the numerical simulation. The comparing results of the numerical simulation with that of variational approach are given, it is found that the even square fluctuation of pulse-width is approximately proportional to the propagation distance, and the result of the numerical simulation is in agreement with that of variational approach. The stochastic perturbation leads to disintegration of soliton, and the length of integration relates to its strength. It is also found that stochastic perturbation enhances the interaction between solitons even if the separation between solitons is large enough (for example, larger than five times of pulse-width), and the length of interaction relates to its strength and the separation between solitons. The nonlinear gain combined with filter can suppress effectively the influence of the stochastic perturbation.

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