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"Cold" dispersion relation of corrugated waveguide filled with plasma immersed in a finite magnetic field

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A general dispersion equation of a partially filled plasma corrugated waveguide immersed in a finite magnetic field is presented. When the guiding magnet $B_0 \to \infty$ or 0, this equation can be reduced to the results obtained in previous works.

Keywords: finite field, plasma-filled waveguide, relativistic electron beam, ion-channel
PACC: 5000, 5200, 5240D

1. Introduction

One of the very promising schemes for generating high-power microwave pulses is based on the interaction of an intense relativistic electron beam (IREB) with the slow electromagnetic wave in a rippled wall waveguide.[1−8] Such a beam can be transported through a waveguide if the electron space charge is neutralized. Neutralization can be arranged by filling the system with the plasma which is denser in comparison with the beam. The use of a background plasma in a backward wave oscillator (BWO) or travelling wave tube (TWT)[9−11] can increase the space charge limiting current by a factor $(1-n_{\rm i}/n_{\rm e})^{-1}$; $n_{\rm i}/n_{\rm e}$ is the number density of the background plasma normalized to the number density in the electron beam.

As is known, the TM and TE modes are always coupled to form the HE and EH hybrid modes in a magnetized plasma-filled waveguide. Thus the field expressions and dispersion relations are complicated. Moreover, when the effect of ion-channel is considered, the dispersion relations are far more complicated. During the beam head propagating through the plasma, it continuously expels the plasma electrons from the beam volume, leaving the relatively immobile ions fixed to provide focusing for the remainder of the beam. These un-neutralized ions occupy a cylindrical volume of radius $R_{\rm i} = R_{\rm b}(n_{\rm b}/n_{\rm p})^{1/2}$, and this is the so-called "ion-channel".[7]

In this paper, a more general dispersion relation is obtained by taking into account a finite external magnetic field and the effect of ion-channel.

2. Basic equations

We consider an infinitely long axisymmetric corrugated-wall waveguide as shown in Fig.1.

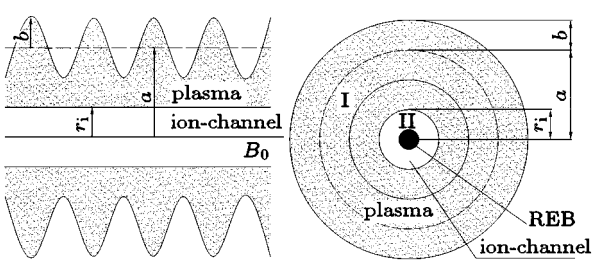


Fig.1. The uniform magnetized plasma-filled sinusoidally corrugated slow-wave structure with a perfectly conducting inner wall. The ion-channel effect is taken into account.

The metal wall is assumed to be perfectly conducting. The wall radius $R(z)$ varies according to the relation $R(z) = a + b\cos(k_0 z)$, where a is the average radius, b is the corrugation amplitude, $k_0 = 2\pi/z_0$, and z_0 is the axial length of the corrugation period. A solid IREB of density $n_{\rm b}$ and radius $R_{\rm b}$ propagates through the waveguide, which is loaded completely

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with a cold magnetized collisionless plasma column of uniform density $n_{\rm p}$. The dielectric tensor of the magnetized plasma is given by [1]

$$\overleftrightarrow{\varepsilon}=\varepsilon_0\begin{bmatrix}\varepsilon_1 & {\rm j}\varepsilon_2 & 0\\ -{\rm j}\varepsilon_2 & \varepsilon_1 & 0\\ 0 & 0 & \varepsilon_3\end{bmatrix}, \tag{1}$$

where $\varepsilon_1=1-\omega_{\rm p}^2/(\omega^2-\omega_{\rm c}^2)$, $\varepsilon_2=-\omega_{\rm p}^2\omega_{\rm c}^2/[\omega(\omega^2-\omega_{\rm c}^2)]$, $\varepsilon_3=1-\omega_{\rm p}^2/\omega^2$; and $\omega_{\rm p}=(n_{\rm p}e^2/m_0\varepsilon_0)^{1/2}$ is the electron plasma frequency, $\omega_{\rm c}=eB_0/m_0$ is the electron cyclotron frequency, and ω is the angular frequency of the electromagnetic wave, j is the unit imaginary number, m_0 is the static mass. B_0 is the axial magnetic field. $\varepsilon_{\rm D}$ is the dielectric constant, and $r_{\rm i}$ is the radius of ion-channel.

For every model, there are many space harmonic waves in the corrugated waveguide. The phase constant is expressed as[1−5]

$$k_{z,n}=k_z+\frac{2\pi n}{z_0}\quad(n=0,\pm1,\pm2,\cdots). \tag{2}$$

For the corrugated waveguide, the field components should be expanded into space harmonics according to Floquet's theorem, e.g.

$$\begin{cases}E_z=\sum\limits_n E_{z,n}\exp[{\rm j}(\omega t-k_{z,n}z+m\theta)],\\ H_z=\sum\limits_n H_{z,n}\exp[{\rm j}(\omega t-k_{z,n}z+m\theta)],\end{cases} \tag{3}$$

$$\begin{cases}E_{z,n}=A_{1,n}{\rm J}_m(p_{1,n}r)+A_{2,n}{\rm J}_m(p_{2,n}r),\\ H_{z,n}=A_{1,n}h_{1,n}{\rm J}_m(p_{1,n}r)+A_{2,n}h_{2,n}{\rm J}_m(p_{2,n}r),\end{cases} \tag{4}$$

$E_{z,n}$ and $H_{z,n}$ must satisfy the following equations that are obtained from Maxwell equations and Eq.(1)[1]

$$\begin{cases}\sum\limits_n(\nabla_\perp^2E_{z,n}+a_nE_{z,n})\\ =\sum\limits_n\left(b_nH_{z,n}+{\rm j}\omega\mu_0J_{z,n}-\dfrac{{\rm j}k_{z,n}}{\varepsilon_0\varepsilon_1}\rho_n\right),\\ \sum\limits_n(\nabla_\perp^2H_{z,n}+c_nH_{z,n})\\ =\sum\limits_n\left(d_nE_{z,n}-(\nabla\times\boldsymbol{J})_z-\dfrac{\omega\varepsilon_2}{\varepsilon_0}\rho_n\right),\end{cases} \tag{5}$$

where

$$\nabla_\perp^2=\nabla^2-\partial^2/\partial z^2,\quad a_n=(k^2\varepsilon_1-k_{z,n}^2)\varepsilon_3/\varepsilon_1,$$

$$k^2=\omega^2/c^2,\quad b_n={\rm j}k_{z,n}\omega\mu_0\varepsilon_2/\varepsilon_1,$$

$$c_n=-k_{z,n}^2+k^2(\varepsilon_1^2-\varepsilon_2^2)/\varepsilon_1,$$

$$d_n=-{\rm j}k_{z,n}\omega\varepsilon_0\varepsilon_2\varepsilon_3/\varepsilon_1, \tag{6}$$

E_r, E_θ, H_r and H_θ have the same forms and can be expressed as the following with $E_{z,n}$.

$$E_{r,n}=\frac{1}{D_n}\left[{\rm j}k_{z,n}K_n^2\frac{\partial E_{z,n}}{\partial r}+\omega\mu_0k_g^2\frac{\partial H_{z,n}}{\partial r}\right],$$

$$E_{\theta,n}=\frac{1}{D_n}\left[{\rm j}k_{z,n}k_g^2\frac{\partial E_{z,n}}{\partial r}+{\rm j}\omega\mu_0K_n^2\frac{\partial H_{z,n}}{\partial r}\right],$$

$$H_{r,n}=\frac{1}{D_n}\left[-\omega\varepsilon_0\varepsilon_gk_{z,n}^2\frac{\partial E_{z,n}}{\partial r}-{\rm j}k_{z,n}^2K_n^2\frac{\partial H_{z,n}}{\partial r}\right],$$

$$H_{\theta,n}=\frac{1}{D_n}\left[-{\rm j}\omega\varepsilon_0(\varepsilon_1K_n^2-\varepsilon_2k_g^2)\frac{\partial E_{z,n}}{\partial r}+k_{z,n}k_g^2\frac{\partial H_{z,n}}{\partial r}\right],$$

where $D_n=K_n^4-k_g^4$, $K_n^2=k_{z,n}^2-k^2\varepsilon_1$, $k_g^2=k^2\varepsilon_g$, $\varepsilon_2=-{\rm j}\varepsilon_g$.

From Eq.(5) we can see that TE and TM modes are coupled; they cannot exist independently. This coupling generates the hybrid HE and EH modes. J_z and ρ are electron-beam-current density and charge density respectively. By using the electronic kinetic equation and contiguous equation, they can be expressed as follows with E_z

$$J_{z,n}=-\frac{{\rm j}\omega\varepsilon_0\omega_{\rm b}^2}{\gamma_0^3(\omega-k_{z,n}v_0)^2}E_{z,n},$$

$$\rho_n=k_{z,n}J_{z,n}/\omega,$$

where

$$\omega_{\rm b}=(N_{\rm b}e^2/m_0\varepsilon_0)^{1/2}$$

is the electronic frequency of IREB, and v_0 is the z axial velocity of IREB.

In the case without IREB, there are $J_{z,n}$=0, ρ_n=0, and a_n, b_n, c_n and d_n are changed into a_{n0}, b_{n0}, c_{n0}, d_{n0}; H_{z0} and E_{z0} are the fields without IREB. Thus we have[1,2]

$$\nabla_\perp^2E_{z0}+a_{n0}E_{z0}=b_{n0}H_{z0}, \tag{7}$$

$$\nabla_\perp^2H_{z0}+c_{n0}H_{z0}=d_{n0}E_{z0}. \tag{8}$$

3. "Cold" dispersion equations

From Eqs.(7) and (8) we can obtain the field components in each region as follows:

For region I (plasma region, $r_{\rm i}\le|r|\le R(z)$), we have

$$\begin{aligned}E_{z,n}^{\rm I}=&A_{1,n}{\rm J}_0(p_{1,n}r)+A_{2,n}{\rm N}_0(p_{1,n}r)\\ &+A_{3,n}{\rm J}_0(p_{2,n}r)+A_{4,n}{\rm N}_0(p_{2,n}r),\end{aligned} \tag{9}$$

$$\begin{aligned}H_{z,n}^{\rm I}=&[A_{1,n}{\rm J}_0(p_{1,n}r)+A_{2,n}{\rm N}_0(p_{1,n}r)]h_{1,n}\\ &+[A_{3,n}{\rm J}_0(p_{2,n}r)+A_{4,n}{\rm N}_0(p_{2,n}r)]h_{2,n}.\end{aligned} \tag{10}$$

For region II (ion-channel region, $0 \le |r| \le r_{\rm i}$), all electrons are expelled from this region, so we have

$$E^{\rm II}_{z,n} = A_{5,n}{\rm J}_0(k_{c,n}r), \tag{11}$$

$$H^{\rm II}_{z,n} = A_{6,n}{\rm J}_0(k_{c,n}r), \tag{12}$$

$$\begin{aligned}(p_{1,2})_n^2 =& \frac{1}{2\varepsilon_1}[-k_{z,n}^2(\varepsilon_1+\varepsilon_2)+k^2(\varepsilon_1\varepsilon_3+\varepsilon_1^2-\varepsilon_2^2)]\\ &\pm\frac{1}{2\varepsilon_1}\{[-k_{z,n}^2(\varepsilon_3-\varepsilon_1)\\ &+k^2(\varepsilon_1\varepsilon_3-\varepsilon_1^2+\varepsilon_2^2)]^2+4k^2k_{z,n}^2\varepsilon_2^2\varepsilon_3\}^{1/2},\end{aligned}$$

$$(h_{1,2})_n = \frac{(-k_{z,n}^2+k^2\varepsilon_1)\varepsilon_3-\varepsilon_1(p_{1,2})_n^2}{{\rm j}\omega\mu_0 k_{z,n}\varepsilon_2},$$

$$k_{c,n}^2 = k^2 - k_{z,n}^2,$$

where $\rm J_0$ and $\rm N_0$ are the zeroth-order Bessel and Neumann functions, respectively, $\rm J_1$ and $\rm N_1$ are the first-order Bessel and Neumann functions; $E_{r,n}$, $E_{\theta,n}$, $H_{r,n}$ and $H_{\theta,n}$ are omitted.

The field components must satisfy the following continuity and boundary conditions:

3.1. For $|r| = r_{\rm i}$, we have

$$E_z^{\rm I} = E_z^{\rm II}, \quad E_\theta^{\rm I} = E_\theta^{\rm II},$$

$$H_z^{\rm I} = H_z^{\rm II}, \quad H_\theta^{\rm I} = H_\theta^{\rm II}. \tag{13}$$

3.2. For $|r| = R(z) = a + b\cos(k_0 z)$, we have

$$[E_z^{\rm I} - E_r^{\rm I}k_0 b\sin(k_0 z)]_{|r=a+b\cos(k_0 z)} = 0,$$

$$[H_z^{\rm I}k_0 b\sin(k_0 z) + H_r]_{|r=a+b\cos(k_0 z)} = 0. \tag{14}$$

Substituting Eqs.(9)–(12) into Eqs.(13) and (14) and using Eq.(3), we can obtain the following matrix expressions[1,2]

$$\begin{bmatrix} (F_{m,1})_{n,s} & (F_{m,2})_{n,s} \\ (G_{m,1})_{n,s} & (G_{m,2})_{n,s} \end{bmatrix} \begin{bmatrix} A_{1,n} \\ A_{2,n} \end{bmatrix} = 0. \tag{15}$$

In the above derivation, we have applied the same mathematical process as that in Ref.[4]. Thus, $(F_{m,q})_{n,s}$ (q=1,2) and $(G_{m,q})_{n,s}$ (q=1,2) can be expressed in the following forms

$$(F_{m,1})_{n,s} = W_1 + C_1\cdot W_3 + C_2\cdot W_4,$$

$$(F_{m,2})_{n,s} = W_2 + B_1\cdot W_3 + B_2\cdot W_4,$$

$$(G_{m,1})_{n,s} = L_1 + C_1\cdot L_3 + C_2\cdot L_4,$$

$$(G_{m,2})_{n,s} = L_2 + B_1\cdot L_3 + B_2\cdot L_4,$$

where

$$\begin{aligned}C_1 =& \frac{\pi r_{\rm i}}{2M_3}\Bigg[\frac{D(H_2-H_1)\omega^2{\rm J}_0(p_1r_{\rm i}){\rm J}_1(pr_{\rm i}){\rm N}_0(p_2r_{\rm i})}{p{\rm J}_0(pr_{\rm i})}\\ &+M_1p_1{\rm J}_1(p_1r_{\rm i}){\rm N}_0(p_2r_{\rm i})-M_4p_2{\rm J}_0(p_1r_{\rm i}){\rm N}_1(p_2r_{\rm i})\\ &+\frac{M_2(H_1-H_2)pp_1p_2{\rm J}_1(p_1r_{\rm i}){\rm J}_0(pr_{\rm i}){\rm N}_1(p_2r_{\rm i})}{{\rm J}_1(pr_{\rm i})}\Bigg],\end{aligned}$$

C_2 is obtained by using –J to replace N. B_1 can be obtained by using $\rm N_0$ $(p_1r_{\rm i})$ and $\rm N_1$ $(p_1r_{\rm i})$ to replace $\rm J_0$ $(p_1r_{\rm i})$ and $\rm J_1(p_1r_{\rm i})$ respectively.

$$\begin{aligned}B_2 =& \frac{\pi r_{\rm i}}{2M_3}\Bigg[\frac{D(H_1-H_2)\omega^2{\rm N}_0(p_1r_{\rm i}){\rm J}_1(pr_{\rm i}){\rm J}_0(p_2r_{\rm i})}{p{\rm J}_0(pr_{\rm i})}\\ &-M_1p_1{\rm N}_1(p_1r_{\rm i}){\rm J}_0(p_2r_{\rm i})+M_4p_2{\rm N}_0(p_1r_{\rm i}){\rm J}_1(p_2r_{\rm i})\\ &-\frac{M_2(H_1-H_2)pp_1p_2{\rm N}_1(p_1r_{\rm i}){\rm J}_0(pr_{\rm i}){\rm J}_1(p_2r_{\rm i})}{{\rm J}_1(pr_{\rm i})}\Bigg],\end{aligned}$$

where

$$\begin{aligned}M_1 =& (K^2H_1+k_zk_g^2)\omega^2\\ &+DH_2[(-k_zH_1+\varepsilon_g\omega^2)k_g^2-\varepsilon_1K^2\omega^2],\end{aligned}$$

$$M_2 = k_z^2k_g^4 + K^2(\varepsilon_gk_g^2-\varepsilon_1K^2)\omega^2,$$

$$\begin{aligned}M_3 =& k_zk_g^2(DH_2^2-\omega^2)\\ &+H_2[K^2(\varepsilon_1D-1)-D\varepsilon_gk_g^2]\omega^2,\end{aligned}$$

$$\begin{aligned}M_4 =& (K^2H_2+k_zk_g^2)\omega^2\\ &+DH_1[(-k_zH_2+\varepsilon_g\omega^2)k_g^2-\varepsilon_1K^2\omega^2].\end{aligned}$$

In all the above expressions H_1 and H_2 have the forms as follows: $H_1 = {\rm j}\omega\mu_0h_1$ and $H_2 = {\rm j}\omega\mu_0h_2$.

$$W_1 = [1+Q_1(n-s)]C_{ns1}^J,$$

$$W_2 = [1+Q_1(n-s)]C_{ns1}^N,$$

$$W_3 = [1+Q_2(n-s)]C_{ns2}^J,$$

$$W_4 = [1+Q_2(n-s)]C_{ns2}^N,$$

$$Q_1 = \frac{k_0(k_zK^2+{\rm j}\omega\mu_0h_1k_g^2)}{D},$$

$$Q_2 = \frac{k_0(k_zK^2+{\rm j}\omega\mu_0h_2k_g^2)}{D},$$

$$L_1 = \frac{p_1(\omega\varepsilon_0\varepsilon_gk_z^2+K^2{\rm j}k_zh_1)}{D}S_{ns1}^J + {\rm j}(n-s)\frac{h_1k_0}{p_1}T_{ns1}^J,$$

$$L_2 = \frac{p_1(\omega\varepsilon_0\varepsilon_gk_z^2+K^2{\rm j}k_zh_1)}{D}S_{ns1}^N + {\rm j}(n-s)\frac{h_1k_0}{p_1}T_{ns1}^N,$$

$$L_3 = \frac{p_2(\omega\varepsilon_0\varepsilon_gk_z^2+K^2{\rm j}k_zh_2)}{D}S_{ns2}^J + {\rm j}(n-s)\frac{h_2k_0}{p_2}T_{ns2}^J,$$

$$L_4 = \frac{p_2(\omega\varepsilon_0\varepsilon_gk_z^2+K^2{\rm j}k_zh_2)}{D}S_{ns2}^N + {\rm j}(n-s)\frac{h_2k_0}{p_2}T_{ns2}^N,$$

$$C^J_{ns1}=\sum_{q=0}^{\infty}\frac{(p_1b)^{2q+|n-s|}\mathrm{J}_0^{(2q+|n-s|)}(p_1a)}{2^{2q+|n-s|}q!(q+|n-s|)!},$$

$$C^N_{ns1}=\sum_{q=0}^{\infty}\frac{(p_1b)^{2q+|n-s|}\mathrm{N}_0^{(2q+|n-s|)}(p_1a)}{2^{2q+|n-s|}q!(q+|n-s|)!},$$

$$C^J_{ns2}=\sum_{q=0}^{\infty}\frac{(p_2b)^{2q+|n-s|}\mathrm{J}_0^{(2q+|n-s|)}(p_2a)}{2^{2q+|n-s|}q!(q+|n-s|)!},$$

$$C^N_{ns2}=\sum_{q=0}^{\infty}\frac{(p_2b)^{2q+|n-s|}\mathrm{N}_0^{(2q+|n-s|)}(p_2a)}{2^{2q+|n-s|}q!(q+|n-s|)!},$$

$$S^J_{ns1}=\sum_{q=0}^{\infty}\frac{(p_1b)^{2q+|n-s|}\mathrm{J}_0^{(2q+|n-s|+1)}(p_1a)}{2^{2q+|n-s|}q!(q+|n-s|)!},$$

$$S^N_{ns1}=\sum_{q=0}^{\infty}\frac{(p_1b)^{2q+|n-s|}\mathrm{N}_0^{(2q+|n-s|+1)}(p_1a)}{2^{2q+|n-s|}q!(q+|n-s|)!},$$

$$S^J_{ns2}=\sum_{q=0}^{\infty}\frac{(p_2b)^{2q+|n-s|}\mathrm{J}_0^{(2q+|n-s|+1)}(p_2a)}{2^{2q+|n-s|}q!(q+|n-s|)!},$$

$$S^N_{ns2}=\sum_{q=0}^{\infty}\frac{(p_2b)^{2q+|n-s|}\mathrm{N}_0^{(2q+|n-s|+1)}(p_2a)}{2^{2q+|n-s|}q!(q+|n-s|)!},$$

$$T^J_{ns1}=\sum_{q=1}^{\infty}\frac{(p_1b)^{2q+|n-s|}\mathrm{J}_0^{(2q+|n-s|-1)}(p_1a)}{2^{2q+|n-s|}q!(q+|n-s|)!},$$

$$T^N_{ns1}=\sum_{q=1}^{\infty}\frac{(p_1b)^{2q+|n-s|}\mathrm{N}_0^{(2q+|n-s|-1)}(p_1a)}{2^{2q+|n-s|}q!(q+|n-s|)!},$$

$$T^J_{ns2}=\sum_{q=1}^{\infty}\frac{(p_2b)^{2q+|n-s|}\mathrm{J}_0^{(2q+|n-s|-1)}(p_2a)}{2^{2q+|n-s|}q!(q+|n-s|)!},$$

$$T^N_{ns2}=\sum_{q=1}^{\infty}\frac{(p_2b)^{2q+|n-s|}\mathrm{N}_0^{(2q+|n-s|-1)}(p_2a)}{2^{2q+|n-s|}q!(q+|n-s|)!}.$$

So the general dispersion relation for the magnetized-plasma-filled BWO or TWT is expressed by the determinant equation

$$\det[D]=\begin{vmatrix}(F_{m,1})_{n,s} & (F_{m,2})_{n,s}\\ (G_{m,1})_{n,s} & (G_{m,2})_{n,s}\end{vmatrix}=0. \tag{16}$$

4. Discussion and conclusions

We can see that the dispersion relations become more complicated when the finite external magnetic field and ion-channel are taken into consideration simultaneously. However, we can learn the radiation electromagnetic wave frequency characteristics and interaction growth rate from these dispersion relations in a magnetized plasma BWO or TWT. In the following, we discuss the two degenerated cases: i.e. one with an infinitely large external guiding magnetic field, and the other without taking into account the ion-channel. And the above dispersion relations serve as comparisons.

In the former case, the symmetric modes (m=0) and the dispersion relations are simplified as[1]

$$\begin{vmatrix}(F_{0,1})_{n,s} & (F_{0,2})_{n,s}\\ (G_{0,1})_{n,s} & (G_{0,2})_{n,s}\end{vmatrix}=0. \tag{17}$$

In the latter case, the guiding magnetic field is $B_0\to\infty$ or $B_0\to 0$, then we can obtain ε_2=0. So the dispersion equations reduce to two independent equations for TM and TE modes. For TM modes, there are

$$|(F_{0,1})_{n,s}|=0. \tag{18}$$

For TE modes, there are

$$|(G_{0,2})_{n,s}|=0. \tag{19}$$

4.1. For $B_0\to\infty$, we can obtain: $\varepsilon_1=1$, $\varepsilon_2=0$, $D_n=K_n^4$, $k_c^2=p^2=p_{1,n}^2=K_n^2=k^2-k_{z,n}^2$, $p_2^2=K_n^2\left(1-\dfrac{\omega_{\mathrm{p}}^2}{\omega^2}\right)$, H_1=0 and $\varepsilon_3=1-\dfrac{\omega_{\mathrm{p}}^2}{\omega^2}$, $k_g=0$.

So the dispersion relation can be expressed as

$$\begin{aligned}(F_{0,1})_{n,s}=&\left[1+(n-s)\frac{k_{z,n}k_0}{K^2}\right]C^J_{ns1}\\&+\frac{\pi r_{\mathrm{i}}}{2p}[-K^2\mathrm{J}_1(pr_{\mathrm{i}})\mathrm{N}_0(p_2r_{\mathrm{i}})\\&+pp_2\mathrm{J}_0(pr_{\mathrm{i}})\mathrm{N}_1(p_2r_{\mathrm{i}})]\\&\times\left[1+(n-s)\frac{k_{z,n}k_0}{K^2}\right]C^J_{ns2}\\&+\frac{\pi r_{\mathrm{i}}}{2p}[K^2\mathrm{J}_1(pr_{\mathrm{i}})\mathrm{J}_0(p_2r_{\mathrm{i}})\\&-pp_2\mathrm{J}_0(pr_{\mathrm{i}})\mathrm{J}_1(p_2r_{\mathrm{i}})]\\&\times\left[1+(n-s)\frac{k_{z,n}k_0}{K^2}\right]C^N_{ns2},\end{aligned} \tag{20}$$

$$\begin{aligned}(G_{0,2})_{n,s}=&\frac{\pi r_{\mathrm{i}}}{2p(K^4-1)\mathrm{J}_0(pr_{\mathrm{i}})\mathrm{J}_1(pr_{\mathrm{i}})}\\&\times[K^2\mathrm{J}_1(pr_{\mathrm{i}})\mathrm{N}_0(p_2r_{\mathrm{i}})-pp_2\mathrm{J}_0(pr_{\mathrm{i}})\mathrm{N}_1(p_2r_{\mathrm{i}})]\\&\times[\mathrm{J}_1(pr_{\mathrm{i}})\mathrm{N}_0(pr_{\mathrm{i}})-K^4\mathrm{J}_0(pr_{\mathrm{i}})\mathrm{N}_1(pr_{\mathrm{i}})]\\&\times\left[\frac{k_{z,n}p_2}{\omega K^2}S^J_{ns2}+(n-s)\frac{k_0}{p_2\omega}T^J_{ns2}\right]\\&+\frac{\pi r_{\mathrm{i}}}{2p(K^4-1)\mathrm{J}_0(pr_{\mathrm{i}})\mathrm{J}_1(pr_{\mathrm{i}})}\\&\times[K^2\mathrm{J}_1(pr_{\mathrm{i}})\mathrm{J}_0(p_2r_{\mathrm{i}})-pp_2\mathrm{J}_0(pr_{\mathrm{i}})\mathrm{J}_1(p_2r_{\mathrm{i}})]\\&\times[-\mathrm{J}_1(pr_{\mathrm{i}})\mathrm{N}_0(pr_{\mathrm{i}})+K^4\mathrm{J}_0(pr_{\mathrm{i}})\mathrm{N}_1(pr_{\mathrm{i}})]\\&\times\left[\frac{k_{z,n}p_2}{\omega K^2}S^N_{ns2}+(n-s)\frac{k_0}{p_2\omega}T^N_{ns2}\right].\end{aligned} \tag{21}$$

4.2. For $B_0 \to \infty$, and ion-channel being not taken into account ($r_{\rm i} \to 0$), then for TM modes, Eqs.(20) and (21) can be changed into

$$(F_{0,1})_{n,s} = \left[1 + (n-s)\frac{k_{z,n}k_0}{k^2 - k_{z,n}^2}\right] C_{ns1}^J, \tag{22}$$

$$(G_{0,2})_{n,s} = \frac{(k^2 - k_z^2)\omega_{\rm p}^2}{[(k^2 - k_z^2)^2 - 1]\omega^2} \times \left[\frac{k_z\sqrt{1 - \dfrac{\omega_{\rm p}^2}{\omega^2}}}{\sqrt{k^2 - k_z^2}} S_{ns2}^N + (n-s)\frac{k_0}{\sqrt{k^2 - k_z^2}\sqrt{\left(1 - \dfrac{\omega_{\rm p}^2}{\omega^2}\right)}} T_{ns2}^N\right], \tag{23}$$

and Eq.(22) is the results obtained in Refs.[3], [5] and [6].

4.3. For B_0=0, we can have: $\varepsilon_2 = 0$, $\varepsilon_1 = \varepsilon_3 = 1 - \dfrac{\omega_{\rm p}^2}{\omega^2}$, $D_n = K_n^4$, $k_c^2 = p^2 = k^2 - k_{z,n}^2$, $p_{2,n}^2 = p_{1,n}^2 = K_n^2 = \varepsilon_1 k^2 - k_{z,n}^2$, H_1=0, k_g=0; then for TM and TE modes we have

$$\begin{aligned}(F_{0,1})_{n,s} =& \left[1 + (n-s)\frac{k_{z,n}k_0}{K^2}\right] C_{ns1}^J + \frac{\pi r_{\rm i}}{2p(\varepsilon_1 K^2 - 1){\rm J}_0(pr_{\rm i}){\rm J}_1(pr_{\rm i})} \\ &\times [{\rm J}_1(pr_{\rm i}){\rm J}_0(p_2 r_{\rm i}) - K^2\varepsilon_1 p p_2 {\rm J}_0(pr_{\rm i}){\rm J}_1(p_2 r_{\rm i})][K^2{\rm J}_1(pr_{\rm i}){\rm N}_0(p_2 r_{\rm i}) - pp_2{\rm J}_0(pr_{\rm i}){\rm N}_1(p_2 r_{\rm i})] \\ &\times \left[1 + (n-s)\frac{k_{z,n}k_0}{K^2}\right] C_{ns2}^J + \frac{\pi r_{\rm i}}{2p(\varepsilon_1 K^2 - 1){\rm J}_0(pr_{\rm i}){\rm J}_1(pr_{\rm i})} \\ &\times [K^2{\rm J}_1(pr_{\rm i}){\rm J}_0(p_2 r_{\rm i}) - pp_2{\rm J}_0(pr_{\rm i}){\rm J}_1(p_2 r_{\rm i})] \times [-{\rm J}_1(pr_{\rm i}){\rm J}_0(p_2 r_{\rm i}) \\ &+ K^2\varepsilon_1 p p_2 {\rm J}_0(pr_{\rm i}){\rm J}_1(p_2 r_{\rm i})] \times \left[1 + (n-s)\frac{k_{z,n}k_0}{K^2}\right] C_{ns2}^N,\end{aligned} \tag{24}$$

$$\begin{aligned}(G_{0,2})_{n,s} =& \left[\frac{k_{z,n}p_1}{\omega K^2} S_{ns1}^N + (n-s)\frac{k_0}{p_1\omega} T_{ns1}^N\right] + \frac{-\pi r_{\rm i}}{2p(\varepsilon_1 K^2 - 1){\rm J}_0(pr_{\rm i}){\rm J}_1(pr_{\rm i})} \\ &\times [K^2{\rm J}_1(pr_{\rm i}){\rm N}_0(p_2 r_{\rm i}) - pp_2{\rm J}_0(pr_{\rm i}){\rm N}_1(p_2 r_{\rm i})][-{\rm J}_1(pr_{\rm i}){\rm N}_0(p_2 r_{\rm i}) + K^2\varepsilon_1 p p_2 {\rm J}_0(pr_{\rm i}){\rm N}_1(p_2 r_{\rm i})] \\ &\times \left[\frac{k_{z,n}p_2}{\omega K^2} S_{ns2}^J + (n-s)\frac{k_0}{p_2\omega} T_{ns2}^J\right] + \frac{\pi r_{\rm i}}{2p(\varepsilon_1 K^2 - 1){\rm J}_0(pr_{\rm i}){\rm J}_1(pr_{\rm i})} \\ &\times [K^2{\rm J}_1(pr_{\rm i}){\rm J}_0(p_2 r_{\rm i}) - pp_2{\rm J}_0(pr_{\rm i}){\rm J}_1(p_2 r_{\rm i})] \times [-{\rm J}_1(pr_{\rm i}){\rm N}_0(p_2 r_{\rm i}) \\ &+ K^2\varepsilon_1 p p_2 {\rm J}_0(pr_{\rm i}){\rm N}_1(p_2 r_{\rm i})] \times \left[\frac{k_{z,n}p_2}{\omega K^2} S_{ns2}^N + (n-s)\frac{k_0}{p_2\omega} T_{ns2}^N\right].\end{aligned} \tag{25}$$

4.4. For B_0=0 and ion-channel being not considered ($r_{\rm i} \to 0$), then for TM mode, Eq.(24) is changed into

$$(F_{0,1})_{n,s} = \left[1 + (n-s)\frac{k_{z,n}k_0}{\left(1 - \dfrac{\omega_{\rm p}^2}{\omega^2}\right)k^2 - k_{z,n}^2}\right] C_{ns1}^J. \tag{26}$$

For TE mode Eq.(25) is changed into

$$\begin{aligned}(G_{0,2})_{n,s} =& \left[-\frac{K^2(K^2 + k_z^2 - k^2)(\omega^2 - \omega_{\rm p}^2)}{(-1 + K^4)\omega^2 - K^4\omega_{\rm p}^2}\right] \\ &\times \left[\frac{k_{z,n}}{K} S_{ns2}^N + (n-s)\frac{k_0}{K} T_{ns2}^N\right].\end{aligned} \tag{27}$$

Although Eq.(26) has the same form as Eq.(22), there are different p_1 and p_2 in C_{ns1}^J and C_{ns2}^J; for $B_0 \to \infty$, there are: $k_c^2 = p^2 = p_{1,n}^2 = K_n^2 = k^2 - k_{z,n}^2$, $p_2^2 = K_n^2\left(1 - \dfrac{\omega_{\rm p}^2}{\omega^2}\right)$; and for B_0=0, there are: $p_{2,n}^2 = p_{1,n}^2 = K_n^2 = \varepsilon_1 k^2 - k_{z,n}^2$, $k_c^2 = p^2 = k^2 - k_{z,n}^2$. Thus Eq.(26) is just the relation shown in Refs.[3], [5] and [6].

In conclusion, we have derived the general dispersion relation from a BWO or TWT driven by a solid intense relativistic electron beam with magnetized-plasma- filled uniformly in a sinusoidally corrugated slow wave structure with ion-channel being taken into account. We find that the dispersion relations are more complicated due to the coupling of TM and TE modes. The comparisons of our results with the previous works show a good agreement.

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