Vol 12 No 5, May 2003
1009-1963/2003/12(05)/0502-05
Chinese Physics
© 2003 Chin. Phys. Soc.
and IOP Publishing Ltd

A modified split-step Fourier method for optical pulse propagation with polarization mode dispersion*

Rao Min(饶 敏), Sun Xiao-Han(孙小菡), and Zhang Ming-De(张明德)

Department of Electronics Engineering, Southeast University, Nanjing 210096, *China*

(Received 15 January 2003)

A modified split-step Fourier method (SSFM) is presented to solve the coupled nonlinear Schrödinger equation (CNLS) that can be used to model high-speed pulse propagation in optical fibres with polarization mode dispersion (PMD). We compare our approach with the SSFM and demonstrate that our approach is much faster with no loss of accuracy. We discuss the pulse distortion and system Q-factor of non-return-to-zero (NRZ), return-to-zero (RZ) and pre-chirped RZ (CRZ) formats in the presence of high PMD through this approach. The simulation results show that CRZ pulses are the most tolerant to high PMD values and the extinct ratio has a great impact on the transmission performance.

Keywords: polarization mode dispersion, non-return-to-zero, return-to-zero, pre-chirped RZ
PACC: 4225B, 4281F, 4230L

1. Introduction

As the bit rate of a single channel in wavelength-division multiplexing (WDM) system is increased to 40Gb/s or beyond, polarization mode dispersion (PMD) impairs the transmission performance more severely and has become one of the critical hurdles for next-generation optical transmission systems with high bit rate.[1−3] A great deal of discussions about the generation of PMD and optical compensation techniques have been reported.[4−6] However, it is more important to understand the effect of PMD on the optical pulses with different modulation formats[7−9] for realizing the high-speed optical transmission system, which has been attracting more and more interest recently.[9,10]

To investigate the PMD effect, it is necessary to track the propagation of the high-speed optical pulse in long-haul optical links by using the nonlinear optical propagation equation. At present the split-step Fourier method (SSFM)[11,12] is used frequently to solve nonlinear Schrödinger equation.[13−16] This method may lead to errors if the step size is not small enough, thus it requires large run-times to yield accurate results.[12] Previous studies on the improvement of SSFM focus largely on the simplification for some parts of the system,[17,18] which cannot be used in general cases. In this paper, a novel modified SSFM is proposed. This approach can improve the simulation speed greatly with no loss of accuracy by using an interpolation technique. The pulse distortion and Q-factor of non-return-to-zero (NRZ), return-to-zero (RZ) and pre-chirped RZ (CRZ) formats in 10 and 40Gb/s systems are calculated through this method, respectively. The modified SSFM can be used widely in the numerical simulation of high-speed long-haul optical transmission systems, which is of great importance to the investigation of the optical transmission impairments and the compensation techniques.

2. Theoretical model

2.1. Propagation equation and SSFM

Consider the chromatic dispersion, PMD, and nonlinear effects, the coupled nonlinear Schrödinger (CNLS) equations describing a fibre with a relatively high birefringence can be written as[13−16]

$$\begin{aligned}&\frac{\partial A_x}{\partial z}+\beta'_x\frac{\partial A_x}{\partial t}+\frac{\mathrm{i}}{2}\beta''\frac{\partial^2 A_x}{\partial t^2}+\frac{\alpha}{2}A_x\\&=\mathrm{i}\gamma\left(|A_x|^2+\frac{2}{3}|A_y|^2\right)A_x,\\&\frac{\partial A_y}{\partial z}+\beta'_y\frac{\partial A_y}{\partial t}+\frac{\mathrm{i}}{2}\beta''\frac{\partial^2 A_y}{\partial t^2}+\frac{\alpha}{2}A_y\\&=\mathrm{i}\gamma\left(|A_y|^2+\frac{2}{3}|A_x|^2\right)A_y,\end{aligned}\tag{1}$$

*Project supported by the National Natural Science Foundation of China (Grant No 60272048).

where A_x, A_y are slowly varying amplitudes of the two polarization components, $\beta'_x - \beta'_y$ is wavevector mismatch due to linear birefringence, β'' is the group velocity dispersion parameter, α is the absorption coefficient, the nonlinearity coefficient $\gamma = n_2\omega_0/A_{\text{eff}}c$, where the Kerr coefficient $n_2 = 2.6 \times 10^{-16}\text{cm}^2/\text{W}$, ω_0 is the centre frequency, the effective mode area $A_{\text{eff}} = 50\mu\text{m}^2$, and c is the speed of light.

SSFM assumes that the dispersion and fibre nonlinearity act one at a time over the elementary step Δz. The expression of the spatial derivative of the mode amplitudes is then broken down into two operators written as $A(z + \Delta z, t) \approx \exp(\Delta z\hat{D})\exp(\Delta z\hat{N})A(z, t)$, where $\hat{D}$ is the linear differential operator responsible for the dispersion and absorption, $\hat{N}$ is the nonlinear operator responsible for fibre nonlinearity. In this scheme, it is important to select an appropriate value of Δz. In the parameter regime in which present-day communication systems operate, the dispersion and nonlinear scale lengths are hundreds of kilometres, while the birefringence is large in the sense that the beat lengths are typically 10–100m. Δz should be selected as a fraction of the smallest of the above characteristic lengths, so 10^{-2}km scale is usually used, which is computationally expensive.

2.2. Modified SSFM

Using the moving coordinate system with an average velocity $\overline{v}_{\text{g}} = \dfrac{1}{\overline{\beta'}} = \dfrac{2}{\beta'_x + \beta'_y}$, defining $T = t - z/\overline{v}_{\text{g}}$, Eq.(1) can be written as[11]

$$\begin{aligned}
&\frac{\partial A_x}{\partial z} + \frac{\beta'_x - \beta'_y}{2}\frac{\partial A_x}{\partial T} + \frac{\mathrm{i}}{2}\beta''\frac{\partial^2 A_x}{\partial T^2} + \frac{\alpha}{2}A_x \\
&= \mathrm{i}\gamma\left(|A_x|^2 + \frac{2}{3}|A_y|^2\right)A_x, \\
&\frac{\partial A_y}{\partial z} - \frac{\beta'_x - \beta'_y}{2}\frac{\partial A_y}{\partial T} + \frac{\mathrm{i}}{2}\beta''\frac{\partial^2 A_y}{\partial T^2} + \frac{\alpha}{2}A_y \\
&= \mathrm{i}\gamma\left(|A_y|^2 + \frac{2}{3}|A_x|^2\right)A_y. \qquad (2)
\end{aligned}$$

To reduce the times of fast Fourier transform (FFT), the step size is selected at 1 km scale. At the same time, we subdivide each interval Δz into N segments in polarization maintain (PM) fibre, the length of which is selected to correspond to the fibre birefringence length typically 0.3 to 300m.[19] To simplify the discussion, we assume that the gain in the amplifiers balances the fibre loss. Defining $b' = (\beta'_x - \beta'_y)/2$, the linear part of Eq.(2) in the Fourier may be written in the form

$$\frac{\partial \boldsymbol{A}}{\partial z} + \mathrm{i}b'\boldsymbol{\sigma}\omega\boldsymbol{A} - \frac{\mathrm{i}}{2}\beta''\omega^2\boldsymbol{A} = 0, \qquad (3)$$

where $\boldsymbol{\sigma} = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$, $\boldsymbol{A} = (A_x, A_y)^t$. In the small subinterval $h = \dfrac{\Delta z}{N}$, the solution to Eq.(3) may be written as

$$\boldsymbol{A}(z + h, \omega) = \exp\left(\frac{\mathrm{i}}{2}\beta''\omega^2 h\right)\begin{bmatrix} \exp(-\mathrm{i}b'\omega) & 0 \\ 0 & \exp(\mathrm{i}b'\omega) \end{bmatrix}\boldsymbol{A}(z, \omega). \qquad (4)$$

Consider the random coupling between the PMF sections, in each interval Δz, the transfer matrix of the fibre can be written as $\boldsymbol{T}(\omega) = \exp\left(\dfrac{\mathrm{i}}{2}\beta''\omega^2\Delta z\right)\boldsymbol{M}(\omega)$, where $\boldsymbol{M}(\omega)$ is represented by

$$\boldsymbol{M}(\omega) = \prod_{n=1}^{N}\begin{bmatrix} \exp\left[-\mathrm{i}\left(\dfrac{\Delta\varphi}{2} + \phi_n\right)\right] & 0 \\ 0 & \exp\left[\mathrm{i}\left(\dfrac{\Delta\varphi}{2} + \phi_n\right)\right] \end{bmatrix} \cdot \begin{bmatrix} \cos\theta_n & \sin\theta_n \\ -\sin\theta_n & \cos\theta_n \end{bmatrix}, \qquad (5)$$

where $\Delta\varphi = \sqrt{\dfrac{3\pi h}{8}}D_{\text{PMD}}\omega$ is the differential phase delay (DPD) of each segments,[20] D_{PMD} $(\text{ps}/\text{km}^{1/2})$ is the PMD coefficient of the fibre and may be written as $D_{\text{PMD}} = 2b'\sqrt{\dfrac{8h}{3\pi}}$, ϕ_n and θ_n denote the random phase and coupling angle in the nth subinterval respectively, and are randomly generated following a uniform distribution in $[0, 2\pi)$.

Since FFT will be used in each step, if we calculate $\boldsymbol{M}(\omega)$ at each of the frequencies, the procedure will be slow. However, the variation of $\boldsymbol{M}(\omega)$ as a function of frequency is very slow on the length scale given by Δz, and it is only necessary to evaluate $\boldsymbol{M}(\omega)$ at a few frequencies and to interpolate between them. This approach will increase the calculation speed significantly. Figure 1 shows the slow variation of $\boldsymbol{M}(\omega)$

with frequency. Here we consider an example in which Δz=1km, h=20m, and $D_{\rm PMD}$=3ps/km$^{1/2}$. In Fig.1, Re(M_{ij}) and Im(M_{ij}) represents the real and imaginary parts of the elements in $\boldsymbol{M}(\omega)$ respectively.

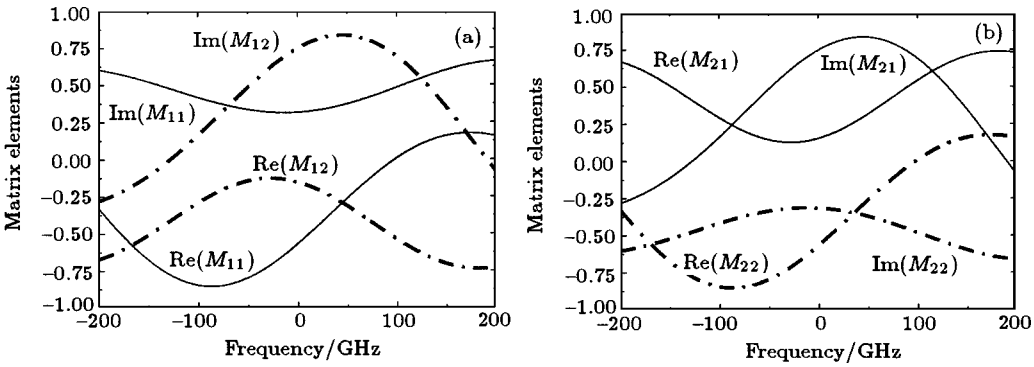


Fig.1. The slow variation of the elements of M-matrix with frequency.

Dealing with the nonlinear terms in the time domain is quite straightforward, and we use Eq.(29) in Ref.[19].

2.3. Q-factor and system penalty

Consider the PMD degradation, the system Q-factor can be calculated using

$$Q = \frac{U_1 - U_0}{\sqrt{N_1} + \sqrt{N_0}}, \tag{6}$$

where N_1 and N_0 denote the total noise of data "1" and "0", respectively, including thermal noise, shot noise, signal-spontaneous beat noise, and spontaneous–spontaneous beat noise contributions. Their average powers are reported in Ref.[21] Eqs.(8)–(11). Assuming all of the noises could be denoted by Gaussian distribution, the total noise power can be expressed as $N_{1/0} = N_{\rm th} + N_{{\rm shot},1/0} + N_{s-sp,1/0} + N_{sp-sp}$. U_1 and U_0 are the average received optical power of data "1" and "0", respectively. It can be found that the bit error rate (BER) is mainly determined by "010" and "101" in these two three-bit patterns, which corresponds to the minimum eye opening. So considering the extreme case, U_1 and U_0 can be expressed as

$$U_1 = \int_{t_{\rm d}-\frac{T}{2}}^{t_{\rm d}+\frac{T}{2}} |\boldsymbol{E}_{\rm b}(t)|^2 {\rm d}t, \tag{7}$$

$$U_0 = \int_{t_{\rm d}-\frac{T}{2}}^{t_{\rm d}+\frac{T}{2}} (|\boldsymbol{E}_{\rm b}(t-T)|^2 + |\boldsymbol{E}_{\rm b}(t+T)|^2) {\rm d}t, \tag{8}$$

where $t_{\rm d}$ is the decision time, which is determined at the middle of the bit duration in this paper. Due to the statistical character of PMD, the Q-factor will vary with the time. Here we choose the average value of Q-factor. The PMD induced Q-factor penalty is defined as $p_Q = 20\log(Q/Q_0)$, and Q_0 is the Q-factor without PMD. The relationship between the Q-factor penalty and the propagation length can be obtained through the model above.

3. Simulation results and analysis

Dispersion compensation fibre (DCF) is usually used to compensate the chromatic dispersion in long haul optical communication systems. We use here post-compensation scheme consisting of a periodic sequence of 80km single-mode fibre (SMF) section and of 20km compensation section. The parameters we used are listed in Table 1.

Table 1. Parameters of SMF and DCF.

	SMF	DCF
Dispersion @ 1550nm/(ps/(km·nm))	–2.125	17
Length/km	80	10
Nonlinear refractive index n_2	2.6	2.6
Effective core area/μm^2	80	30

We assume that the input field is a Gaussian pulse given by $E_a(t) = A_0 \exp\left[-\frac{1+{\rm i}C}{2}\left(\frac{t}{T_0}\right)^{2m}\right]$, where C is the initial chirp factor (C=0 for NRZ and RZ formats), A_0 is the peak amplitude of the pulse, T_0 denotes the pulse width parameter, given by $T_0 = \Delta T/[2(\ln 2)^{1/2m}]$. The NRZ pulse, with ΔT equal to the bit duration, is given by m=1.436 (super-Gaussian) while the RZ and CRZ formats, with ΔT equal to the full width at half maximum of the pulse, is given by m=1. The ratio between ΔT and the bit

period is the extinct ratio, the value of which is 1 for NRZ format. How to choose the extinct ratio of RZ and CRZ is related to the system parameters and will affect the transmission performance significantly.

3.1. The simulation speed of SSFM and modified SSFM

The standard SSFM and modified SSFM are used to simulate the NRZ pulses transmission in 40Gb/s systems. The output pulses after 360 km propagation are shown in Fig.2 (a), in which the dashed line and solid line represent the input and output pulses respectively. The shape of the pulses is strongly dependent on the random coupling angle between the subinterval PMF sections. The overshoot seen at the edges of the pulses can be somewhat larger or smaller. It can be found that both approaches produce similar output, but the running time of SSFM program is more than 6 times of the modified SSFM, which is shown in Fig.2 (b). The calculation is carried out using a personal computer with 800-MHz Pentium III processor.

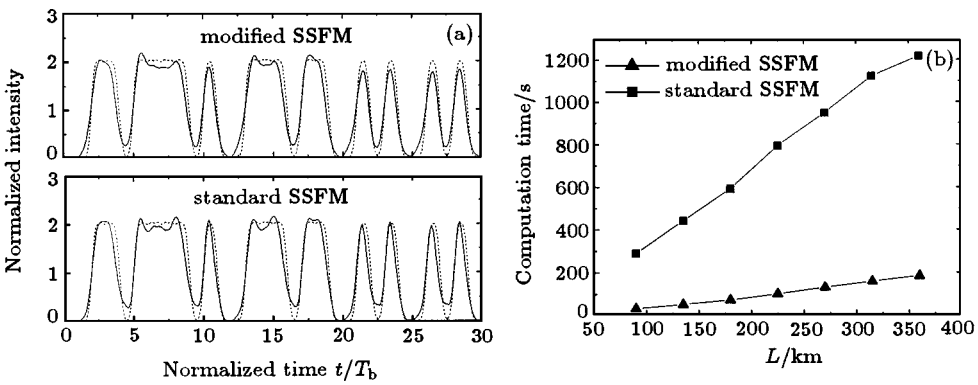


Fig.2. NRZ pulses transmission performance simulation in a 40Gb/s system through standard SSFM and modified SSFM. (a) output pulses; (b) running time comparison.

3.2. The output pulses and eye diagrams

Figure 3 shows the shape of output pulses and eye diagrams for different modulation formats in a 40Gb/s systems after 360km propagation. The extinct ratio value for RZ and CRZ formats is 0.75. NRZ pulses are severely distorted and the eye diagram is almost closed. RZ pulses split and the interference occurs but the eye is still open. For the CRZ format with an initial chirp $C = -1$, compression helps the pulses cancel out part of the broadening caused by PMD. Therefore, a wide-open eye is observed. These results are in good agreement with the experimental conclusions found in Ref.[22].

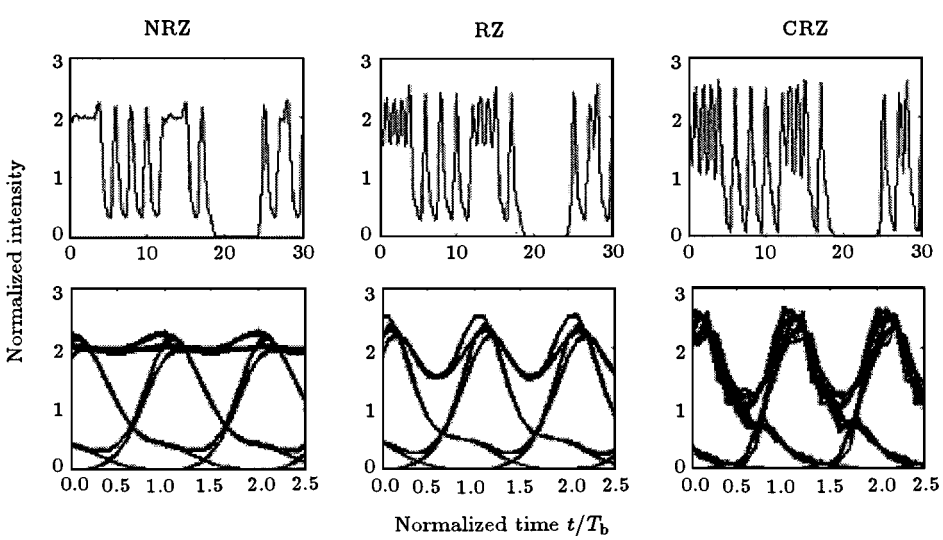


Fig.3. Output pulses and eye diagrams for different modulation formats in a 40Gb/s system after 360km propagation.

3.3. Q-factor penalty induced by PMD

Figure 4 shows the Q-factor penalty versus distance for the 10 and 40Gb/s systems with different modulation formats. Here D_{PMD} is assumed to be 1ps/km$^{1/2}$. In the 10Gb/s system, the CRZ format has the best performance and the RZ format takes the second place. When the extinct ratio is 0.5, the Q-factor penalty will be lower. Furthermore, the curves have the similar variation period with the dispersion map. In the 40Gb/s system, the Q-factor penalties are much larger than that of the 10Gb/s system. The performance of NRZ and RZ formats are close to each other, while the CRZ format with 0.75 extinct ratio still has the lowest penalty. Therefore, to improve the system performance, it is important to adjust the extinct ratio value according to the system bit rate.

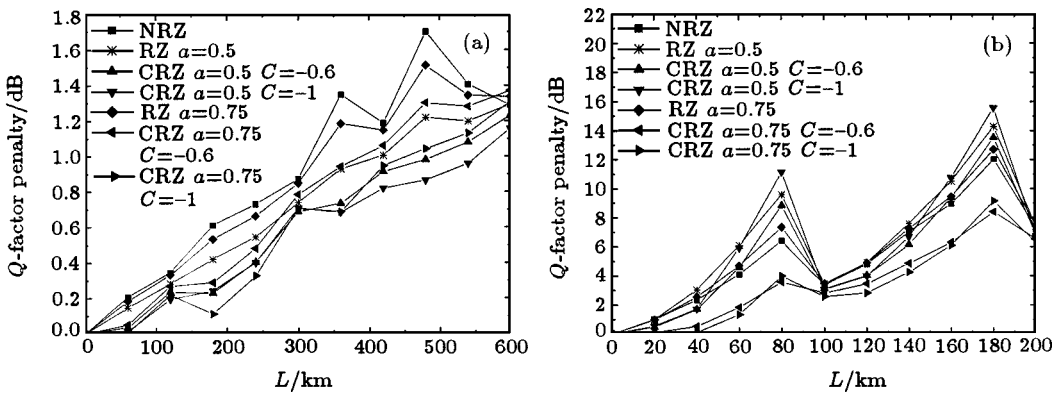


Fig.4. The effect of PMD on Q-factor penalty. (a) 10Gb/s system; (b) 40Gb/s system.

4. Conclusion

A modified SSFM is proposed to solve CNLS equation. Compared with the standard SSFM, this approach can improve the simulation efficiency greatly without losing accuracy. The pulse distortion and system Q-factor of NRZ, RZ and CRZ formats in the presence of high PMD are calculated through this approach. The simulation results show that CRZ pulses are the most tolerant to high PMD values and the extinct ratio has great impact on the transmission performance. The modified SSFM can be widely used in the numerical simulation of high-speed long-haul optical transmission systems to study the optical pulses transmission impairments and the compensation technique.

References

[1] Francia C, Bruyère F, Penninckx D and Chbat M 1998 *IEEE Photon. Technol. Lett.* **10** 1739

[2] Bruyère F 1996 *Opt. Fibre Technol.* **2** 269

[3] Xu W C, Chen W C, Zhang S M, Luo A P and Liu S H 2002 *Chin. Phys.* **11** 352

[4] Takahashi T, Imai T and Aiki M 1994 *Electron. Lett.* **30** 348

[5] Noe R, Sandel D, Weyrauch T and Haase W 1999 *J. Lightwave Technol.* **17** 1602

[6] Shtaif M *et al* 2000 *IEEE Photon. Technol. Lett.* **12** 434

[7] Taga H, Suzuki M and Namihira Y 1998 *Electron. Lett.* **34** 2098

[8] Marcuse D and Menyuk C R 1999 *J. Lightwave Technol.* **17** 564

[9] Khosravani R and Willner A E 2000 *OFC'*2000 Paper WL5

[10] Karlsson M 2002 *OFC'* 2002 Paper WI1 231

[11] Agrawal G P 1995 *Nonlinear Fibre Optics* (New York: Academic)

[12] Mu R M, Grigoryan V S, Menyuk C R, Carter G M and Jacob J M 2000 *IEEE J. Select. Topics Quantum Electron.* **6** 248

[13] Khosravani R and Willner A E 2001 *IEEE Photon. Technol. Lett.* **13** 296

[14] Matsumoto M, Akagi Y and Hasegawa A 1997 *J. Lightwave Technol.* **15** 584

[15] Liu C Y, Guo H, Hu W and Deng D M 2002 *Acta Phys. Sin.* **51** 524 (in Chinese)

[16] Tang Y and Yan X H 2000 *Chin. Phys.* **9** 565

[17] Jorge R C and Carlos R P 2001 *IEEE J. Quantum Electron.* **37** 145

[18] Norimatsu S and Yamamoto T 2001 *J. Lightwave Technol.* **19** 159

[19] Marcuse D, Menyuk C R and Wai P K A 1997 *J. Lightwave Technol.* **15** 1735

[20] Forno A O *et al* 2000 *IEEE Photon. Technol. Lett.* **12** 296

[21] Vu V S and Hoang V V 2000 *Opt. Commun.* **6** 71

[22] Favre F, Guen D L and Georges T 1998 *Electron. Lett.* **34** 201