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Single mode rate equations for two sections self-pulsating DFB laser

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We propose a set of single mode rate equations for multi-section distributed feedback (DFB) lasers. On the basis of the rate equations, the self-pulsation in the two-section DFB lasers with asymmetrical injection current was explained. It was found that the dynamic distribution of the power in two sections play key role in the self-pulsation running.

Keywords: distributed feedback laser, self-pulsations, single mode rate equations
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There are important uses of the distributed feedback (DFB) lasers in the optical time division multiplex system and optical chaotic communication systems.[1,2] Multi-section DFB lasers will show high frequency self-pulsations when the laser is subjected to asymmetrical injection. The pulsation frequency can be tuned by changing the injection current. The self-pulsation laser will be a useful device for 3R regeneration in high-speed optical time division multiplex system. Clock-extraction and the potential for optical clock distribution by synchronization of the laser diode self-pulsation to applied electrical and optical signals have already been demonstrated.[3] According to the theoretical study,[4,5] the dispersive Q-switching is the physical mechanism of the high frequency self-pulsation in two-section DFB lasers. The detuning of the Bragg wavelengths of the two DFB sections induced by injection current is introduced to explain the self-pulsation phenomenon in the theoretical explanation.

The self-pulsating DFB laser was usually described by travelling-wave equations. A set of single mode rate equations for photons and carriers were proposed in paper,[6] where a longitudinal analogy of Peterman's factor of excessive spontaneous emission has been thought to be the driving force for the sustained relaxation oscillations.

In this paper, we deduce a new set of single mode rate equations for photons from travelling wave equations. It was found that the longitudinal fill factor plays key role in the self-pulsations in two-section DFB lasers with asymmetrical injected current.

In the DFB structure, the electric field of the TE mode in the cavity can be expressed by the combination of the forward and reverse waves[7]

$$E(z,t) = [F(z,t)\exp(\mathrm{i}\beta_0 z) + R(z,t)\exp(-\mathrm{i}\beta_0 z)] \times \exp\left(\mathrm{i}\omega_0 t + \mathrm{i}\int \Delta\omega(t)\mathrm{d}t\right), \quad (1)$$

where the ω_0 is the reference frequency, $\beta_0 = 2\pi/\Lambda_0$, Λ_0 is the Bragg wavelength. The amplitudes of the forward and reverse waves can be described by the time-dependent coupled wave equations

$$\frac{1}{c_\mathrm{g}}\frac{\partial F}{\partial t} + \frac{\partial F}{\partial z} = \mathrm{i}kR - \left(\mathrm{i}\delta + \mathrm{i}\frac{\Delta\omega}{c_\mathrm{g}} - g - a_\mathrm{s}\right)F, \quad (2)$$

$$\frac{1}{c_\mathrm{g}}\frac{\partial R}{\partial t} - \frac{\partial R}{\partial z} = \mathrm{i}kF - \left(\mathrm{i}\delta + \mathrm{i}\frac{\Delta\omega}{c_\mathrm{g}} - g - a_\mathrm{s}\right)R, \quad (3)$$

where c_g is the group velocity of light in the active region, and a_s is the loss caused by the free-electron absorption and scattering. The grating-coupling coefficient k is related to the overlap integral of the forward and reverse wave fields and the Fourier coefficient of the grating. Because the spontaneous noise has only minor effect on the self-pulsations,[8] it was neglected in the travelling wave equations. The complex fields F and R can be replaced by their modulus and phase[9]

$$F = A_\mathrm{f}\exp(\mathrm{i}\phi_\mathrm{f}), \quad (4)$$

$$R = A_\mathrm{r}\exp(\mathrm{i}\phi_\mathrm{r}). \quad (5)$$

Then we can obtain the following equations

$$\frac{1}{c_g}\frac{\partial U_f}{\partial t}+\frac{\partial U_f}{\partial z}=2A_fA_rk\sin(\phi_f-\phi_r)+2(g-a_s)U_f, \quad (6)$$

$$\frac{1}{c_g}\frac{\partial U_r}{\partial t}-\frac{\partial U_r}{\partial z}=-2A_fA_rk\sin(\phi_f-\phi_r)+2(g-a_s)U_r, \quad (7)$$

where $U_f=A_f^2$, $U_r=A_r^2$. Following Eqs.(6) and (7), we can obtain

$$\begin{aligned}&\frac{1}{c_g}\frac{d}{dt}\int_0^l U_z dz\\ &=\int_0^{l_1}2(g_1-a_1)U_z dz\\ &\quad+\int_{l_1}^{l_2}2(g_2-a_2)U_z dz-\int_0^l\frac{\partial(U_f-U_r)}{\partial z}dz, \quad (8)\end{aligned}$$

where $U_z=U_f+U_r$, $l=l_1+l_2$, l_1 and l_2 are the lengths of the gain section and reflection section, respectively. Equation (8) can be expressed as

$$\begin{aligned}\frac{1}{c_g}\frac{dU_s}{dt}=&2(g_1-a_1)\Gamma_1U_s+2(g_2-a_2)\Gamma_2U_s\\ &-\int_0^l\frac{\partial(U_f-U_r)}{\partial z}dz, \quad (9)\end{aligned}$$

where $U_s=\int_0^l U_z dz$; g_1, a_1 and g_2, a_2 represent the gain and loss in the two sections, respectively. Γ_1, Γ_2 are the longitudinal fill factors, i.e. the relative portion of the power in each section[6]

$$\Gamma_k=\frac{\int_k^l U_z}{\int_0^l U_z}, \quad k=1,2,$$

where k=1,2 denote gain section and reflection section, respectively. They are the solutions of the instantaneous coupled mode equations. From Eqs.(8) and (9), we neglect spatial hole burning and assume that the carrier density are uniform in each section. The third item in the right of Eq.(9) represents the output loss from two outer facets of the laser. If the two outer facets of the lasers are anticoated, it can be expressed as

$$\int_0^l\frac{\partial(U_f-U_r)}{\partial z}dz=\gamma U_s, \quad (10)$$

where $\gamma=(U_f(l)+U_b(0))/U_s$. In two-section lasers with asymmetrical injection current, the reflection section usually works at transparency carrier density, so $g_2\approx 0$. Generally, the internal losses in the two sections are equal, so the loss coefficient can be represented by a. Because $S\propto U_s$, the rate equation for the photon number can be written as

$$\frac{dS}{dt}=(g_1-a)\Gamma_1c_gS-\Gamma_2ac_gS-\gamma c_gS. \quad (11)$$

The carrier density in each section can be written as

$$\frac{dN_k}{dt}=\frac{I_k}{q\nu}-\frac{N_k}{\tau}-g_k\Gamma_kS, \quad (12)$$

where N_k and I_k are the carrier density and injection current in section k, respectively (k=1,2). The time evolution of the number of photons and carrier density in each section are governed by the rate Eqs. (11) and (12).

Compared with the conventional photon number rate equation, an additional loss (Γ_2ac_gS) appears in the rate Eq. (11). The effective gain can be written as $G=c_g(\Gamma_1g_1-a)$. The effective gain in the cavity changes with the longitudinal fill factors of the gain section. This clearly shows that gain Q-switching process happens in self-pulsation process. The new photon rate equations are different from that proposed in Ref.[6], where longitudinal analogy of Peterman's factor of excess spontaneous emission (K) was introduced in single mode rate equation to explain the self-pulsation. In that paper, the dynamic distribution of the power in two sections was not considered and the variation of the effective gain was neglected, so the K factor was introduced to balance the net gain and the output loss. In this paper, the longitudinal fill factor is introduced to describe the variation of the power distribution in two sections, so, the K factor does not appear in the photon number rate equations. From the Eq.(9), it can be seen that for self-pulsation to appear, the reflection section must work below the threshold, if the reflection section is biased above the threshold ($g_2>a_2$), the additional loss will not exist. This can explain why the reflection section is normally biased at transparency carrier density in experiments.

The condition for self-sustained relaxation oscillation can be obtained by small signal stability analysis of the rate Eqs. (11) and (12). Because the reflection section operates near to its gain transparency, the carrier-photon coupling of this section can be

neglected.[6] Then the damping rate of the relaxation oscillations and the small signal resonance frequency can be expressed as follow

$$G = c_{\mathrm{g}}(\Gamma_1 g - a),$$

$$\Gamma_{\mathrm{damp}} = \frac{1}{2}\left[\frac{1}{\tau} + G_N S - G_S S\right], \tag{13}$$

$$\Omega = \left[(G+G_S S)(G_N S) - \frac{1}{4}\left(\frac{1}{\tau}+G_N S+G_S S\right)^{1/2}\right]^{1/2}, \tag{14}$$

where G is the effective net gain, Γ_{damp} and Ω are damping rate of the relaxation oscillation and the small signal resonance frequency. The indices N, S denote the derivative with respect to the number of carriers and the number of photons in the gain section. From Eq.(13), the condition for the self-sustained relaxation oscillation is

$$G_S S > G_N S + \frac{1}{\tau}. \tag{14}$$

In fact, the longitudinal fill factors depend on the effective reflection coefficient of the reflection section. The effective reflection coefficient is only effected by the difference of carrier densities between two sections. So we can assume $\Gamma_{1S} = 0$. Then an indispensable condition for the self-sustained relaxation oscillations is deduced from Eq.(14)

$$\Gamma_{1N} < 0. \tag{15}$$

It can be seen that the longitudinal fill factors influence both the damping rate and the resonance frequency. If the longitudinal fill factors remain constant ($\Gamma_{1N} = 0$, $\Gamma_{1S} = 0$) then the damping rate of the relaxation oscillations and the small signal resonance frequency are same with normal lasers. The sustained relaxation oscillation will not appear. From Eq.(15), it can be seen that the self-sustained pulsation will not appear, if the derivative of longitudinal fill factor to carrier density in gain section is larger than zero.

The self-pulsation process is numerically simulated. The parameters used in the simulation are typical values for two-section DFB lasers. The time evolution of the longitudinal fill factor, output optical power, and the carrier density in the gain section, are showed in Fig.1. The bias currents in the gain section and reflection section are 150 and 10mA, respectively.

It can be seen from curves *a* and *c* in Fig.1 that the evolution of longitudinal fill factor and carrier density in gain sections are antiphased. The condition in the Eq.(15) for self-pulsation is confirmed by the numerical simulation.

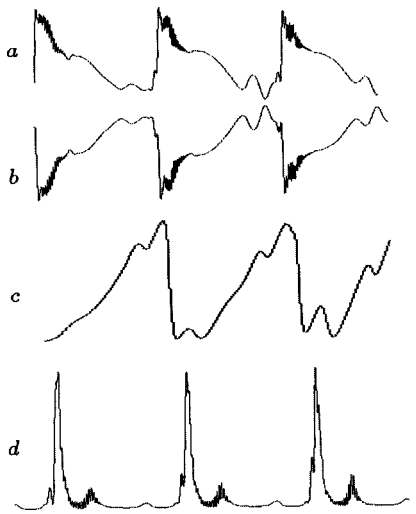


Fig.1. Time evolution trace of longitudinal fill factor in gain section (curve *a*); Time evolution trace of longitudinal fill factor in reflection section (curve *b*); Normalized time evolution trace of carrier density in gain section (curve *c*); Normalized time evolution trace of output optical power (curve *d*).

We can see that curves *a*, *b*, *c* and *d* in Fig.1 clearly show a gain *Q*-switching process. When the longitudinal fill factor in the gain section decreases, the longitudinal fill factor in the reflection section increases at the same time. Hence the additional loss increases and net gain decreases in the gain section. In this stage, the carrier density in the gain section increases (curve *c*). The high carrier density in the gain section results in increasing of the detuning of the Bragg wavelengths between the gain section and the reflection section. Eventually, at a critical point, where the detuning of the Bragg wavelength between two sections and the oscillation mode in the cavity satisfy the reflection condition, the optical power that gets away from the gain section is reflected back to the gain section. In this stage, longitudinal fill factor in the reflection section reaches its maximum value, the power portion absorbed by the reflection section stops increasing. Which results in decreasing the threshold in the gain section. At the same time, the high carrier density in the gain section brings up the photon density rapidly and high photon density saturates the gain rapidly and the optical power reaches the maximum value. As the carrier density in the gain section decreases, the detuning of the Bragg wavelengths between two sections decreases, the optical power in the gain section begins to diffuse to the reflection section. These two factors cause the power in the gain section

to decrease and form the falling edge of the optical pulses.

In conclusion, a new set of rate equations was deduced from the travelling wave equations. The condition for self-pulsation in multi-section DFB lasers was given. It was found that the longitudinal fill factor in the gain section plays key role in the self-pulsation process.

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