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Effects of transverse profile of pump field on second harmonic generation in periodic nonlinear materials*

Zhang Han-Zhuang(张汉壮)†, Yang Jian-Bing(杨建冰), and Gao Jin-Yue(高锦岳)

College of Physics, Jilin University, Changchun 130023, *China*

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We report on a theoretical analysis of the effects of a converging pump field of Gaussian transverse profile on second harmonic generation in a periodic nonlinear material with quasi-phase-matching. The outputs of the centre intensity and the intensity flux for second harmonic generation are derived by simulation, based on the parameters of quasi-phase-mismatch, the waist and focus positions of the input pump beam. The results show that when the transverse profile of the pump field is taken into account, the quasi-phase-match value and focus position of input beam for maximal second harmonic generation follow new criteria.

Keywords: periodic nonlinear materials, second harmonic generation, quasi-phase-matching
PACC: 4265K, 4265B

1. Introduction

Photonic crystals are optical superlattices with a periodic dielectric function (linear photonic crystal), or with a periodic nonlinear optical susceptibility function (periodic nonlinear material). One intriguing issue of linear photonic crystal is the existence of photonic band gaps, i.e. frequency intervals where propagation of light is forbidden inside the structure. In periodic nonlinear materials,[1−4] light frequency conversion, such as second and third harmonic generation (SHG, THG) and optical parametric oscillation, can be realized by quasi-phase-matched technique. Study on periodic nonlinear materials has been the subject of a numbers of theoretical[5−9] and experimental[10−21] works. As far as we know, most papers on this subject are based on the plane wave approximation for the pump field. However, in more realistic situations, the input beam is often focused, consequently the pump field has a converging Gaussian transverse profile in space. In this paper, we discuss theoretically the effects of such a transverse field on the second harmonic generation in a periodic nonlinear material by quasi-phase-match technique.

2. Theoretical equations

Consider firstly the coupling between fundamental and second harmonic wave fields in a periodically poled optical crystal based on quasi-phase-match technique. In the slowly varying envelope approximation, the coupling equations can been written as[22]

$$\frac{\partial F_2}{\partial \eta} = \mathrm{i}\frac{\beta}{4}\nabla_\perp^2 F_2 + \mathrm{i}\alpha_2 f F_1^2 \exp(\mathrm{i}\Delta k\eta), \tag{1a}$$

$$\frac{\partial F_1}{\partial \eta} = \mathrm{i}\frac{1}{4\beta}\nabla_\perp^2 F_1 + \mathrm{i}\alpha_1 f F_1^* F_2 \exp(-\mathrm{i}\Delta k\eta), \tag{1b}$$

$$\nabla_\perp^2 F = \frac{\partial^2 F}{\partial \rho^2} + \frac{1}{\rho}\frac{\partial F}{\partial \rho}, \tag{1c}$$

where

$$\eta = \frac{Z}{L}, \quad \rho = \sqrt{\frac{\pi}{\lambda L}}r, \quad \lambda = \sqrt{\lambda_1\lambda_2}, \quad \beta = \sqrt{\frac{\lambda_2}{\lambda_1}},$$

$$\alpha_1 = 2\alpha_2 = \frac{2\pi L}{\lambda_1}, \quad \Delta k = \left(\frac{2\pi}{\lambda_2} - \frac{2\pi}{\lambda_1}\right)L,$$

$$F_i = E_i d_{\mathrm{eff}} (i = 1, 2).$$

Here L is the length of the crystal sample. λ_1 and λ_2 are, respectively, the wavelengths of fundamental and second harmonic waves. Δk is the wavevector mismatch (dimensionless). η and ρ are the longitudinal and transverse dimensionless variables, respectively, as shown in Fig.1. d_{eff} is the effective second-order nonlinear optical susceptibility coefficient of the ferroelectric crystal. $F_i (i = 1, 2)$ is the dimensionless

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†E-mail: Hzzhang@mail.jlu.edu.cn

variable that is proportional to the amplitude of the fundamental (E_1) and the second harmonic (E_2) wave fields. In order to simulate the periodic domains in a ferroelectric crystal, we divide the sample into N (integer) domains, and let f=1 if the domain number from the entrance of the crystal is even, otherwise, $f=-1$. Therefore, the periodicity of the reciprocal lattice vector in such a photonic superlattice crystal is given by $G_m = m\frac{2\pi}{(2/N)} = mN\pi$, where $m = 0, \pm1, \pm2, \cdots$ The difference $\Delta k - G_m$ is hence the quasi-phase-mismatch, which reflects the periodicity of the periodic nonlinear material in Eqs.(1) and (2). For simplicity, we ignore the possible angular variations of the fundamental and the second harmonic fields with respect to the angular coordinate around the longitudinal axis of the crystal. The transverse profile of the fields is embodied in the transverse Laplacian operators in Eq.(1). In the following, we show that the transverse profile plays an important role when the input beam is highly focused.

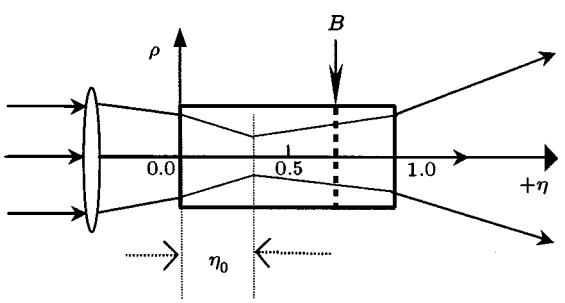


Fig.1. Schematic diagram of the crystal sample with dimensionless coordinates.

At the entrance of the crystal, we assume that the second harmonic wave field has zero input, whereas the fundamental wave field has a Gaussian transverse profile,[22] i.e.

$$F_2(\rho, \eta = 0) = 0, \tag{2a}$$

$$F_1(\rho, \eta = 0) = \frac{\chi_1\sqrt{P_1}}{\sigma(\eta_0)}\exp\left(-\frac{\rho^2}{\sigma^2(\eta_0)}\right) \times \exp\left(\mathrm{i}\frac{\rho^2}{R(\eta_0)}\right)\exp(-\mathrm{i}\Phi(\eta_0)), \tag{2b}$$

where

$$\sigma(\eta_0) = \sqrt{\left(1+\frac{\eta_0^2}{\sigma_0^4}\right)}\sigma_0, \quad \sigma_0 = \sqrt{\frac{\pi}{\lambda L}}\omega_0,$$

$$\frac{1}{R(\eta_0)} = \frac{\eta_0}{\eta_0^2+\sigma_0^4},$$

$$\Phi(\eta_0) = \tan^{-1}\left(\frac{\eta_0}{\sigma_0^4}\right), \quad \chi_1 = \sqrt{\frac{Z_0}{2n_1\lambda L}}d_{\mathrm{eff}},$$

$$\chi_2 = \sqrt{\frac{Z_0}{2n_2\lambda L}}d_{\mathrm{eff}}, \quad Z_0 = 377.$$

Here $n_i (i = 1, 2)$ is the refractive index of the crystal for the fundamental (i=1) and the second harmonic (i=2) waves. We choose a suitable expression for χ_1, so that P_1 is proportional to the input power of the fundamental wave field. η_0 is the dimensionless coordinate of the focus position on the longitudinal axis as labelled in Fig.1. The parameter σ_0 is the dimensionless minimum spot size, which is the "waist" of the light beam. We solve the intensity flux $S_i = \int\int |F_i|^2 \mathrm{d}s = \int 2\pi\rho|F_i|^2\mathrm{d}\rho (i = 1, 2)$ and the centre intensity $|F_i(\rho = 0)|^2 (i = 1, 2)$ for the fundamental (i=1) and the second harmonic (i=2) wave fields from the Eqs.(1a), (1b) and (1c). The numerical procedure is based on the discretization of the transverse Laplacian in terms of fourth-order differences, and the integration of the first-order differential equation is obtained by the fourth-order Runge–Kutta method.

3. Numerical calculation and discussion

In order to explore the effects of transverse profile of the pump field on SHG, we firstly calculate and plot the outputs of the intensity flux $S_2(\eta = 1.0)$ (Figs.2(a) and 2(c)) and the centre intensity $|F_2(\eta = 1.0, \rho = 0.0)|^2$ (Figs.2(b) and 2(d)) for the SHG under first-order quasi-phase-matched condition in the plane wave approximation, ignoring the terms $\nabla_\perp^2 F_1$ and $\nabla_\perp^2 F_2$ in Eqs.(1a) and (1b), with the initial conditions

$$F_2(\rho, \eta = 0) = 0, \tag{3a}$$

$$F_1(\rho, \eta = 0) = \frac{\chi_1\sqrt{P_1}}{\sigma(\eta_0)}\exp\left(-\frac{\rho^2}{\sigma^2(\eta_0)}\right). \tag{3b}$$

We next consider the Laplacian operators of $\nabla_\perp^2 F_1$ and $\nabla_\perp^2 F_2$ in Eqs.(1a) and (1b) with the initial condition of Eqs.(2a) and (2b). The plots are presented in Fig.3 in which the parameters used are the same as in Fig.2.

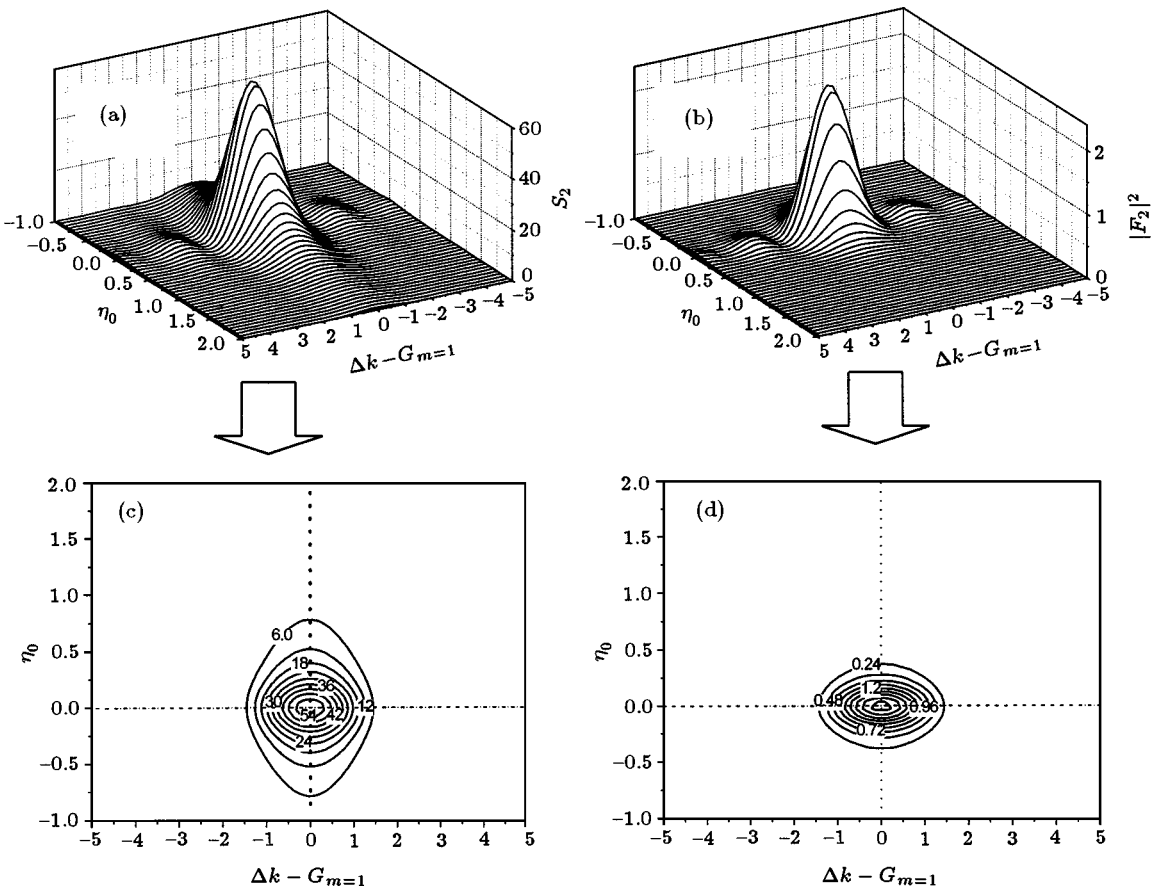


Fig.2. (a) and (b) are respectively the plots of output intensity flux $S_2(\eta = 1.0) \times 10^{-12}$ and the centre intensity $|F(\rho = 0, \eta = 1)|^2 \times 10^{-12}$ for SHG versus focus position η_0 and quasi-phase-mismatch $\Delta k - G_m$ (in π) in the vicinity of the first-order reciprocal lattice vector of a photonic crystal. The simulation assumes the plane wave approximation, i.e. $\nabla_\perp^2 F_1 = 0$ and $\nabla_\perp^2 F_2 = 0$ in Eqs.(1a) and (1b). (c) and (d) are the top views of (a) and (b), respectively. The parameters used are: the length of the crystal L=1.0cm, the refractive index for the fundamental and the second harmonic waves n_1=2.1 and n_2=2.2, P_1=50.0 and P_2=0.0, fundamental wavelength λ_1=1090nm, first-order reciprocal lattice vector $G_{m=1} = 1000\pi$ and the waist of input beam σ_0=0.5.

Comparing Fig.2 and Fig.3, we see that the transverse profiles introduce important effects. Firstly, the intensity flux and the centre intensity for the SHG become asymmetrical with respect to quasi-phase-mismatch. Secondly, the value of the total quasi-phase-mismatch $\Delta k - G_{m=1}$ for maximal SHG has a small shift from zero, and is a function of the waist of the input beam as shown in Fig.6. The results may be explained by a calculation of the phases of the SHG on a plane perpendicular to the axis as shown in Fig.4. It is seen that the phases vary with positions ρ due to the transverse profile. From Eq.(1), when the transverse profile is included, we see that the signal at a particular point is related not only to the signals at the points with the same ρ in the crystal, but also to those with different ρ. In contrast to plane wave approximation, the SHG at a particular point consists of contributions from neighbouring fields with different phases.

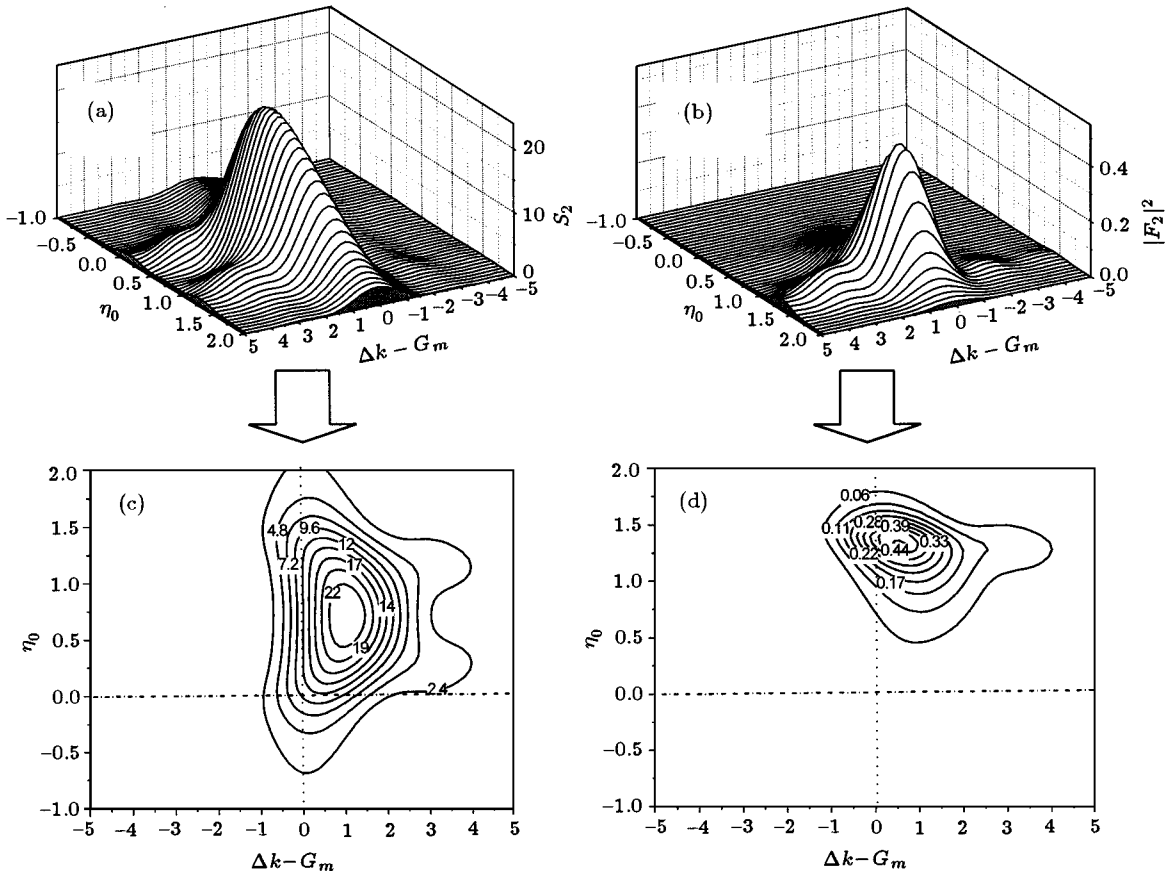


Fig.3. Same parameters as Fig.2, except that $\nabla_\perp^2 F_1 \neq 0$ and $\nabla_\perp^2 F_2 \neq 0$ in Eqs.(1a) and (1b), with the initial conditions given by Eqs.(2a) and (2b).

Finally, from Fig.3, we see that for maximal centre intensity of SHG, the focus position should be closer to the exit of the crystal. On the other hand, for maximal intensity flux, it should be at the vicinity of the centre of the crystal. Fig.3 also show that the centre intensity is more sensitive to the change of focus position than the intensity flux. The reason stems from the evolutions of the fields in the crystal as shown in Fig.5. Further calculated results under higher-order quasi-phase-matched conditions are similar to those obtained under first-order condition. Furthermore, when the waist of the pump field is large, the results tend towards that in the plane wave approximation, as shown in Fig.6. In practice, how narrow the beam has to be focused to see these effects depends on the parameters of the length of the crystal sample and the wavelength of fundamental wave. The longer the length of the crystal and the wavelength of fundamental wave, the larger the focused waist of input beam should be. As an example, if the crystal length and the wavelength of fundamental wave are 2 cm and 1093nm, respectively, the focused waist of input beam is estimated to be about 80μm.

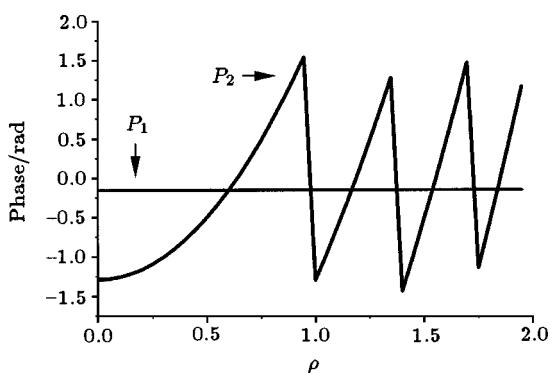


Fig.4. The phases of SHG fields on the plane B (as indicated in Fig.1) as a function of the transverse variable ρ. P_1 is for plane wave approximation and P_2 is for transverse profile. The value of the phase varies between $-\frac{\pi}{2}$ and $\frac{\pi}{2}$ in one period. The parameters used are: waist $\sigma_0 = 0.5$, focus position $\eta_0 = 0.5$, wavevector mismatch $\Delta k = 1000\pi$, the longitudinal coordinate of plane $B\eta = 0.8$. Other parameters are the same as in Fig.2.

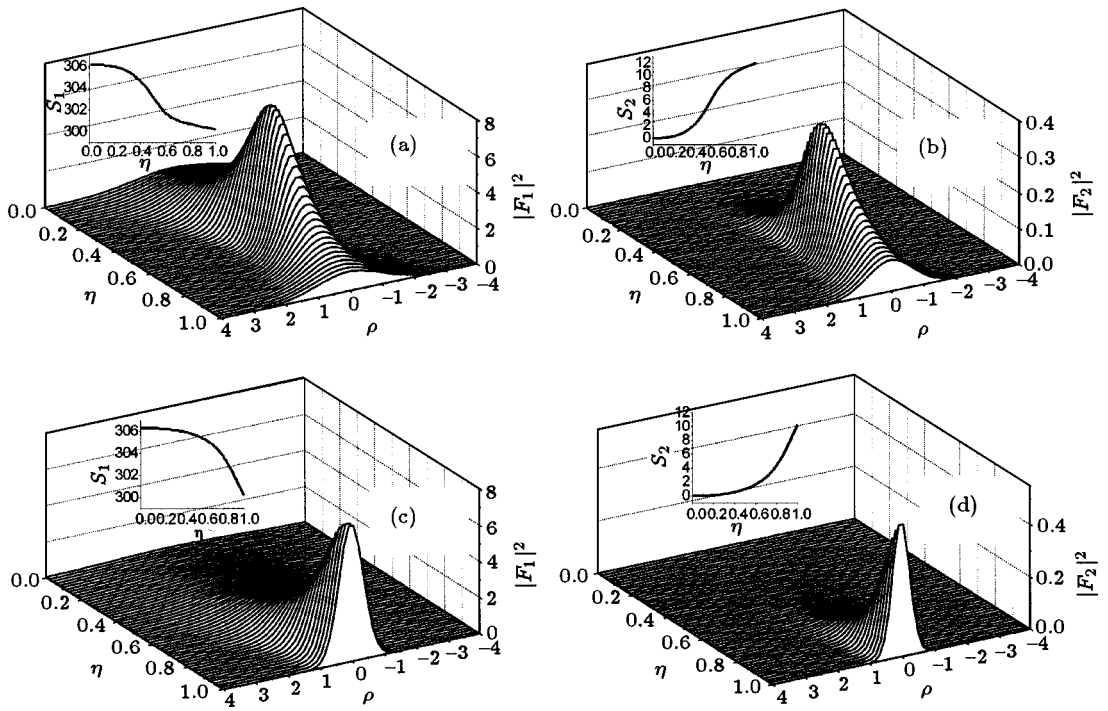


Fig.5. (a) and (c) represent the curves of fundamental wave intensity corresponding to the longitudinal coordinate and the transverse coordinate of a crystal. (b) and (d) are the corresponding second harmonic intensity plots. The parameters used are: waist $\sigma_0 = 0.5$, focus positions $\eta = 0.8$ for (a) and (b), and $\eta = 1.4$ for (c) and (d). Other parameters are the same as in Fig.2.

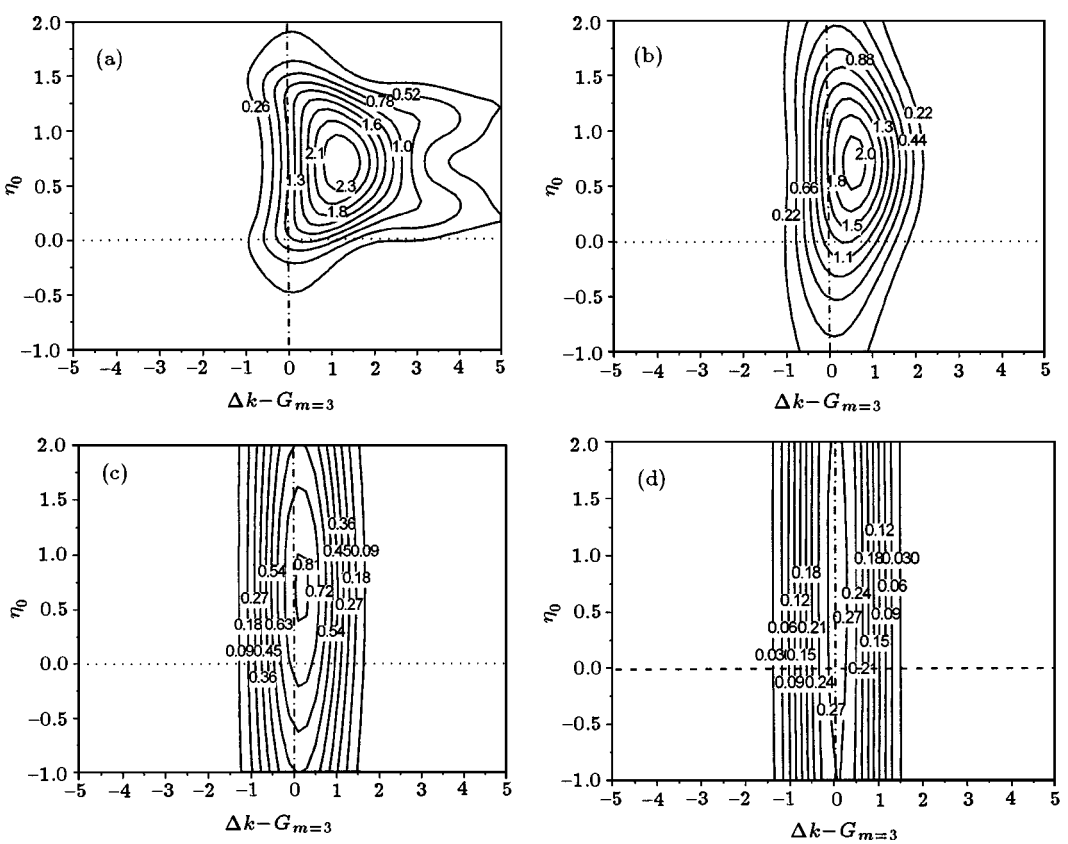


Fig.6. Plots of output intensity flux in the vicinity of the third-orders reciprocal lattice vector $G_{m=3} = 3000\pi$ for different values of waist. The results for σ_0=0.4, 0.8, 1.5, 3.0 are shown in (a), (b), (c) and (d), respectively. Other parameters are the same as in Fig.2.

4. Conclusion

In conclusion, when the pump beam is highly focused, the intensity flux and the centre intensity for the SHG are asymmetric with respect to quasi-phase-mismatch, and the total quasi-phase-matched value has a small shift from zero. For maximal output of the centre intensity, the optimal focus position should be closer to the exit of the crystal in the selected parameters of this paper. For the intensity flux, the optimal focus position should be at the vicinity of the centre of the crystal. The centre intensity is more sensitive to a change of focus position than intensity flux. When the waist of pump field is large, the results tend towards that in the plane wave approximation.

References

[1] Armstrong J A, Bloembergen N, Ducuing J and Pershan P S 1962 *Phys. Rev.* **127** 1918

[2] Fejer M M, Magel G A, Jundt D H and Byer R L 1992 *IEEE J. Quantum Electron.* **28** 2631

[3] Mizuuchi K and Yamamoto K 1996 *Opt. Lett.* **21** 107

[4] Zhu S and Cao W W 1997 *Phys. Rev. Lett.* **79** 2558

[5] Arbore M A, Marco O and Fejer M M 1997 *Opt. Lett.* **22** 865

[6] Berger V 1998 *Phys. Rev. Lett.* **81** 4136

[7] Huang X G and Wang M R 1998 *Opt. Commun.* **150** 235

[8] Liu X, Qian L J and Wise F 1999 *Opt. Commun.* **164** 69

[9] Hsu C W and Yang C C 1999 *Opt. Lett.* **24** 540

[10] Pruneri V, Koch R, Kazansky P G, Clarkson W A, Russell P St J and Hanna D C 1995 *Opt. Lett.* **20** 2375

[11] Lu Y Q, Lu Y L, Xue C C, Zheng J J, Chen X F, Ming N B, Feng B H and Zhang X L 1996 *Appl. Phys. Lett.* **69** 3155

[12] Arie A, Rosenman G, Mahal V, Skliar A, Oron M, Katz M and Eger D 1997 *Opt. Commun.* **142** 265

[13] Zhu S N, Zhu Y Y, Qin Y Q, Wang H F, Ge C and Ming N B 1997 *Phys. Rev. Lett.* **78** 2752

[14] Zhu S N, Zhu Y Y and Ming N B 1997 *Science* **278** 843

[15] Taverner D, Britton P, Smith P G R, Richardson D J, Ross G W and Hanna D G 1998 *Opt. Lett.* **23** 162

[16] Ross G W, Pollnau M, Smith P G R, Clarkson W A, Britton P E and Hanna D C 1998 *Opt. Lett.* **23** 171

[17] Mizuuchi K and Yamamoto K 1998 *Opt. Lett.* **23** 1880

[18] Clausen C B, Kivshar Y S, Bang O and Christiansen P L 1999 *Phys. Rev. Lett.* **83** 4740

[19] Belmonte M, Skettrup T and Pedersen C 1999 *J. Opt. A: Pure Appl. Opt.* **1** 60

[20] Lu Y Q, Zheng J J, Lu Y L and Ming N B 1999 *Appl. Phys. Lett.* **74** 123

[21] Broderick N G R, Ross G W, Offerhause H L, Richardson D J and Hanna D C 2000 *Phys. Rev. Lett.* **84** 4345

[22] Boyd R W 1992 *Nonlinear Optics* (San Diego: Academic) p90