Vol 12 No 5, May 2003
1009-1963/2003/12(05)/0538-04
Chinese Physics
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# Effect of dust charge variation on dust-acoustic solitary waves in a magnetized two-ion-temperature dusty plasma*

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(Received 26 August 2002; revised manuscript received 26 November 2002)

The effect of dust charge variation on the dust-acoustic solitary structures is investigated in a warm magnetized two-ion-temperature dusty plasma consisting of a negatively and variably charged extremely massive dust fluid and ions of two different temperatures. It is shown that the dust charge variation as well as the presence of a second component of ions would modify the properties of the dust-acoustic solitary structures and may excite both dust-acoustic solitary holes (soliton waves with a density dip) and positive solitons (soliton waves with a density hump).

**Keywords:** dust charge variation, dust-acoustic solitary waves, magnetized two-ion-temperature dusty plasma
**PACC:** 5235S, 5235

## 1. Introduction

During the recent few years, nonlinear wave phenomena in dusty plasmas have received great attention because of its importance in the Earth's environment and laboratory.[1−5] It has been shown that the inclusion of static-charged dust grains in plasma modifies the existing plasma wave spectra and introduces new eigenmodes.[6,7] Theoretical studies[8−10] reveal that a finite dust temperature, the second component of light ion impurities, obliqueness, and the external magnetic field are important factors affecting the dust-acoustic solitary wave structures. But in all these investigations, the dust charge is assumed to be a constant. Recent studies[11−13] have indicated that the variation of dust charge would affect the characteristic collection motion of the plasma and modify the dust-acoustic soliton properties. These previous works on the effects of dust charge variation, however, are focused on the unmagnetized dusty plasma. Therefore, in the present study we extend these earlier works to the obliquely propagating dust-acoustic solitary structures in a warm magnetized two-ion-temperature dusty plasma which consists of a negatively and variably charged extremely massive dust fluid, and ions of two different temperatures. The effects of variable dust charge, the presence of a second component of light ion impurities, obliqueness, and external magnetic field on the dust-acoustic solitary waves are discussed in detail.

## 2. Governing equations and derivation of the Korteweg–de Vries equation

We consider the obliquely travelling dust-acoustic waves in a magnetized dusty plasma. The plasma system consists of the warm, extremely massive, micro-sized, negatively charged dust fluid and ions of two different temperatures $T_{\rm ic}$ and $T_{\rm ih}$, in the presence of an external static magnetic field directed along $z$-axis ($\boldsymbol{B}_0||\boldsymbol{z}_0$). Thus, the normalized equations governing the dust-acoustic waves are

$$\begin{cases} \dfrac{\partial n}{\partial t}+\nabla\cdot(n\boldsymbol{V})=0, \\ \dfrac{\partial \boldsymbol{V}}{\partial t}+\boldsymbol{V}\cdot\nabla\boldsymbol{V}+\dfrac{\sigma}{n}\nabla n=Z_{\rm d}[\nabla\phi-\omega_{\rm cd}(\boldsymbol{V}\times\boldsymbol{z}_0)], \\ \nabla^2\phi=Z_{\rm d}n-\dfrac{1}{1+\mu}\exp(-\alpha_{\rm c}\phi)-\dfrac{\mu}{1+\mu}\exp(-\alpha_{\rm h}\phi), \end{cases} \tag{1}$$

*Project supported by the Foundation of Northwest Normal University, China (Grant No KJCXGC-215).

where

$$\boldsymbol{V}=(u,v,w),\quad \nabla=\left(\frac{\partial}{\partial x},\frac{\partial}{\partial y},\frac{\partial}{\partial z}\right).$$

The variables of time $t$, space $(x,y,z)$, dust fluid particle number density $n$, dust fluid velocity $\boldsymbol{V}$, and electrostatic wave potential $\phi$ are normalized by the dust plasma period $\omega_{\rm pd}^{-1}=\sqrt{m_{\rm d}/4\pi n_{\rm d0}Z_{\rm d0}^2e^2}$, effective Debye length $\lambda_{\rm eff}=\sqrt{T_{\rm eff}/4\pi n_{\rm d0}Z_{\rm d0}e^2}$, unperturbed equilibrium dust density $n_{\rm d0}$, dust-acoustic velocity $C_{\rm d}=\sqrt{Z_{\rm d0}T_{\rm eff}/m_{\rm d}}$, and $T_{\rm eff}/e$, respectively. $\sigma=T_{\rm d}/(Z_{\rm d0}T_{\rm eff})$ is the dust temperature ratio. $T_{\rm eff}=n_{\rm d0}/(n_{\rm i0c}/T_{\rm ic}+n_{\rm i0h}/T_{\rm ih})$, $\alpha_{\rm c}=T_{\rm eff}/T_{\rm ic}=\alpha(1+\mu)/(\alpha+\mu)$ and $\alpha_{\rm h}=T_{\rm eff}/T_{\rm ih}=(1+\mu)/(\alpha+\mu)$ with $\alpha=T_{\rm ih}/T_{\rm ic}$ and $\mu=n_{\rm i0h}/n_{\rm i0c}$. $\omega_{\rm cd}=(Z_{\rm d0}eB_0/m_{\rm d})/\omega_{\rm pd}$ is the dust cyclotron frequency normalized by $\omega_{\rm pd}$. $Z_{\rm d}$ is the number of electrons residing on a dust grain.

To derive the Korteweg–de Vries (KdV) equation that governs the behaviours of small but finite amplitude waves, the independent variables are stretched as $\xi=\varepsilon^{1/4}(l_xx+l_yy+l_zz-v_0t)$ and $\tau=\varepsilon^{3/2}t$. Here $\varepsilon$ is a small parameter measuring the strength of the nonlinearity, $v_0$ is the phase velocity of waves, $l_x,l_y$ and $l_z$ are the directional cosines of the wavevector $\boldsymbol{k}$ along $x$-, $y$-, and $z$- axes, respectively, so that $l_x^2+l_y^2+l_z^2=1$. The dependent variables are expanded as

$$\begin{cases} n=1+\varepsilon n_1+\varepsilon^2n_2+\cdots,\\ u=\varepsilon^{3/2}u_1+\varepsilon^2u_2+\cdots,\\ v=\varepsilon^{3/2}v_1+\varepsilon^2v_2+\cdots,\\ w=\varepsilon w_1+\varepsilon^2w_2+\cdots,\\ \phi=\varepsilon\phi_1+\varepsilon^2\phi_2+\cdots,\\ Z_{\rm d}=1+\gamma_1\varepsilon\phi_1+\varepsilon^2(\gamma_1\phi_2+\gamma_2\phi_1^2)+\cdots, \end{cases}\tag{2}$$

where $\gamma_1$ and $\gamma_2$ are two parameters representing the dust charge variation. Substituting Eqs.(2) into Eqs.(1) and collecting the terms in the different powers of $\varepsilon$, to lowest order in $\varepsilon$, we obtain

$$\begin{cases} n_1=-(1+\gamma_1)\phi_1,\\ w_1=-v_0/l_z(1+\gamma_1)\phi_1, \end{cases}\tag{3}$$

where $v_0=l_z\left(\dfrac{1}{1+\gamma_1}+\sigma\right)$. The first-order $x$- and $y$-components of the momentum equation can be written as

$$\begin{cases} u_1=-\dfrac{l_y}{\omega_{\rm cd}}[\sigma(1+\gamma_1)+1]\dfrac{\partial\phi_1}{\partial\xi},\\ v_1=-\dfrac{l_x}{\omega_{\rm cd}}[\sigma(1+\gamma_1)+1]\dfrac{\partial\phi_1}{\partial\xi}. \end{cases}\tag{4}$$

For the next higher order, we can obtain

$$\begin{cases} u_2=-\dfrac{l_xv_0}{\omega_{\rm cd}^2}[\sigma(1+\gamma_1)+1]\dfrac{\partial^2\phi_1}{\partial\xi^2},\\ v_2=-\dfrac{l_yv_0}{\omega_{\rm cd}^2}[\sigma(1+\gamma_1)+1]\dfrac{\partial^2\phi_1}{\partial\xi^2},\\ \dfrac{\partial^2\phi_1}{\partial\xi^2}=n_2-(1+\gamma_1)\phi_2-\gamma_1n_1\phi_1\\ \qquad\qquad -\left[\gamma_2-\dfrac{1}{2}\dfrac{\alpha_{\rm c}^2+\alpha_{\rm h}^2\mu}{1+\mu}\right]\phi_1^2. \end{cases}\tag{5}$$

Similarly, following the same procedure one can obtain the next higher-order continuity equation and $z$-component of the momentum equation as

$$\begin{cases} \dfrac{\partial n_1}{\partial\tau}-v_0\dfrac{\partial n_2}{\partial\xi}+l_x\dfrac{\partial u_2}{\partial\xi}\\ +l_y\dfrac{\partial v_2}{\partial\xi}+l_z\dfrac{\partial(w_2+n_1w_1)}{\partial\xi}=0,\\ \dfrac{\partial w_1}{\partial\tau}-v_0\dfrac{\partial w_2}{\partial\xi}+l_z\left[w_1\dfrac{\partial w_1}{\partial\xi}-\dfrac{\partial\phi_2}{\partial\xi}\right.\\ \left.-\gamma_1\phi_1\dfrac{\partial\phi_1}{\partial\xi}+\sigma\left(\dfrac{\partial n_2}{\partial\xi}-n_1\dfrac{\partial n_1}{\partial\xi}\right)\right]=0. \end{cases}\tag{6}$$

Now, using Eqs.(3)–(6) and eliminate $n_2,u_2,v_2,w_2$, and $\phi_2$, one obtains the KdV equation

$$\frac{\partial\phi_1}{\partial\tau}+A\phi\frac{\partial\phi}{\partial\xi}+D\frac{\partial^3\phi}{\partial\xi^3}=0,\tag{7}$$

where the coefficients $A$ and $D$ are

$$\begin{aligned} A=&\frac{l_z}{2(1+\gamma_1)\sqrt{\dfrac{1}{1+\gamma_1}+\sigma}}\\ &\times\left\{(1+\gamma_1)^2\left[\sigma-3\left(\frac{1}{1+\gamma_1}+\sigma\right)\right]+3\gamma_1\right.\\ &\left.-\frac{2}{1+\gamma_1}\left(\gamma_2-\frac{1}{2}\frac{\alpha_{\rm c}^2+\alpha_{\rm h}^2\mu}{1+\mu}\right)\right\}, \end{aligned}\tag{8}$$

$$\begin{aligned} D=&\frac{l_z}{2\sqrt{\dfrac{1}{1+\gamma_1}+\sigma}}\\ &\times\left\{\frac{1-l_z^2}{\omega_{\rm cd}^2}\left(\frac{1}{1+\gamma_1}+\sigma\right)^2+\frac{1}{(1+\gamma_1)^2}\right\}. \end{aligned}\tag{9}$$

To obtain the steady-state solution of this KdV equation, we transform the variable $\xi$ to $\eta=\xi-U_0\tau$ and impose the appropriate boundary conditions, namely $\phi\to0,{\rm d}\phi_1/{\rm d}\eta\to0,{\rm d}^2\phi_1/{\rm d}\eta^2\to0$ at $\eta\to\pm\infty$. Thus, we can express the steady-state solution $\phi_1=\phi$ of this KdV equation as

$$\phi=\phi_{\rm m}{\rm sech}^2[(\xi-U_0\tau)/\delta],\tag{10}$$

where the amplitude $\phi_{\rm m}$ and the width $\delta$ are given by

$$\begin{cases} \phi_{\rm m}=3U_0/A,\\ \delta=\sqrt{4D/U_0}. \end{cases}\tag{11}$$

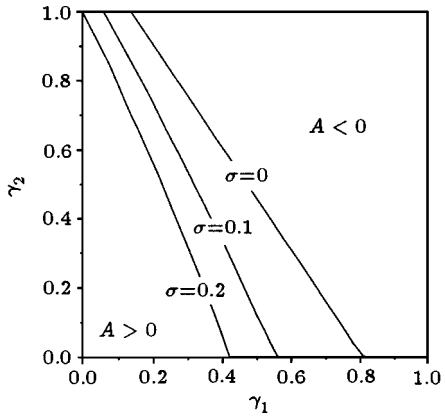



**Fig.1.** Plot of $A$=0 for different $\sigma$ in $(\gamma_1, \gamma_2)$ plane $(\alpha = 20, \mu = 15)$.

It is noticed that if $A < 0$, there exist solitary waves with negative potential or compressive solitary waves, i.e. solitary waves with density hump referred to as positive (density) solitons; if $A > 0$, there exist solitary waves with positive potential or rarefactive solitary waves, i.e. solitary waves with a density dip, referred to as solitary dust-acoustic (density) holes. Equation (8) indicates that the sign of $A$ is related to $\sigma, \alpha, \mu, \gamma_1$, and $\gamma_2$, but is independent of $l_z(\cos\theta)$ and $\omega_{cd}$. It is clear from Eq.(8) that for $\mu = 0, \gamma_1 = 0$, and $\gamma_2 = 0$ (meaning that there is only one type of ion species and the dust charge is constant), $A$ is always negative, i.e. there exist solitary waves with negative potential or compressive solitary waves, i.e. positive (density) solitons only. But for suitable values of $\mu, \alpha, \gamma_1$, and $\gamma_2$, $A$ may become positive, i.e. there exist solitary waves with positive potential or rarefactive solitary waves, i.e. dust-acoustic solitary (density) holes. We can see from the following discussion that the dust charge variation $(\gamma_1, \gamma_2)$, as well as the presence of a second component of ions $(\mu, \alpha)$, would affect the sign of $A$ and the soliton character significantly. Figure 1 illustrates the curve of $A$=0 in $(\gamma_1, \gamma_2)$ plane and shows the two regions of $A < 0$ and $A > 0$. The $A < 0$ region corresponds to dust-acoustic compressive solitary structures, and $A > 0$ region corresponds to dust-acoustic rarefactive solitary structures. It is shown here that as we increase the dust fluid temperature, we need lowering the values of $\gamma_1$ or $\gamma_2$ in order to have dust-acoustic solitary holes in such a dusty plasma system. Figure 2 shows the curve of $A$=0 in $(\mu, \alpha)$ plane, and there also exist two regions. We can see that when $\gamma_1$ increases, we need higher values of $\alpha$ or $\mu$ in order to have dust-acoustic solitary holes.

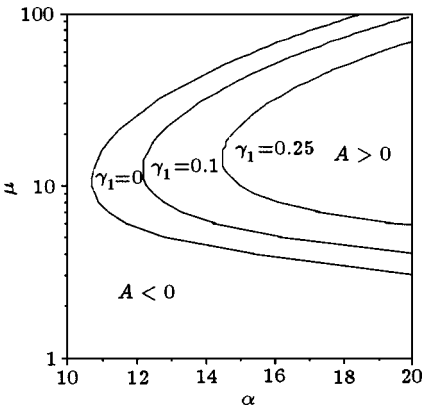



**Fig.2.** Plot of $A$=0 for different $\gamma_l$ in $(\alpha, \mu)$ plane $(\sigma = 0.1, \gamma_2 = 0)$.

It is obvious from Eqs.(8) and (11) that the amplitude of the solitary waves is dependent on $l_z, \sigma, \alpha, \mu, \gamma_1$, and $\gamma_2$, but is independent of the external magnetic field $\omega_{cd}$. Figure 3 shows how the amplitude of the solitary structures changes with $\sigma$ and $\gamma_1$. It is seen that the amplitude increases with $\sigma$ and $\gamma_1$. We also see that the amplitude against $\sigma$ increases with increasing $\gamma_1$. It is found from Eqs.(9) and (11) that the width of the solitary waves is a nonlinear function of $\omega_{cd}$, $l_z(\cos\theta)$, $\sigma$, and $\gamma_1$, but it is independent of $\alpha$, $\mu$, and $\gamma_2$. Figure 4 illustrates how the width of the solitary waves varies with $\omega_{cd}$ and $\gamma_1$. It is clear that the width decreases with $\gamma_1$, but increases with $\omega_{cd}$. The effect of $\gamma_2$ upon the amplitude and width has the similar characters of $\gamma_1$.

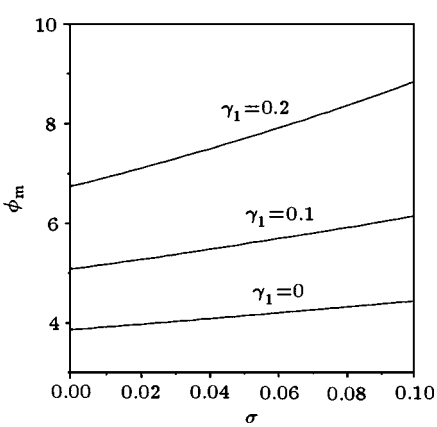



**Fig.3.** Variation of $\phi_m$ against $\sigma$ for different $\gamma_1$ $(\alpha = 20, \mu = 15, l_z = 0.96, \gamma_2 = 0)$.

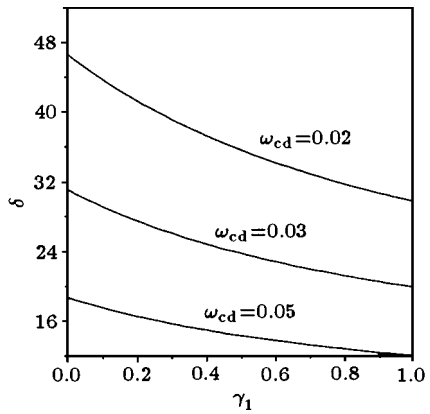



**Fig.4.** Variation of $\delta$ against $\gamma_1$ for different $\omega_{\rm cd}$ ($\sigma = 0.1, l_z = 0.96$).

In conclusion, the effects of the dust charge variation on the solitary wave structures in a warm magnetized dusty plasma have been investigated by the reductive perturbation method. It has been found that the dust charge variation $(\gamma_1, \gamma_2)$, as well as the presence of a second component of ions $(\mu, \alpha)$, would modify the properties of the dust-acoustic solitary structures and may excite both dust-acoustic solitary holes (soliton waves with density dip) and positive solitons (soliton waves with density hump). The dust charge variation also has significant effects on the solitary wave amplitude and width. We note that the external magnetic field only affects the solitons width.

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