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The effects of dust temperature and the dust charge variation in a dusty plasma with vortex-like ion distribution*

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By considering both the dust temperature and the dust charge variation in dusty plasma with vortex-like ion distribution, we obtained a modified Korteweg–de Vries equation. It indicates that the effect of dust charge variation can cause the one-dimensional soliton amplitude to become larger, and the dust temperature can cause the soliton amplitude to become larger as well. Moreover, as the dust temperature increases, the soliton amplitude will increase.

Keywords: dusty plasma, soliton, mKdV equation
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1. Introduction

Charged dust grains constitute an important component of matter in space and astrophysical systems such as the planetary rings, asteroid zone, cometary tails, interstellar clouds, as well as the lower parts of the Earth's ionosphere.[1−3] Dust grains are also found in the laboratory environments.[4] The trapped dust impurities can lead to serious contamination problems during the industrial manufacture of semiconductor devices using plasma-based techniques. Dust grains immersed in ambient plasmas and radiative environments are electrically charged due to various processes, such as plasma (electron and ion) current, photo-emission, secondary emission and field emission. Charged dust grains could react with electromagnetic and gravitational fields, and give rise to a host of low-frequency collective phenomena in dusty plasmas. Over the last decade, the field of dusty plasmas has grown rapidly with applications to space and laboratory systems.

In recent years, a large number of authors have investigated theoretically various aspects of linear and nonlinear wave propagation in homogeneous dusty plasmas. In practical situations, the grain sizes are usually in the micron range, and hence the dust particles are many orders of magnitude heavier than the plasma ions. Consequently, the characteristic time scales associated with the dust and the ions are very much different from each other and, therefore, it is possible to separate the various plasma normal modes arising due to their dynamics. In fact, dusty plasmas open up an ultra-low-frequency regime for the existence of novel types of wave modes, which otherwise are not possible in the usual electron–ion plasmas, but with different ion species. For example, the existence of an ultra-low-frequency electrostatic acoustic-like mode, called the dust acoustic wave (DAW), was firstly predicted by Rao *et al*[5] for demagnetized dusty plasma and D'Angelo[6] for magnetized dusty plasma independently. Their model consists of Boltzmann-distributed thermal electrons and ions which provide the restoring force, while the inertia arises due to the heavier dust component. On the other hand, at a higher frequency, Shukla and Silin showed the existence of dust-ion-acoustic waves (DIAWs) where the electrons are Boltzmann-distributed while the inertia is provided by the ions in the presence of stationary dust grains.[7] Recent laboratory experiments on dusty plasmas have confirmed the existence of DAWs and DIAWs.[8,9] Low-frequency electrostatic ion-acoustic and ion-cyclotron waves have

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been studied by D'Angelo[6] in a magnetized dusty plasma. Nonlinear propagation of the low-frequency DAW leads to the formation of coherent structure such as solitons. Recently, Nejoh[10] pointed out that the dust charge variation with some parameters would affect the characteristic collection motion of the plasma. Ma and Liu[11] and Xie and co-workers[12,13] have considered this effect and showed the existence of dust acoustic solitons. We note that laboratory observations of low phase velocity DAWs are associated with a significant depletion of the electron number density,[8] suggesting that the wave dynamics is governed by the inertia of the dust fluid and the pressure of inertialess ions only.[14] Recent numerical simulation studies[15] on linear and nonlinear DAWs exhibit a significant amount of ion trapping in the wave potential. Clearly, there is a departure from the Boltzmann ion distribution, and one encounters a vortex-like ion distribution in phase space. In this paper, we investigate the properties of nonlinear DAWs by incorporating the effects of the vortex-like ion distribution, the charge variations of dust grains, and the non-zero dust temperature. It is found that the vortex-like ion distribution results in a modified Korteweg–de Vries (mKdV) equation which is the governing equation for this system. We also studied the effects of both the dust temperature and the charge variation of dust grains.

2.Governing equation

We now assume that the plasma is demagnetized and the wave propagates in the x-direction. The dusty plasma consists of two components, i.e. the dust grains and the ions. The electrons in the dusty plasmas are completely depleted. If we consider that both the dust temperature and the charge variations of dust grains we have the one-dimensional (1D) set of equations as follows[12,16,17]

$$\frac{\partial n_{\rm d}}{\partial t}+\frac{\partial}{\partial x}(n_{\rm d}u_{\rm d})=0, \tag{1}$$

$$\frac{\partial u_{\rm d}}{\partial t}+u_{\rm d}\frac{\partial u_{\rm d}}{\partial x}+\frac{\sigma}{n_{\rm d}}\frac{\partial p_{\rm d}}{\partial x}=Z_{\rm d}\frac{\partial \phi}{\partial x}, \tag{2}$$

$$p_{\rm d}=c_0 n_{\rm d}^{\gamma}, \tag{3}$$

$$\frac{\partial^2\phi}{\partial x^2}=Z_{\rm d}n_{\rm d}-n_{\rm i}, \tag{4}$$

where $n_{\rm d}$, $p_{\rm d}$ and $u_{\rm d}$ are the dimensionless number density, the pressure, and the velocity of the dust grains respectively. $Z_{\rm d}$ is the charge number residing in the dust grains, c_0 is a constant, ϕ the electrical potential, $n_{\rm i}$ the number density of ions. For simplicity, we have normalized all the quantities. The density of ions are normalized by $Z_{\rm d0}n_{\rm d0}$, the dust density by $Z_{\rm d0}$, x by $\lambda_{\rm Dd}=(T_{\rm i}/4\pi Z_{\rm d0}n_{\rm d0}e^2)^{1/2}$, t by $\omega^{-1}=(m_{\rm d}/4\pi n_{\rm d0}Z_{\rm d0}^2e^2)^{1/2}$, velocity by $c_{\rm d}=(Z_{\rm d0}T_{\rm i}/m_{\rm d})^{1/2}$, ϕ by $T_{\rm i}/e$, pressure by $n_{\rm d0}T_{\rm d}$, where $n_{\rm d0}$, $Z_{\rm d0}$ are the unperturbed values of $n_{\rm d}$ and $Z_{\rm d}$. $\sigma=T_{\rm d}/T_{\rm i}$, where $T_{\rm d}$ and $T_{\rm i}$ are the dust temperature and the ion temperature respectively. $n_{\rm e}=0$ ($n_{\rm e}$ is the number density of electrons) since we have assumed that the electrons are significantly depleted by dust fluid. For the isothermal case $\gamma=1$, but for the adiabatic case $\gamma\approx 3$. Moreover, we know that $Z_{\rm d}=1+\gamma_1\phi+\gamma_2\phi^2$.[12] There are many research works on the DAWs and DIAWs. Most of these works assumed that the electrons satisfy Boltzmann distribution,[11–13,17], however, this is based on two reasonable conditions. One is that the mass of electrons is small enough, the other is that the electrons are isothermal.[18] On the other hand, most of these works also assumed that the ions in the dusty plasma satisfy Boltzmann distribution. However, this assumption should have some restrictions. The DAW only exists in a dusty plasma on a very slow time scale involving the motion of dust grains. The time scales of any oscillation modes for electrons, ions and dust grains are completely different. They are relatively quick, slow and much slow for electrons, ions and dust grains, respectively, namely $t_{\rm d}\gg t_{\rm i}\gg t_{\rm e}$. Sometimes, the differences between $t_{\rm d}$ and $t_{\rm i}$ are not very large. We now make a comparison as follows: $t_{\rm d}$ and $t_{\rm i}$ can be written as

$$t_{\rm d}^2=\frac{m_{\rm d}}{4\pi n_{\rm d0}Z_{\rm d0}^2e^2},$$

$$t_{\rm i}^2=\frac{m_{\rm i}}{4\pi n_{\rm i0}e^2}.$$

For a typical dusty plasma, we let $m_{\rm d}\sim 10^7 m_{\rm i}$, $n_{\rm d0}\sim 10^{-5}n_{\rm i0}$, $Z_{\rm d0}\sim 10^5$, then $t_{\rm d}\sim 10t_{\rm i}$. It seems that for higher frequency oscillation modes, for example, $m_{\rm d}/m_{\rm i}<10^7$ or $n_{\rm d0}/n_{\rm i0}>10^{-5}$, $t_{\rm d}$ and $t_{\rm i}$ will be nearly the same, then we should consider the ion dynamics. Some authors have studied the higher frequency oscillations of DIAW for this kind of cases. In contrast, for lower frequency oscillation modes, for example, $m_{\rm d}/m_{\rm i}>10^7$ or $n_{\rm d0}/n_{\rm i0}<10^{-5}$, $t_{\rm d}$ is much larger than $t_{\rm i}$. This is also the case of much slow time scale for which DAW exist. For this case, if the gradient of ion pressure can be neglected, then both the electrons and the ions satisfy the Boltzmann distribution. We can find this case by the following esti-

mations. For 1D case, the ion velocity satisfies the following equation of motion

$$\frac{\partial v_{\rm i}}{\partial t}+v_{\rm i}\frac{\partial v_{\rm i}}{\partial x}=\frac{e}{m_{\rm i}}E-\frac{\gamma kT_{\rm i}}{m_{\rm i}n_{\rm i}}\frac{\partial n_{\rm i}}{\partial x}. \tag{5}$$

The convective term $v_{\rm i}\dfrac{\partial v_{\rm i}}{\partial x}$ can often be neglected because it is much smaller than the $\dfrac{\partial v_{\rm i}}{\partial t}$ term. Since we have $\dfrac{\partial v_{\rm i}}{\partial t}\sim\omega v_{\rm i}$, where ω is the frequency of the oscillation mode, for lower frequencies, $\omega\sim\dfrac{1}{t_{\rm d}}$, Eq.(5) can be approximately estimated as follows

$$\omega v_{\rm i}=\frac{e}{m_{\rm i}}E-\frac{\gamma kT_{\rm i}}{m_{\rm i}n_{\rm i}}\frac{\partial n_{\rm i}}{\partial x}. \tag{6}$$

We find from Eq.(6)

$$\frac{\omega v_{\rm i}}{eE/m_{\rm i}}\sim\frac{m_{\rm i}v_{\rm i}}{t_{\rm d}eE}=\frac{t_{\rm i}}{t_{\rm d}}\frac{m_{\rm i}v_{\rm i}}{t_{\rm i}eE}\ll\frac{m_{\rm i}v_{\rm i}}{t_{\rm i}eE}.$$

If we neglect the contribution by the gradient of ion pressure to $m_{\rm i}v_{\rm i}$, then $\dfrac{m_{\rm i}v_{\rm i}}{t_{\rm i}eE}\sim 1$. Finally, Eq.(6) can be written as

$$\frac{e}{m_{\rm i}}E\sim\frac{\gamma kT_{\rm i}}{m_{\rm i}n_{\rm i}}\frac{\partial n_{\rm i}}{\partial x}. \tag{7}$$

For the isothermal case, $\gamma=1$, thus we can obtain the Boltzmann distribution for ions

$$n_{\rm i}=n_{\rm i0}\exp(e\phi/kT_{\rm i}). \tag{8}$$

Now we find that there are some restriction conditions for Boltzmann ion distribution. It is reported that in some cases the ion contribution does not satisfy the Boltzmann distribution. For example, there are vortex-like ion distributions, in which the ion distribution satisfy the following equation[16]

$$n_{\rm i}=1-\phi-\frac{4}{3}a(-\phi)^{3/2}+\frac{1}{2}\phi^2. \tag{9}$$

By using the reductive perturbation method to obtain a mKdV equation, we have the following stretched variables

$$\xi=\epsilon^{1/4}(x-v_0t), \tag{10}$$

$$\tau=\epsilon^{3/4}t, \tag{11}$$

and the following transformations

$$\begin{pmatrix} n_{\rm d}\\ u_{\rm d}\\ p_{\rm d}\\ \phi\\ Z_{\rm d}\end{pmatrix}=\begin{pmatrix}1\\0\\1\\0\\1\end{pmatrix}+\epsilon\begin{pmatrix} n_1\\ u_1\\ p_1\\ \phi_1\\ \gamma_1\phi_1\end{pmatrix}+\epsilon^{3/2}\begin{pmatrix} n_2\\ u_2\\ p_2\\ \phi_2\\ \gamma_1\phi_2\end{pmatrix}+\cdots \tag{12}$$

Substituting Eqs.(10)–(12) into Eqs.(1)–(4) and collecting the terms in the same power of ϵ, we have for the lowest order

$$\begin{pmatrix} n_1\\ u_1\\ p_1\end{pmatrix}=-(1+\gamma_1)\begin{pmatrix}1\\ v_0\\ \gamma\end{pmatrix}\phi_1, \tag{13}$$

where

$$v_0^2=\gamma\sigma+\frac{1}{1+\gamma_1}. \tag{14}$$

For the higher order of ϵ, we have

$$-v_0\frac{\partial n_2}{\partial\xi}+\frac{\partial n_1}{\partial\tau}+\frac{\partial u_2}{\partial\xi}=0, \tag{15}$$

$$-v_0\frac{\partial u_2}{\partial\xi}+\frac{\partial u_1}{\partial\tau}+\sigma\frac{\partial p_2}{\partial\xi}-\frac{\partial\phi_2}{\partial\xi}=0, \tag{16}$$

$$-v_0\frac{\partial p_2}{\partial\xi}+\frac{\partial p_1}{\partial\tau}+\gamma\frac{\partial u_2}{\partial\xi}=0, \tag{17}$$

$$-\frac{\partial^2\phi_1}{\partial\xi^2}+n_2+(1+\gamma_1)\phi_2+\frac{4}{3}a(-\phi_1)^{3/2}=0. \tag{18}$$

We finally obtain the following mKdV equation from Eqs.(15)–(18)

$$\frac{\partial\phi_1}{\partial\tau}+A\sqrt{-\phi_1}\frac{\partial\phi_1}{\partial\xi}+B\frac{\partial^3\phi_1}{\partial\xi^3}=0, \tag{19}$$

where

$$A = \frac{a}{(1+\gamma_1)^{3/2}[1+\gamma\sigma(1+\gamma_1)]^{1/2}}, \quad (20)$$

$$B = \frac{1}{2}\frac{1}{(1+\gamma_1)^{3/2}[1+\gamma\sigma(1+\gamma_1)]^{1/2}}. \quad (21)$$

3. Discussion

The mKdV Eq. (19) has a soliton solution of

$$\phi_1 = -\phi_m \mathrm{sech}^4[(\xi - u_0\tau)/w], \quad (22)$$

where $\phi_m = \frac{15u_0}{8A}$ and $w = \sqrt{8/u_0}$ are the amplitude and the width of a soliton solution respectively. It is found that as A, the coefficient of the nonlinear term in the mKdV equation, increases, the amplitude of the soliton decreases, while the width of the soliton is irrelevant with respect to both the coefficients of the nonlinear term A and the dispersive term B in the mKdV Eq. (19). Because γ_1 and γ_2 are usually positive,[12] A is always positive. Therefore, the soliton is always a rarefactive wave. Now we discuss several special situations:

(1) If the dust temperature is zero and the dust charge is a constant, i.e. $\sigma = 0, \gamma_1 = \gamma_2 = 0$, then we have

$$A = a,$$

$$B = \frac{1}{2}.$$

This indicates that the soliton amplitude depends only on u_0 and a.

(2) If the dust temperature is not zero, while the dust charge is a constant, we have $\gamma_1 = \gamma_2 = 0$, and $\sigma \neq 0$. Then

$$A = \frac{a}{(1+\gamma\sigma)^{1/2}},$$

$$B = \frac{1}{2(1+\gamma\sigma)^{1/2}}.$$

It seems that as the dust temperature increases, the soliton amplitude will increase.

(3) If the dust temperature is zero, while the dust charge is not a constant, we find that

$$A = \frac{a}{(1+\gamma_1)^{3/2}},$$

$$B = \frac{1}{2(1+\gamma_1)^{3/2}}.$$

This suggests that the soliton amplitude will be larger than that where we neglect the dust charge variations. Generally, both the dust temperature and the dust charge variations will cause the soliton amplitude to become larger. A soliton is a rarefactive waves in dusty plasmas with vortex-like ion distributions.

References

[1] Whipple E C, Northrop T G and Mendia D A 1985 *J. Geophys. Res.* **99** 7405

[2] de Angelis U, Formisano V and Giordano M 1988 *J. Plasma. Phys.* **40** 339

[3] Goertz C K 1989 *Rev. Geophys.* **27** 271

[4] Chu J H, Du J B and Lin I 1994 *J. Phys. D: Appl. Phys.* **27** 196

[5] Rao N N, Shukla P K and Yu M Y 1990 *Planet. Space. Sci.* **38** 543

[6] D' Angelo N 1990 *Planet. Space. Sci.* **38** 1143

[7] Shukla P K and Silin V P 1992 *Phys. Scr.* **45** 508

[8] Barkan A, Merlino R L and D'Angelo N 1995 *Phys. Plasmas* **2** 3563

[9] Merlino R L, Barkan A, Thompson C and D' Angelo N 1998 *Phys. Plasmas* **5** 1607

[10] Nejoh Y N 1997 *Phys. Plasmas* **4** 2813

[11] Ma J X and Liu J Y 1997 *Phys. Plasmas* **4** 253

[12] Xie B S, He K F and Huang Z Q 1998 *Phys. Lett.* A **247** 403

[13] Xie B S and He K F 1999 *Phys. Plasmas* **6** 3808

[14] Mamun A A, Cairns R A and Shukla P K 1996 *Phys. Plasmas* **3** 702

[15] Winske D *et al* 1995 *Geophys. Res. Lett.* **22** 2069

[16] Mamun A A 1998 *J. Plasma Phys.* **59** 575

[17] Duan W S 2001 *Chin. Phys. Lett.* **18** 1088

[18] Pu W, Sun A P, Zhao Q, Geng M and Qiu X M 2000 *Plasma Sci. Technol.* **2** 257