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Solitons in a generalized (2+1)-dimensional Ablowitz–Kaup–Newell–Segur system*

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In the previous Letter (Zheng C L and Zhang J F 2002 *Chin. Phys. Lett.* **19** 1399), a localized excitation of the generalized Ablowitz–Kaup–Newell–Segur (GAKNS) system was obtained via the standard Painlevé truncated expansion and a special variable separation approach. In this work, starting from a new variable separation approach, a more general variable separation excitation of this system is derived. The abundance of the localized coherent soliton excitations like dromions, lumps, rings, peakons and oscillating soliton excitations can be constructed by introducing appropriate lower-dimensional soliton patterns. Meanwhile we discuss two kinds of interactions of solitons. One is the interaction between the travelling peakon type soliton excitations, which is not completely elastic. The other is the interaction between the travelling ring type soliton excitations, which is completely elastic.

Keywords: GAKNS system, variable separation approach, soliton
PACC: 0340, 0365, 0545

1. Introduction

Soliton is one important aspect of nonlinear science and has been widely applied in natural science such as chemistry, biology, mathematics, communication and particularly in almost all branches of physics like fluid dynamics, plasma physics, field theory, optics, and condensed matter physics, etc.[1–3] Consequently, many researchers pay their attention to searching for exact and explicit solutions of a nonlinear physical model, especially for new exponentially localized structures, like soliton or dromion solutions, since it is an important work. In a previous Letter,[4] we obtained a localized excitation of the generalized Ablowitz–Kaup–Newell–Segur (GAKNS) system[5–7] via the standard Painlevé truncated expansion and a special variable separation approach. However, in recent studies, we find that a more general variable separation excitation of the system can be derived by means of a new variable separation approach. The goal of this paper is to conduct further investigations of the GAKNS system

$$\mathrm{i}q_t + aq_{xx} + bq_{yy} + 2(aw + bv)q = 0, \tag{1}$$

$$-\mathrm{i}r_t + ar_{xx} + br_{yy} + 2(aw + bv)r = 0, \tag{2}$$

$$v_x + \lambda(qr)_y = 0, \quad w_y + \lambda(qr)_x = 0, \tag{3}$$

where a, b are arbitrary constants and $\lambda = \pm 1$. More attention is paid on its localized coherent soliton structures and interactions. As a detailed physical background of the GAKNS system has been given in Ref.[4], we omit the corresponding description. For simplification of discussion, we only analyse the case of $\lambda = 1$ in this paper. If $\lambda = -1$, we can make a simple variable transformation $x \to -x$ (or $y \to -y$), then the situation will be identical with the λ=1 case.

This paper is organized as follows. In section 2, we outline the main procedures of the variable separation approach and apply this approach to the GAKNS

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system. In section 3, we discuss the abundant structures of the solution based on the result obtained by the variable separation approach. A simple discussion and summary is given in the previous section.

2. Variable separation approach and its application to the (2+1)-dimensional GAKNS system

2.1. General theory of the variable separation approach

Solving nonlinear physical models is far more difficult than solving linear ones. In linear physics the Fourier transformation (FT) and the variable separation approach are the two most important methods. It is known that the celebrated inverse scattering transformation (IST) can be viewed as an extension of the FT method. However, it is difficult to extend the variable separation approach to nonlinear physics. Recently, two kinds of "variable separating" procedures have been established. The first kind is called symmetry constraints or nonlinearization of the Lax pairs[8−10] since this type of procedure is only used for integrable models which possess Lax pairs. Lou and Chen[11,12] have also extended the method to some nonintegrable models. The independent variables of a reduced field in the method have not totally been separated though the reduced field satisfies some lower-dimensional equations, so this approach is equivalently called the "formal variable separation approach"(FVSA). The second kind of variable separation method is established first for the (2+1)-dimensional Davey–Stewartson (DS) equations and asymmetric Davey–Stewartson (ADS) equation,[13] and then revised and developed recently for various (2+1)-dimensional models like the (2+1)-dimensional Korteweg–de Vries (KdV) equation, the Nizhnik–Novikov–Vesselov (NNV) equation, the asymmetric Nizhnik–Novikov–Vesselov (ANNV), the nonlinear Schrödinger equation and the usual Ablowitz–Kaup–Newell–Segur (AKNS) system, etc.[14−22] The main idea is by solving the bilinear equations or higher multi-linear equations of the original models and introducing an *a prior* ansatz. Some special types of exact solution of the (2+1)-dimensional nonlinear models can be obtained from some (1+1)-dimensional variable separation fields. Here we describe the basic procedures of the variable separation approach briefly. For a general nonlinear physics system

$$P(v) \equiv P(x_0 = t, x_1, x_2, \cdots, x_n, v, v_{x_i}, v_{x_i x_j}, \cdots), \tag{4}$$

where $v = v(v_1, v_2, \cdots, v_q)^{\mathrm{T}}$ (T indicates the transposition of a matrix), $P(v) = (P_1(v), P_2(v), \cdots, P_q(v))^{\mathrm{T}}$ and $P_i(v)$ are polynomials of v_i and their derivatives.

Firstly, we make a Bäcklund transformation

$$v_i = \sum_{j=0}^{\alpha_i} v_{ij} f^{j-\alpha_i}, \quad i = 1, 2, \cdots, q, \tag{5}$$

where $v_{i\alpha_i}$ are arbitrary known seed solutions of Eq.(4). In usual cases, α_i should be chosen as small as possible since substituting Eq.(5) into Eq.(4) can yield relatively simple multi-linear equations in this situation and is determined by the leading term analysis (suppose $f \sim 0$). Inserting Eq.(5) with α_i into Eq.(4) and vanishing the leading and sub-leading terms, then we can derive $\{v_{ij}, j = 1, 2, \cdots, \alpha_i - 1\}$. If the original model is integrable, this procedure will result in its bilinear or higher multi-linear equations.

Secondly, after obtaining the multi-linear equations of the original system, we select an appropriate variable separation hypothesis. For integrable models, it can be chosen as a modifying Horita's multi-soliton form. For instance, we often take f as such an ansatz for many significant physical models[6,7]

$$f = a_0 + a_1 F(x,t) + a_2 G(y,t) + A F(x,t) G(y,t), \tag{6}$$

where the variable separation functions $F(x,t) \equiv F$ and $G(y,t) \equiv G$ are only the functions of (x,t) and (y,t), respectively, a_0, a_1, a_2 and A are arbitrary constants. When F and G are set as exponential functions, Eq.(6) is just the Hirota's two-soliton form.

Finally, we determine the variable separation equations which the variable separation functions F and G should satisfy after substituting the ansatz (6) into the multi-linear equations. It is worthwhile to mention that this procedure is a hard work since different physical models should be disposed in different ways. In the next subsection, we apply this approach to the (2+1)-dimensional GAKNS system. The variable separation approach shows its efficiency in searching for localized excitations of nonlinear physical models.

2.2. Variable separation approach for the (2+1)-dimensional GAKNS system

According to the above procedures, we first apply the following Bäcklund transformation to q, r v and

w in Eqs.(1), (2) and (3)

$$q=\sum_{j=0}^{\alpha_1} q_j f^{j-\alpha_1}, \quad r=\sum_{j=0}^{\alpha_2} r_j f^{j-\alpha_2},$$

$$v=\sum_{j=0}^{\alpha_3} v_j f^{j-\alpha_3}, \quad w=\sum_{j=0}^{\alpha_4} w_j f^{j-\alpha_4}, \tag{7}$$

where $q_{\alpha_1}, r_{\alpha_2}, v_{\alpha_3}$ and w_{α_4} are arbitrary seed solutions of the GAKNS system. By using the leading term analysis, we obtain

$$\alpha_1=\alpha_2=1, \quad \alpha_3=\alpha_4=2. \tag{8}$$

Substituting Eqs.(7) and (8) directly into Eqs.(1)–(3) and considering the fact that the functions q_1, r_1, v_2 and w_2 are the seed solutions of the model, we obtain

$$\sum_{i=0}^{3} P_{1i} f^{i-4}=0, \tag{9}$$

$$\sum_{i=0}^{3} P_{2i} f^{i-4}=0, \tag{10}$$

$$\sum_{i=0}^{2} P_{3i} f^{i-3}=0, \quad \sum_{i=0}^{2} P_{4i} f^{i-3}=0, \tag{11}$$

where P_{1i}, P_{2i}, P_{3i} and P_{4i} are functions of $\{v_j, w_j f, j{=}0, 1\}$ and their derivatives. Since the complexity of the expressions of P_{1i}, P_{2i}, P_{3i} and P_{4i}, we omit their concrete forms. Vanishing the leading and subleading terms of Eqs.(9), (10) and (11), the functions $\{v_j, w_j, j=0,1\}$ are determined. Inserting the results into Eq.(7) and rewriting its form, the Bäcklund transformation becomes

$$q=\frac{q_0}{f}+q_1, \quad r=\frac{r_0}{f}+r_1,$$

$$v=(\ln f)_{xx}+v_2, \quad w=(\ln f)_{yy}+w_2. \tag{12}$$

For convenience of discussion, we choose the seed solutions q_1, r_1, v_2 and w_2 to be

$$q_1=r_1=0, \quad w_2=F_0(x,t), \quad v_2=G_0(y,t), \tag{13}$$

where $F_0(x,t)$ and $G_0(y,t)$ are arbitrary functions of the arguments indicated. Simple inspection of the GAKNS system shows that the above seed solutions are identically satisfied.

Now substituting Eqs.(12) and (13) into the system (1)–(3) yields its bilinear form

$$(aD_x^2+bD_y^2+\mathrm{i}D_t)q_0\cdot f+2fq_0(aF_0+bG_0)=0, \tag{14}$$

$$(aD_x^2+bD_y^2-\mathrm{i}D_t)r_0\cdot f+2fr_0(aF_0+bG_0)=0, \tag{15}$$

$$D_xD_yf\cdot f+2q_0r_0=0, \tag{16}$$

where D_x, D_y and D_t are the usual bilinear operators first introduced by Hirota.[23]

In order to find some interesting solutions of Eqs.(14)–(16), we use the following variable separation ansatz

$$f=a_1F+a_2G+AFG,$$

$$q_0=F_1G_1\exp(p+s), \quad r_0=\frac{F_1G_1}{\exp(p+s)}, \tag{17}$$

where a_1, a_2 and A are arbitrary constants and $F\equiv F(x,t)$, $G\equiv G(y,t)$, $F_1\equiv F_1(x,t)$, $G_1\equiv G_1(y,t)$, $p\equiv p(x,t)$, $s\equiv s(y,t)$ are all arbitrary functions of the indicated variables.

Substituting the ansatz (17) into Eq.(16) yields

$$F_1^2G_1^2-a_1a_2F_xG_y=0. \tag{18}$$

Because the functions F, F_1 are only functions of $\{x,t\}$ and the functions G, G_1 are only functions of $\{y,t\}$, Eq.(18) can be solved by the following variable separated equation

$$F_1=\delta_1\sqrt{a_1a_2c_0^{-1}F_x}, \quad G_1=\delta_2\sqrt{c_0G_y} \quad (\delta_1^2=\delta_2^2=1). \tag{19}$$

Similar to the above procedure, inserting the ansatz (17) with Eq.(19) into Eqs.(14) and (15) yields the following variable separated equations

$$F_t=2\mathrm{i}ap_xF_x+c_1(a_2+AF)^2+c_2(a_2+AF)+a_1a_2c_3, \tag{20}$$

$$G_t=2\mathrm{i}bs_yG_y-c_3(a_1+AG)^2-c_2(a_1+AG)-a_1a_2c_1, \tag{21}$$

$$aF_{xx}^2=2aF_xF_{xxx}+4F_x^2(p_t+ap_x^2+2aF_0+c_4), \tag{22}$$

$$bG_{yy}^2=2bG_yG_{yyy}+4G_y^2(s_t+bs_y^2+2bG_0-c_4), \tag{23}$$

where c_0, c_1, c_2, c_3, c_4 are all arbitrary functions of t.

Though it is still difficult to obtain general solutions of Eqs.(20)–(23) for any fixed F_0 and G_0, we can treat the problem in an alternative way. Because F_0 and G_0 are arbitrary seed solutions, we can view F and G as arbitrary functions of $\{x,t\}$ and $\{y,t\}$ respectively. The functions p and s can be expressed by F and G simply by integrations from Eqs.(20) and (21). Then the seed solutions F_0 and G_0 can be fixed by Eqs.(22) and (23).

Finally, substituting Eq.(17) and Eqs.(19)–(23) with (13) into Eq.(12), we obtain a quite general solution of the (2+1)-dimensional GAKNS system

$$q=\frac{\delta_1\delta_2\sqrt{a_1a_2F_xG_y}\exp(p+s)}{a_1F+a_2G+AFG}, \tag{24}$$

$$r = \frac{\delta_1\delta_2\sqrt{a_1a_2F_xG_y}}{(a_1F + a_2G + AFG)\exp(p+s)}, \tag{25}$$

$$w = F_0 + \left(\frac{a_1F_{xx} + AGF_{xx}}{a_1F + a_2G + AFG} - \frac{(a_1F_x + AGF_x)^2}{(a_1F + a_2G + AFG)^2}\right), \tag{26}$$

$$v = G_0 + \left(\frac{a_2G_{yy} + AFG_{yy}}{a_1F + a_2G + AFG} - \frac{(a_2G_y + AFG_y)^2}{(a_1F + a_2G + AFG)^2}\right), \tag{27}$$

with two arbitrary functions F and G, and F_0, G_0 are determined by Eqs.(22) and (23) respectively.

Now, we are interested in the physical quantity qr. From expressions (24) and (25), we have

$$qr = \frac{a_1a_2F_xG_y}{(a_1F + a_2G + AFG)^2} = \frac{a_1a_2K_xL_y}{4\left(A_1\cosh\frac{1}{2}(K+L+C_1) + A_2\cosh\frac{1}{2}(K-L+C_2)\right)^2}, \tag{28}$$

where $F = b_1 + \mathrm{e}^K$, $G = b_2 + \mathrm{e}^L$ and

$$A_1 = \sqrt{A(a_1b_1 + a_2b_2 + Ab_1b_2)},$$

$$A_2 = \sqrt{(a_1 + Ab_2)(a_2 + Ab_1)},$$

$$C_1 = \ln\frac{A}{a_1b_1 + a_2b_2 + Ab_1b_2}, \quad C_2 = \ln\frac{a_1 + Ab_2}{a_2 + Ab_1},$$

for b_1 and b_2 being arbitrary constants. K and L are also arbitrary functions of $\{x,t\}$ and $\{y,t\}$ respectively.

Compare the excitation (28) with the solution (20) of the previous Letter,[4] we find that the solution (20) of Ref.[4] is a special solution of the excitation (28).

3. Solitons in the (2+1)-dimensional GAKNS system

It is interesting that expression (28) is valid for many (2+1)-dimensional models, like the DS equation, NNV system, ANNV equation, ADS model and the dispersive long wave equation, etc.[14,15,17,22] Because of the arbitrariness of the functions F and G included in Eq.(28), the physical quantity qr possesses quite rich structures. For instance, as mentioned in Refs.[14], [15], [17] and [22], when selecting the functions F and G appropriately, we can obtain many kinds of localized solutions like the multi-solitoff solutions, multi-dromion and dromion lattice solutions, multiple ring soliton solutions, and so on. Since these types of localized solutions have been discussed widely for other models, here we only cite some new or interesting examples. For simplification in the following discussion, we set $a_1 = a_2 = 1$ and $A = 2$ in expression (28).

3.1. Peakon soliton excitations

Now we cite the first type of important localized excitation. It is known that the celebrated (1+1)-dimensional Camassa–Holm equation[24]

$$u_t + 2ku_x - u_{xxt} + 3uu_x = 2u_xu_{xx} + uu_{xxx}, \tag{29}$$

possesses a special type of novel solution, namely the peakon solution which is discontinuous at its crest

$$u(x,t) = -k + c\exp(-|x - ct|), \quad k \to 0. \tag{30}$$

Naturally, we may ask: is there any kind of peakon in other types of (2+1)-dimensional model? Although the (1+1)-dimensional Camassa–Holm equation has been extended to (2+1)-dimensions in several possible way,[25,26] we know little of the (2+1)-dimensional peakons which are localized in all directions for the (2+1)-dimensional integrable models. Actually, since the arbitrariness of the functions F and G in expression (28), the (2+1)-dimensional peakons can be derived by selecting appropriate functions. For instance, when F and G take the following simple form

$$F_x = c\exp(-|x - ct + 3|) + d\exp(-|x - dt - 3|),$$
$$G = 1 + \exp(y), \tag{31}$$

and

$$F_x = c\exp(-|x - ct + 4|) + d\exp(-|x - dt - 4|),$$
$$G_y = c\exp(-|y - ct + 4|) + d\exp(-|y - dt - 4|), \tag{32}$$

then we can obtain two types of multi-peakon excitation shown in Figs.1(a) and 1(b) when $c = 1$, $d = 2$ and $t = 0$.

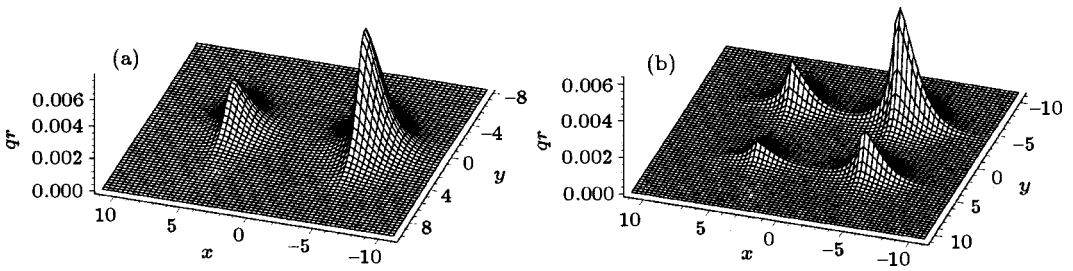


Fig.1. Two plots of multi-peakon solutions determined by solution (28) with selections: (a) related to Eq.(31), (b) related to Eq.(32) and $c = 1, d = 2$ at $t = 0$.

This fact hints that function F or G can be in a more general form, such as piecewise function

$$F = \sum_{i=1}^{M} \begin{cases} X_i(x + c_i t), & x + c_i t \leq 0, \\ -X_i(-x - c_i t) + 2X_i(0), & x + c_i t > 0, \end{cases} \tag{33}$$

$$G = \sum_{i=1}^{N} \begin{cases} Y_i(y + d_i t), & y + d_i t \leq 0, \\ -Y_i(-y - d_i t) + 2Y_i(0), & y + d_i t > 0, \end{cases} \tag{34}$$

where the functions $X_i(x + c_i t) \equiv X_i(\xi_i)$ and $Y_i(y + d_i t) \equiv Y_i(\eta_i)$ are differentiable functions of the indicated arguments and possess the boundary conditions

$$X_i(\pm\infty) = C_{\pm i}, \ i = 1, 2, \ldots, M, \tag{35}$$

$$Y_i(\pm\infty) = D_{\pm i}, \ i = 1, 2, \ldots, N, \tag{36}$$

with $C_{\pm i}$ and $D_{\pm i}$ being constants and/or even infinity while c_i, d_i are arbitrary constants. For example, when taking

$$X_1(\xi_1) = -\ln\left(\tanh\left(\frac{1}{2}(1 + t - x)\right)\right) + \frac{1}{2},$$

$$X_2(\xi_2) = -\ln\left(\tanh\left(\frac{1}{2}(1 - t - x)\right)\right) + \frac{1}{2}, \tag{37}$$

$$Y_1(\eta_1) = -\ln\left(\tanh\left(\frac{1}{2}(1 - y)\right)\right), \tag{38}$$

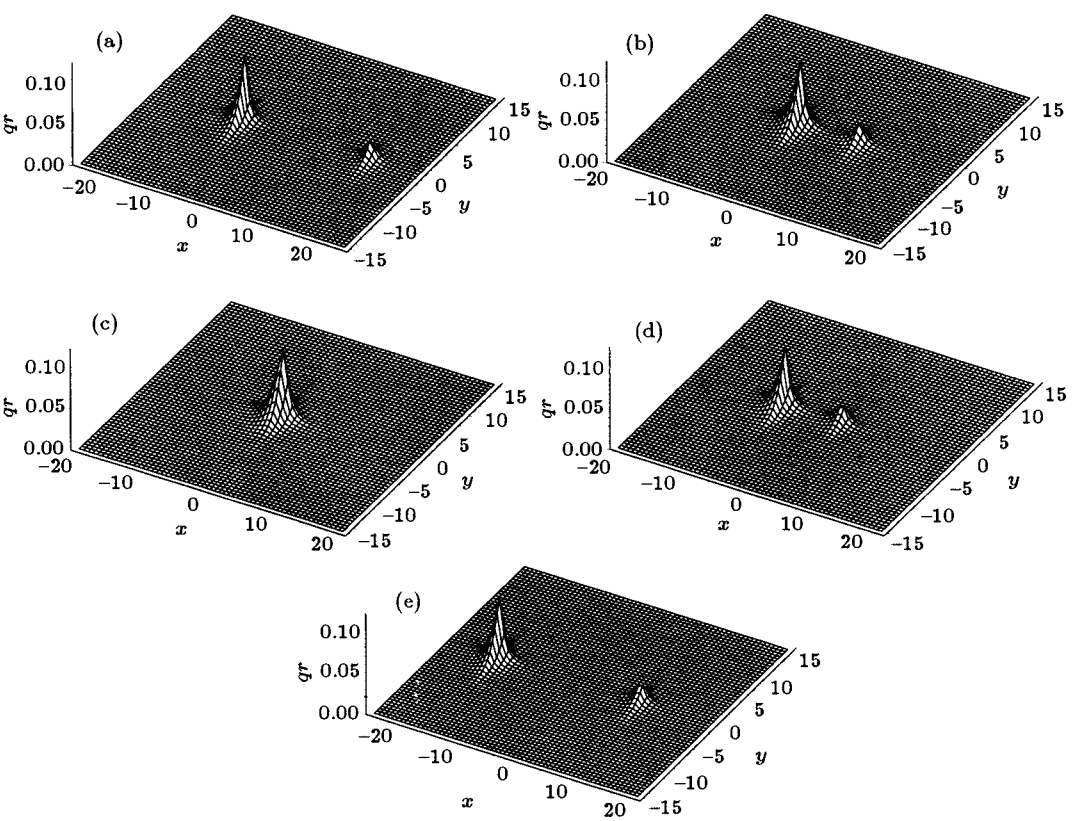


Fig.2. The exhibition of the interaction property between two-peakons related to excitation (28) with conditions (33)–(34) and (37)–(38) is at times (a)$t = -6$, (b)$t = -3$, (c)$t = 0$, (d)$t = 3$ and (e)$t = 6$.

in Eqs.(33) and (34), we can derive a multi-peakon excitation in Fig.2, which evolves at (a)$t = -6$, (b)$t = -3$, (c)$t = 0$, (d)$t = 3$ and (e)$t = 6$. From Figs.2(a)–2(e), we can see that the interaction of two travelling peakons is not completely elastic since the two travelling peakons completely exchange their shapes after the interaction.

3.2 Ring soliton excitations

In high dimensions, in addition to the point-like localized coherent excitations, there may be some other types of physically significant localized excitation. For instance, in (2+1)-dimensional cases, there may be some types of ring soliton solutions which are not equal to zero identically at some closed curves and decay exponentially away from the closed curves.[14−16] In Fig.3, we plot the interaction property of two travelling saddle ring solitons with the condition

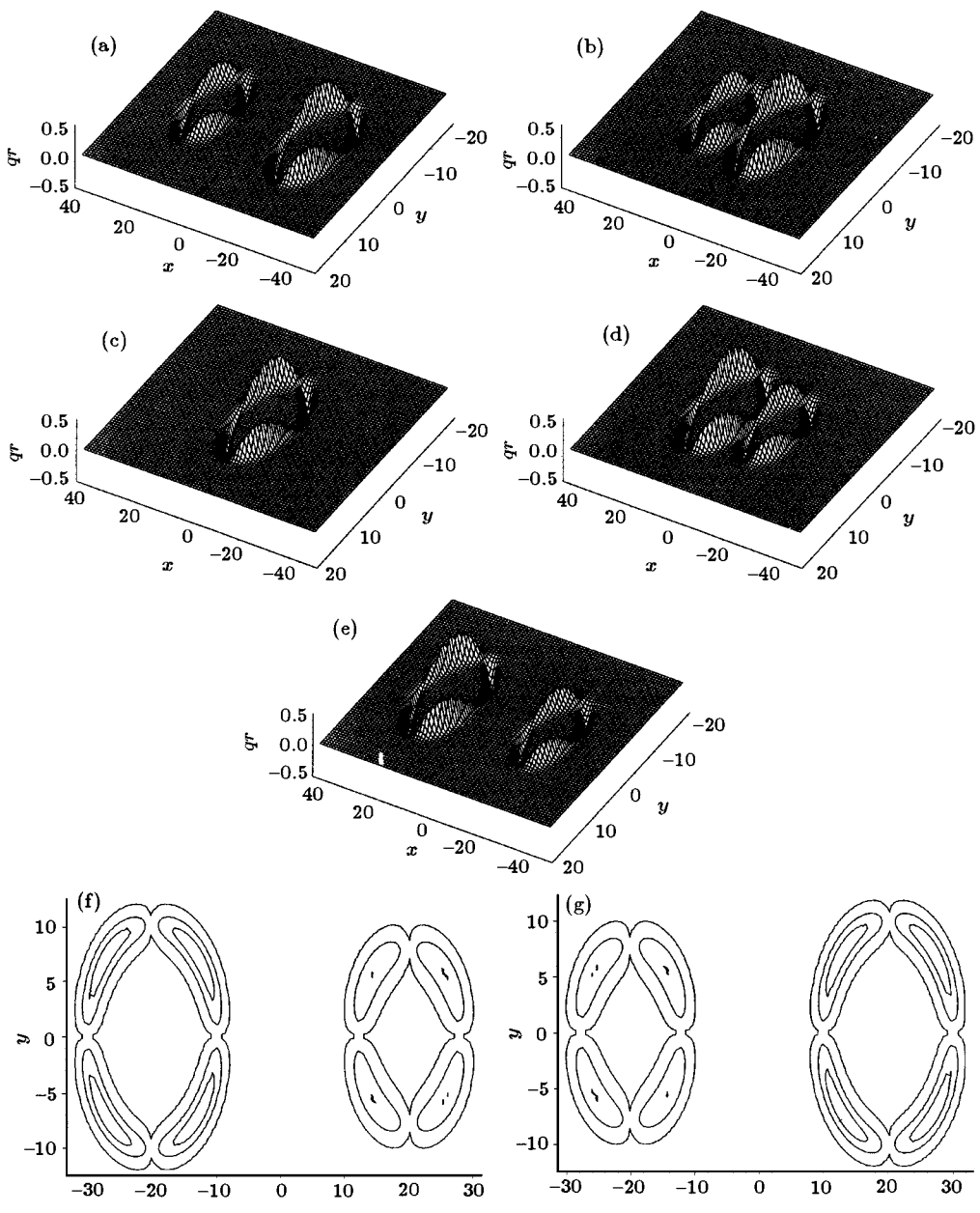


Fig.3. The evolution of two saddle-type ring soliton solutions related to excitation (28) with conditions (39) at (a)$t = -1$, (b)$t = -0.5$, (c)$t = 0$, (d)$t = 0.5$ and (e)$t = 1$. (f) The contour plot related to (a). (g) The contour plot of (e). The values of the contour in these figures from outside to the inside contour are $|qr| = 0.01$, $|qr| = 0.03$ and $|qr| = 0.1$ respectively.

$$F = 1 + \exp\left(-\frac{(x-20t)^2}{10} + 10\right) + \exp\left(-\frac{(x+20t)^2}{10} + 5\right),$$
$$G = 1 + \exp\left(\frac{y^2}{10} - 10\right),\ a_1 = a_2 = 1,\ a_3 = 0. \quad (39)$$

In Fig.3, the evolution of the two ring soliton solutions for the quantity qr expressed by Eq.(28) with the condition (39) are shown at times (a) $t = -1$, (b) $t = -0.5$, (c) $t = 0$, (d) $t = 0.5$ and (e) $t = 1$. From Figs.3(a)–3(e) and through detailed calculation, we find that the interaction of two ring soliton solutions is completely elastic. To see more clearly the completely elastic interaction properties between two travelling ring soliton solutions, two counter plots related to Figs.3(a) and 3(e) are plotted in the Figs.3(f) and 3(g).

4. Summary and discussion

In summary, by means of the new variable separation approach, the (2+1)-dimensional GAKNS system is solved. Abundant localized coherent soliton structures of solution (28), such as multi-dromion, multi-ring, multi-lump solutions, peakons, breathers and instantons, etc., can be easily obtained by selecting appropriate arbitrary functions. Meanwhile we discuss two kinds of interaction of solitons and the result shows that the interaction between the travelling peakons is not completely elastic while the interaction between the travelling rings is completely elastic.

Because of the wide applications of the soliton theory in many physics fields like fluid dynamics, plasma physics, field theory and optics, etc., it is a very significant work to learn more about the localized coherent soliton excitations of some higher-dimensional physical models.

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