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Bifurcation, chaotic phenomena and control of chaos in a one-dimensional discrete Josephson lattice*

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We have investigated the fluxon dynamical behaviour in a one-dimensional parallel array of small Josephson junctions in the presence of an externally applied magnetic field. In the case of high damping, the system is in stable states. On the contrary, in the case of low damping, bifurcation and chaotic phenomena have been observed. Control of chaos is achieved by a delayed feedback mechanism, which drives the chaotic system into a selected unstable periodic orbit embedded within the associated strange attractor. It is attractive to control chaos to a periodic state, rather than operating always outside the device parameter space where chaos dominates.

Keywords: Josephson junction, chaos, bifurcation, controlling chaos
PACC: 7450, 0545, 0547

1. Introduction

Josephson lattices are the basis of superconducting electronics. Applications of discrete Josephson lattice as high-frequency oscillators, voltage standards, digital-to-analogue converters (DAC),[1] phase-mode logic[2] and rapid single flux quantum (RSFQ) circuits for the high-frequency signal output[3] are widely studied. Voltage pulses generated by fluxons motion serve as bits of information in RSFQ circuits. Due to its very high computing speed, the RSFQ logic can also be very promising in combination with optical communication.

Josephson lattices are successfully modelled by using the perturbed sine-Gordon system, which has been studied to understand the magnetic fluxons associated behaviours in long Josephson lattices coupled to external magnetic fields, such as the phase-locking phenomena in the oscillating rf magnetic field and the so-called Fiske step in the dc current-voltage characteristics in the dc magnetic field case.[4−8] In spite of this, there is still a need to acquire a more detailed understanding of the dynamical behaviours about perturbed sine-Gordon systems. A systematic study provides one the knowledge about the stability of many physical objects in one-dimensional (1D) problem, such as charge-density waves, ferromagnetic or antiferromagnetic system, dislocations in solids described by perturbed sine-Gordon system as well as Josephson devices, which is crucial to set a right bias to the systems.

The discrete Josephson lattice circuit could be either in periodical state or chaotic state, depending on the parameters used. The question is whether the chaotic state can be controlled or driven to a stable state. Starting with the pioneering work of Ott *et al.*,[9] various chaos control methods have been proposed[10−12] and applied to physical and nonlinear dynamical systems.[13−15] We will show that the feedback method similar to that in Ref.[9] can control the chaotic behaviour of vortex fluxons, where the feedback is applied as a perturbation to an accessible system variable in such a way that when the system behaves in a desired period manner, the feedback tends to zero.[16]

2. Model

1D parallel biased arrays of small Josephson junctions represent an experimental realization of the spatially discrete sine-Gordon lattice (also known as the Frenkel–Kontorova model). A 1D parallel array of

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Josephson junctions can be described by the discrete version of the perturbed sine-Gordon equation

$$\frac{\mathrm{d}^2\varphi_n}{\mathrm{d}t^2} = \gamma - \alpha\frac{\mathrm{d}\varphi_n}{\mathrm{d}t} - \sin\varphi_n + \frac{1}{a^2}(\varphi_{n-1} - 2\varphi_n + \varphi_{n+1}). \quad (1)$$

We use the resistively and capacitively shunted Josephson junction model for a single cell of the lattice, where each small junction has a critical current I_c, a resistance R and a capacitance C. φ_n is the superconducting phase difference on the nth junction (n=1,2,..., N, with N the number of junctions). The damping, α, is the inverse square root of the Stewart–McCumber parameter. $a=\beta_L^{1/2}=(2\pi LI_c/\Phi_0)^{1/2}$ is the discreteness parameter, where L is the inductance of a single cell and Φ_0 is the magnetic flux quantum. For convenience, dimensionless units have been used; that is, voltage is normalized to I_cR, bias current is normalized to I_c, time is normalized to the inverse plasma frequency $\omega_0^{-1}=(\Phi_0C/2\pi I_c)^{1/2}$ and magnetic flux is normalized to Φ_0. The equivalent circuit of the 1D Josephson array is shown in Fig.1.

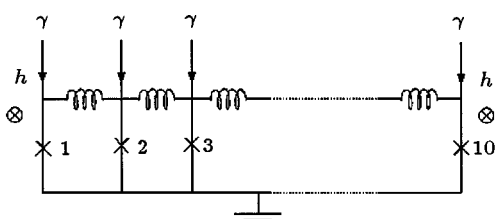


Fig.1. Equivalent circuit of the 1D array of Josephson junctions (N=10).

The static magnetic field boundary conditions for Eq.(1) are

$$\frac{\varphi_2 - \varphi_1}{a} = \frac{\varphi_N - \varphi_{N-1}}{a} = h, \quad (2)$$

where h is the externally applied magnetic field.

3. Numerical simulations

We performed numerical simulations of Eqs.(1) and (2) for an array consisting of 10 cells (N=10) and with a fixed discreteness parameter a=1 to study how the dynamical behaviour of Josephson array changes with the damping α and external current γ. The integration was made using a fourth-order Runge–Kutta scheme with a time step equal to 0.02. The current γ was firstly swept up and then taken down with a step of $\Delta\gamma$=0.0001. For each new bias current, the initial conditions were taken from the steady state achieved in the previous point. In order to eliminate the transient effect due to the change in γ, the first 3000 iterations were dropped at each new γ value.

Consider a high damping case, α=0.2, for instance. Figure 2 shows the I-V curves of the array with h=3.0 and h=4.5. Resonant steps were indicated, which implied the periodical states of the Josephson array. In Fig.3, we plot the trajectory of the 4th junction in the phase space and in time domain with bias current γ=0.5. Clearly, the stable period orbits is indicated. Of course, n, representing the nth junction, can be chosen arbitrarily from 1 to 10, while the physical characteristic will remain the same as that of the 4th junction.

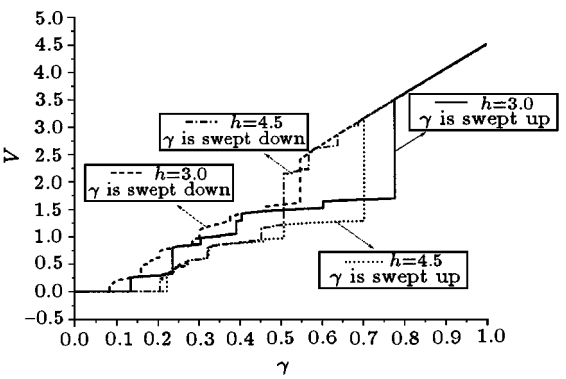


Fig.2. Numerically simulated I-V curves for high damping (α=0.2), showing the resonant flux-flow steps in the 10-cell array. Vertical axis, V, represents the average voltage of the 10 Josephson junctions over 6000 normalized time units.

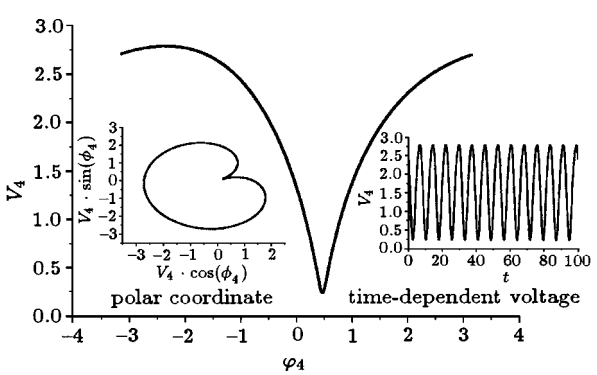


Fig.3. Phase portrait (V-φ) of high damping (α=0.2, h=3.0). We investigated the 4th junction for γ=0.5. The insets are the trajectory of the 4th junction in the phase space (left) and in the time domain (right). All these show the period-1 orbit for high damping.

Obviously, in the case of high damping, the unstable system will settle into a stable orbit over a long time evolution because of the high dissipation coefficient. But in the case of low damping, we predict that the system will be in an unstable state.

Indeed, for sufficiently low damping α=0.08, the situation is quite different. We find that the sharp resonant steps disappear in the whole bias current range, and unstable phenomena emerge instead (see Fig.4). We will focus on the study of the time-space dependent voltage over the current region γ=(0.1336, 0.2325), where chaotic dynamics exists.

The following study will be the representation of Poincaré plots. Since the junction phase enters Eq.(1) only as $\sin\varphi_n$, if φ_n is a solution of Eq.(1), $\varphi_n \pm k(2\pi)$ will also be solutions for an arbitrary integer k. Thus the phase difference φ_n is taken as mod $(\varphi_n, 2\pi)$ and can be restricted to the range from $-\pi$ to π without losing generality.[17] Because time-variable does not enter the equation explicitly, Eq.(1) represents an autonomous system. Given a bias current γ, we sample V_n of a trajectory that intersects with the Poincaré section (plane) which is defined as a constant phase difference φ_n plane in our case. The selection of this reference value is entirely arbitrary, although the visual appearance of the resulting Poincaré orbit, i.e. the sampled V_n points, depends upon the choice made. Unless specified otherwise, a reference phase of $\pi/2$ is used. Since the dynamical system has now been converted into 1D system, period-doubling bifurcation is expected as γ varies.

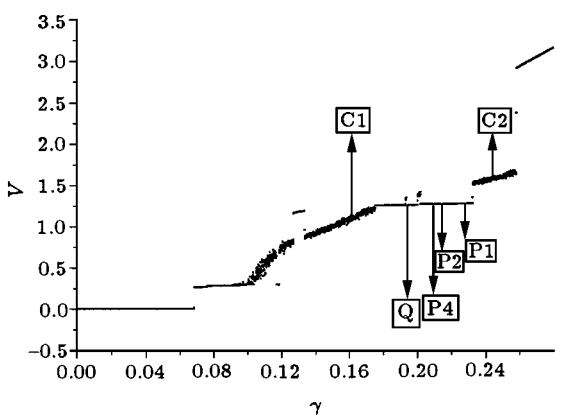


Fig.4. Numerically simulated I-V curve (γ is swept down) for low damping (α=0.08, h=3.0), showing the chaotic dynamics in the 10-cell array. V-axis represents the average voltage of the 10 Josephson junctions over 6000 normalized time units.

Junction voltage of the 4th junction, V_4 versus γ plot is illustrated in Fig.5 and Fig.6. For instance, see Fig.5, in the range of γ=(0.2020, 0.2325), as the bias current is slowly decreased, a period-doubling bifurcation leading to chaos occurs. Period-1 becomes period-2 when γ is equal to 0.2252. Further decrease of γ leads to period-2 becoming period-4 when γ is equal to 0.2119, etc. And finally, the system enters into a chaotic state. Similarly, bifurcations from chaos to period have been observed as bias current is slowly increased as shown in Fig.6. We can also find quasi-period in several different bias current regions, such as γ=(0.1937, 0.1968) in Fig.5 and γ=(0.1809, 0.1829) in Fig.6.

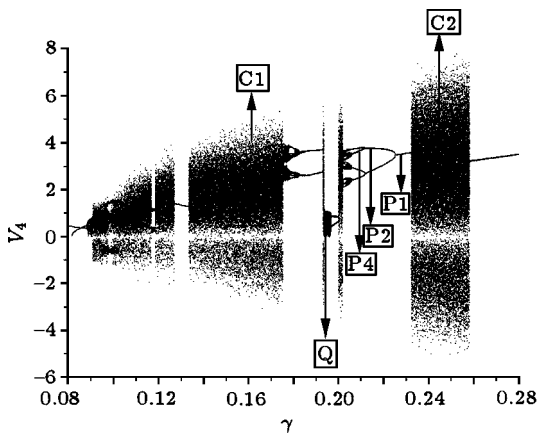


Fig.5. Bifurcation diagram of the 4th Josephson junction (γ is swept down). Poincaré section for a reference phase is $\varphi_4 = \pi/2$ (h=3.0, α=0.08).

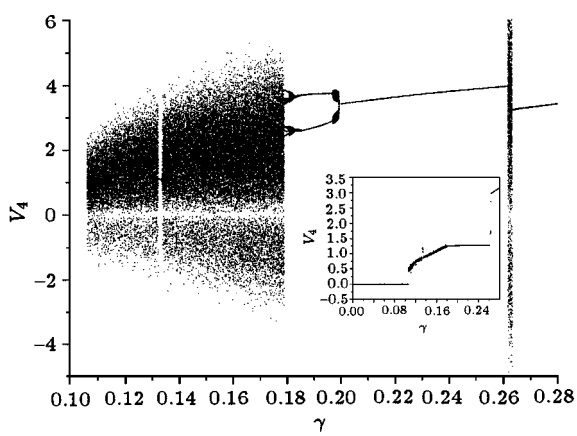


Fig.6. Same as Fig.5, but γ is swept up. The inset is I-V curve.

Time-dependent voltage of the 4th junction at different bias points is shown in Fig.7. Our numerical simulations indicate that the chaotic behaviour, besides the phase-locking phenomenon, of the soliton motion in the perturbed sine-Gordon system is a common feature, in particular, for the case of long Josephson lattice with a relatively low damping.

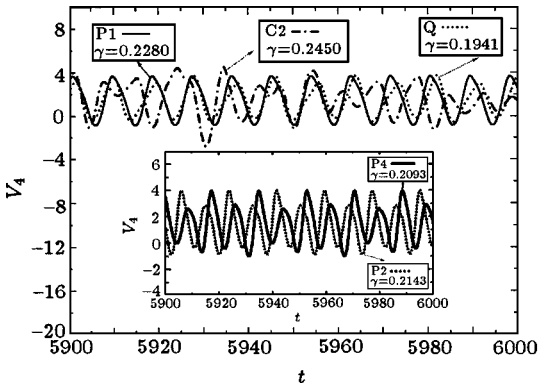


Fig.7. Time-dependent voltage of the 4th junction over the last 100 normalized time units at different bias current points shown in Figs.4 and 5. Period-1 (P1), quasi-period (Q) and chaos (C2). The inset reveals period-2 (P2) and period-4 (P4) (h=3.0, α=0.08).

As lower damping in this kind of arrays is synonymous with higher frequencies of operation for resonant mode and flux flow microwave oscillators, it is attractive to control chaos to a periodic state, rather than operating always outside the device parameter space where chaos dominates.

In order to control chaos, we apply the delayed feedback control strategy[16,18−20] to the array by adding the following feedback function $F(t)$ to the right-hand side of Eq.(1), for index n=4,

$$F(t) = KD(t) = K[V_n(t-\tau) - V_n(t)], \qquad (3)$$

where K is the perturbation weight and τ is the feedback delay time. It is clear that the control term vanishes when a τ-periodic solution is stabilized, i.e. $V_n(t-\tau) - V_n(t) \to 0$. This implies that the stabilized solution is a τ-periodic solution of system (1). It should be noted that since there exist infinite unstable periodic orbits (UPOs) embedded in a chaotic attractor, a prior knowledge about the period of the desired periodical orbit is useful. In our investigation, τ was swept up from zero with a step of $\Delta\tau$=0.1 and the first periodic solution with a minimum period was locked. The electronics for this feedback into the bias current can be realized by today's high-speed semiconductor technology.

As a candidate for control, we choose to control the chaotic state occurring at (point C1) γ=0.1616 in Fig.4 with K=0.4. A right delay time can be chosen as follows. We scan the feedback delay time and measure the variance of the feedback, $< D^2(t) >$, $D(t) = V_4(t-\tau) - V_4(t)$. As stated, a $< D^2(t) >$ zero, implies that the system have been stabilized on to a desired UPO embedded in the chaotic attractor. We plotted these results in Fig.8, which clearly shows the first drop in the feedback by four orders of magnitude at a delay time of τ=9.10.

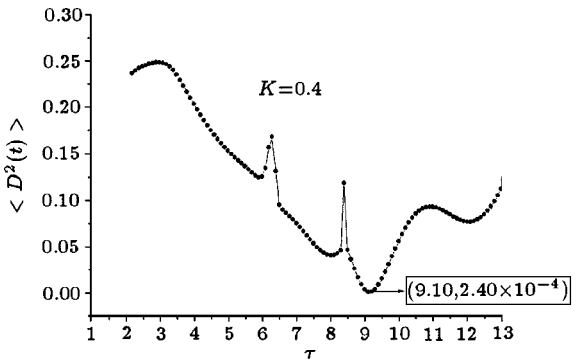


Fig.8. Variation of feedback with delay time, showing the stabilization of UPO at τ=9.10, with γ=0.1616, h=3.0, α=0.08 (C1 in Figs.4 and 5).

Figure 9 shows the time-dependent voltage and phase portraits of the 4th junction in the uncontrolled and controlled cases. A chaos to period-1 transfer is clearly shown with K=0.4 and τ=9.10, giving the controlled period $T_{\rm control}$=τ=9.10. It turns out that this delay-time constant is the period of the target UPO. In Fig.10 we plot the phase portraits of the boundary junctions for the uncontrolled and controlled arrays, indicating that the chaotic attractor is collapse into a limit cycle as a result of the delayed feedback.

Figure 11 shows the controlled I-V curves of the Josephson array from Fig.4. The original chaotic steps become clear resonant flux-flow steps.

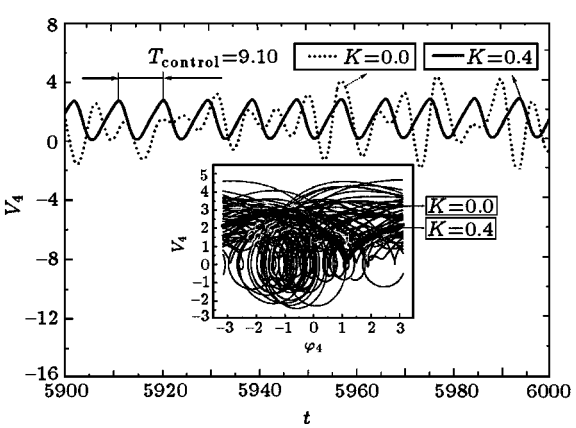


Fig.9. Time-dependent voltage of the 4th junction over the last 100 normalized time units under the conditions of uncontrolled (K=0.0) and controlled (K=0.4) arrays of point C1 (γ=0.1616, h=3.0, α=0.08) in Figs.4 and 5. The inset is the trajectory of the 4th junction in the phase space over the last 500 time units, showing the transfer occurring from chaos to period-1.

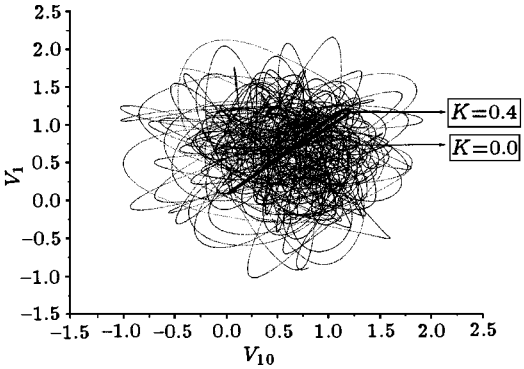


Fig.10. Phase portraits (V_{10}, V_1) for chaotic and controlled arrays of point C1 (γ=0.1616, h=3.0, α=0.08).

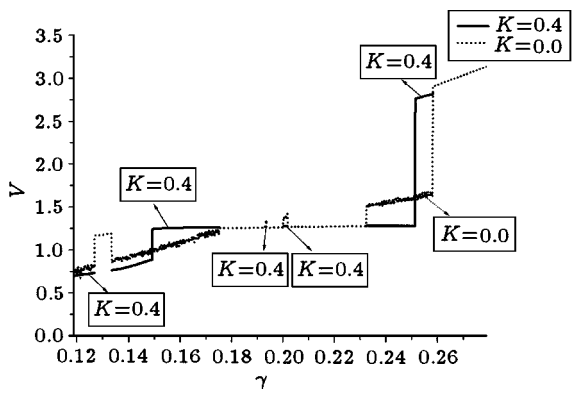


Fig.11. *I-V* curve of uncontrolled (K=0.0) and controlled (K=0.4) arrays for low damping (α=0.08, h=3.0).

The above simulations could also be performed if γ is swept up. Similar results could be obtained.

4. Conclusion

We have investigated the dynamical behaviour of a 1D discrete Josephson lattice. Different phenomena, such as periodic, quasi-periodic and chaotic states as well as bifurcations are revealed. A complete γ-V bifurcation diagram is presented. And we have demonstrated that chaos could be controlled by the application of delayed feedback method to the discrete Josephson array, giving a periodic behaviour. For our example, when the feedback delay time approaches 9.10, the feedback drops by four orders of magnitude, indicating the stabilization of a natural state of the system.

Thus, the system can operate in a wider parameter space than before. The uniqueness of our method is that it does not require full information about the target UPO; instead, it only uses a constant delayed time in the feedback controller. Furthermore, this method stabilizes the UPO embedded within the strange attractor, and in doing so it will not change the basic physics underlying the system. Once under control, the Josephson device can operate inside the parameter space where chaos dominates.

Acknowledgments

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