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Kinetic behaviour of two-species-group aggregation process with complete annihilation*

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We propose a two-species-group aggregation-annihilation model in which an irreversible aggregation reaction occurs between any two clusters of the same species, and an irreversible joint annihilation reaction occurs between two distinct species groups. Based on the mean-field theory, we have investigated the rate equations of the process with constant reaction rates to obtain the asymptotic descriptions of the cluster-mass distributions. The results indicate that for the symmetrical initial cases the kinetic behaviour of the system depends crucially on the ratio of the equivalent aggregation rate to the annihilation rate. The cluster-mass distribution of each species always obeys a conventional or modified scaling law in each individual case. Moreover, all the species cannot finally survive except for the cases in which at least one equivalent aggregation rate is less than twice the annihilation rate.

Keywords: kinetic behaviour, aggregation-annihilation, scaling law
PACC: 8220, 2460, 0570L

1. Introduction

The phenomena of aggregation and annihilation processes are central to many fields of science and technology, such as colloid science, polymerization, biology, and star formation. Much attention has been devoted to understanding the kinetics of aggregation and annihilation processes,[1−15] and theoretical research found that the evolution behaviour of the aggregates approaches a scaling law in the long-time limit.[15−22] In recent years, Krapivsky[6] studied a two-species aggregation process with partial annihilation and analysed the evolution behaviour of the mass distribution. Ben-Naim and Krapivsky[7] investigated the kinetics of a two-species aggregation model with complete annihilation and determined its scaling description of the cluster-mass distribution in the long-time limit. But there are few studies concerning multi-species aggregation processes with annihilation, especially the species-group aggregation-annihilation case. Zhang and Yang[11] investigated several irreversible aggregation-annihilation models of an n-species system and analysed the scaling properties of the cluster-mass distributions in some symmetrical cases. In our previous works, we studied the kinetics of n-species aggregation processes with complete annihilation and found that the evolution of the system obeys a scaling law in most cases.[21]

In this paper, we propose a solvable two-species-group aggregation model with complete annihilation and investigate the competitions between aggregation and annihilation processes of two distinct species-groups, A^m and B^n ($m = 1, 2, \ldots, M$, $n = 1, 2, \ldots, N$). When $M = 1$ or $N = 1$, this model reduces to the multi-species aggregation process with joint annihilation.[22] Here we set $M, N \geq 2$. In our model, an irreversible aggregation reaction occurs only between any two clusters of the same species, $A_i^m + A_j^m \longrightarrow A_{i+j}^m$ and $B_i^n + B_j^n \longrightarrow B_{i+j}^n$, where A_i^m (B_i^n) denotes a cluster consisting of i-mers of A^m (B^n) species. Meanwhile, an irreversible complete annihilation reaction occurs only between A and B group, and each species in one group (A or B) is annihilated jointly by all species in the other group, $A_i^m + B_j^n \longrightarrow$ inert. The rate of aggregation reaction between A_i^m and A_j^m equals $K_m^A(i, j)$ and that between B_i^n and B_j^n equals $K_n^B(i, j)$. The rate of annihilation

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between A_i^m and B_j^n is $J_{mn}(i,j)$.

The present investigation is based on the mean-field theory, which applies to the case with the spatial dimension d of the system greater than or equal to a critical dimension d_c. Thus, the fluctuations in densities of the reactants are ignored and the clusters are considered to be homogeneously distributed in the space throughout the process. It was suggested by Ben-Naim and Krapivsky[7] that for irreversible aggregation-annihilation processes $d_c = 2$. Here, we assume that the spatial dimension of this two-species-group system is greater than 2 and therefore, the mean-field assumption is valid. Based on the mean-field rate equations, we have derived the asymptotic solutions for the cluster-mass distributions under the symmetrical initial conditions. The results show that the cluster-mass distribution of each species obeys a conventional scaling law in general cases and it also possesses peculiar scaling properties in several special cases, which depend strongly on the ratios of the reaction rates.

This paper is organized as follows. In section 2, we describe an irreversible two-species-group aggregation process with complete annihilation and give the corresponding mean-field rate equations. Then we derive the asymptotic solutions of the cluster-mass distributions under the symmetrical initial conditions and analyse their evolution behaviours. A brief summary is given in section 3.

2. Model of irreversible aggregation process with complete annihilation

In our investigation, the theoretical approach to the aggregation process is based on the mean-field rate equation, which assumes that the reaction proceeds at a rate proportional to the reactant concentrations. There are two types of species groups in the system, A group and B group. A group consists of M kinds of distinct species while B group has N kinds. The concentrations of A_k^m and B_k^n clusters are respectively denoted as a_{mk} and b_{nk}, $m = 1, 2, \ldots, M$; $n = 1, 2, \ldots, N$. We consider a simple model with constant reaction rates. The aggregation rates of A^m and B^n species are equal to I_{1m} and I_{2n}, respectively. Meanwhile, all the annihilation reaction rates equal a constant J.

By generalizing the rate equations given by Ref.[7] we write out the corresponding rate equations for this two-species-group aggregation system as follows:

$$\frac{\mathrm{d}a_{mk}}{\mathrm{d}t} = I_{1m}\left(\frac{1}{2}\sum_{i+j=k} a_{mi}a_{mj} - a_{mk}\sum_{j=1}^{\infty} a_{mj}\right) - J\sum_{n=1}^{N}\sum_{j=1}^{\infty} a_{mk}b_{nj},\ m = 1, 2, \ldots, M, \tag{1a}$$

$$\frac{\mathrm{d}b_{nk}}{\mathrm{d}t} = I_{2n}\left(\frac{1}{2}\sum_{i+j=k} b_{ni}b_{nj} - b_{nk}\sum_{j=1}^{\infty} b_{nj}\right) - J\sum_{m=1}^{M}\sum_{j=1}^{\infty} b_{nk}a_{mj},\ n = 1, 2, \ldots, N. \tag{1b}$$

We consider a simplest but important case in which there only exist the monomer-clusters of A^m and B^n species at $t = 0$ and their concentrations are equal to A_{m0} and B_{n0}, respectively. Then the initial conditions are

$$a_{mk}(0) = A_{m0}\delta_{k1}, \quad b_{nk}(0) = B_{n0}\delta_{k1}, \quad m = 1, 2, \ldots, M; \quad n = 1, 2, \ldots, N. \tag{2}$$

Under the above monodisperse initial conditions, the rate Eqs. (1) can be solved with the help of ansatz[6]

$$a_{mk}(t) = A_m(t)[a_m(t)]^{k-1}, \quad b_{nk}(t) = B_n(t)[b_n(t)]^{k-1}, \quad m = 1, 2, \ldots, M;\ n = 1, 2, \ldots, N. \tag{3}$$

Substituting Eqs.(3) into Eqs.(1), we can transform the rate Eqs. (1) into the following differential equations:

$$\frac{\mathrm{d}a_m}{\mathrm{d}t} = \frac{I_{1m}A_m}{2}, \quad \frac{\mathrm{d}A_m}{\mathrm{d}t} = -\frac{I_{1m}A_m^2}{1 - a_m} - JA_m\sum_{n=1}^{N}\frac{B_n}{1 - b_n}, \quad m = 1, 2, \ldots, M, \tag{4a}$$

$$\frac{\mathrm{d}b_n}{\mathrm{d}t} = \frac{I_{2n}B_n}{2}, \quad \frac{\mathrm{d}B_n}{\mathrm{d}t} = -\frac{I_{2n}B_n^2}{1 - b_n} - JB_n\sum_{m=1}^{M}\frac{A_m}{1 - a_m}, \quad n = 1, 2, \ldots, N. \tag{4b}$$

Correspondingly, the initial conditions of Eqs.(4) are

$$a_m = b_n = 0,\ A_m = A_{m0},\ B_n = B_{n0}, \quad m = 1, 2, \ldots, M;\ n = 1, 2, \ldots, N \quad \text{at } t = 0. \tag{5}$$

Introducing new variables $\alpha_m(t)$ and $\beta_n(t)$,

$$\alpha_m = (1 - a_m)^{-1},\ \beta_n = (1 - b_n)^{-1}, \tag{6}$$

we recast Eqs.(4) to the following equations:

$$\frac{\mathrm{d}^2\alpha_m}{\mathrm{d}t^2} = -\frac{\mathrm{d}\alpha_m}{\mathrm{d}t}\sum_{n=1}^{N}\left(\frac{2J}{I_{2n}\beta_n}\frac{\mathrm{d}\beta_n}{\mathrm{d}t}\right),$$

$$m = 1, 2, \ldots, M; \tag{7a}$$

$$\frac{\mathrm{d}^2\beta_n}{\mathrm{d}t^2} = -\frac{\mathrm{d}\beta_n}{\mathrm{d}t}\sum_{m=1}^{M}\left(\frac{2J}{I_{1m}\alpha_m}\frac{\mathrm{d}\alpha_m}{\mathrm{d}t}\right),$$

$$n = 1, 2, \ldots, N, \tag{7b}$$

with the corresponding initial conditions

$$\alpha_m = 1,\ \beta_n = 1,\ \frac{\mathrm{d}\alpha_m}{\mathrm{d}t} = \frac{I_{1m}A_{m0}}{2},\ \frac{\mathrm{d}\beta_n}{\mathrm{d}t} = \frac{I_{2n}B_{n0}}{2},$$

$$m = 1, 2, \ldots, M;\ n = 1, 2, \ldots, N \quad \text{at } t = 0. \tag{8}$$

Then we obtain the integrals of Eqs.(7),

$$\frac{\mathrm{d}\alpha_m}{\mathrm{d}t} = \frac{I_{1m}A_{m0}}{2}\prod_{n=1}^{N}\beta_n^{-2J/I_{2n}}, \quad m = 1, 2, \ldots, M; \tag{9a}$$

$$\frac{\mathrm{d}\beta_n}{\mathrm{d}t} = \frac{I_{2n}B_{n0}}{2}\prod_{m=1}^{M}\alpha_m^{-2J/I_{1m}}, \quad n = 1, 2, \ldots, N. \tag{9b}$$

It is difficult to determine the solutions of Eqs.(9) in general initial cases. In order to derive the analytical solution of the system, we turn to discuss the symmetrical initial case, i.e. $I_{1m}A_{m0} = 2A_0$, $I_{2n}B_{n0} = 2B_0$, $\alpha_m = \alpha$, and $\beta_n = \beta$ for $m = 1, 2, \ldots, M$ and $n = 1, 2, \ldots, N$. Thus Eqs.(9) can be rewritten as

$$\frac{\mathrm{d}\alpha}{\mathrm{d}t} = A_0\beta^{-\sum_{n=1}^{N}2J/I_{2n}}, \quad m = 1, 2, \ldots, M; \tag{10a}$$

$$\frac{\mathrm{d}\beta}{\mathrm{d}t} = B_0\alpha^{-\sum_{m=1}^{M}2J/I_{1m}}, \quad n = 1, 2, \ldots, N, \tag{10b}$$

and the initial conditions become $\alpha = \beta = 1$, $d\alpha/\mathrm{d}t = A_0$ and $d\beta/\mathrm{d}t = B_0$. For convenience, we set $\sum_{m=1}^{M}1/I_{1m} = 1/I_{1e}$ and $\sum_{n=1}^{N}1/I_{2n} = 1/I_{2e}$. Here, I_{1e} and I_{2e} can be considered as the equivalent aggregation rates of A and B groups, respectively. From Eqs.(10) one can obtain the differential equations for $\alpha(t)$ and $\beta(t)$,

$$\left(\frac{\mathrm{d}\alpha}{\mathrm{d}t}\right)^{-I_{2e}/2J}\frac{\mathrm{d}^2\alpha}{\mathrm{d}t^2} = -\frac{2JB_0}{I_{2e}}A_0^{-I_{2e}/2J}\alpha^{-2J/I_{1e}}\frac{\mathrm{d}\alpha}{\mathrm{d}t}, \tag{11a}$$

$$\left(\frac{\mathrm{d}\beta}{\mathrm{d}t}\right)^{-I_{1e}/2J}\frac{\mathrm{d}^2\beta}{\mathrm{d}t^2} = -\frac{2JA_0}{I_{1e}}B_0^{-I_{1e}/2J}\beta^{-2J/I_{2e}}\frac{\mathrm{d}\beta}{\mathrm{d}t}. \tag{11b}$$

In order to thoroughly investigate the kinetic behaviours of the system, we discuss the solutions of Eqs.(11) in several different cases.

case 1: $I_{1e} > 2J$ **and** $I_{2e} > 2J$

In this case, Eqs.(11) can be solved to yield

$$\frac{\mathrm{d}\alpha}{\mathrm{d}t} = \left(C_1\alpha^{1-2J/I_{1e}} + C_2\right)^{-2J/(I_{2e}-2J)}, \tag{12a}$$

$$\frac{\mathrm{d}\beta}{\mathrm{d}t} = \left(C_3\beta^{1-2J/I_{2e}} + C_4\right)^{-2J/(I_{1e}-2J)}, \tag{12b}$$

where $C_1 = [I_{1e}(I_{2e} - 2J)/I_{2e}(I_{1e} - 2J)]B_0A_0^{-I_{2e}/2J}$, $C_2 = A_0^{-I_{2e}/2J}[A_0 - B_0I_{1e}(I_{2e} - 2J)/I_{2e}(I_{1e} - 2J)]$, $C_3 = [I_{2e}(I_{1e} - 2J)/I_{1e}(I_{2e} - 2J)]A_0B_0^{-I_{1e}/2J}$ and $C_4 = B_0^{-I_{1e}/2J}[B_0 - A_0I_{2e}(I_{1e} - 2J)/I_{1e}(I_{2e} - 2J)]$.

We assume that the system reaches its steady state at $t \to \infty$ and the steady conditions are then given as follows:

$$\frac{\mathrm{d}a_m}{\mathrm{d}t} = \frac{1}{\alpha^2}\frac{\mathrm{d}\alpha}{\mathrm{d}t} = 0,$$

$$\frac{\mathrm{d}A_m}{\mathrm{d}t} = \frac{2}{I_{1m}\alpha^2}\frac{\mathrm{d}^2\alpha}{\mathrm{d}t^2} - \frac{4}{I_{1m}\alpha^3}\left(\frac{\mathrm{d}\alpha}{\mathrm{d}t}\right)^2 = 0,$$

$$m = 1, 2, \ldots, M; \tag{13a}$$

$$\frac{\mathrm{d}b_n}{\mathrm{d}t} = \frac{1}{\beta^2}\frac{\mathrm{d}\beta}{\mathrm{d}t} = 0,$$

$$\frac{\mathrm{d}B_n}{\mathrm{d}t} = \frac{2}{I_{2n}\beta^2}\frac{\mathrm{d}^2\beta}{\mathrm{d}t^2} - \frac{4}{I_{2n}\beta^3}\left(\frac{\mathrm{d}\beta}{\mathrm{d}t}\right)^2 = 0,$$

$$n = 1, 2, \ldots, N. \tag{13b}$$

Thus we can conclude that either $\alpha(\beta) \to \infty$ or $\mathrm{d}\alpha/\mathrm{d}t(\mathrm{d}\beta/\mathrm{d}t) \to 0$ at $t \to \infty$. From Eqs.(12) we know that $\alpha \to \infty$ and $\beta \to \infty$ at $t \to \infty$ in the case of $I_{1e} > 2J$ and $I_{2e} > 2J$. Hence, $\alpha \gg 1$ and $\beta \gg 1$ at $t \gg 1$. In the long-time limit, Eqs.(12) can be approximately rewritten as

$$\frac{\mathrm{d}\alpha}{\mathrm{d}t} \simeq \left(C_1\alpha^{1-2J/I_{1e}}\right)^{-2J/(I_{2e}-2J)}, \tag{14a}$$

$$\frac{\mathrm{d}\beta}{\mathrm{d}t} \simeq \left(C_3\beta^{1-2J/I_{2e}}\right)^{-2J/(I_{1e}-2J)}. \tag{14b}$$

One can then determine the following asymptotic solutions of $\alpha(t)$ and $\beta(t)$ in a very long time:

$$\alpha(t) \simeq C_5t^{\gamma_1}, \tag{15a}$$

$$\beta(t) \simeq C_6t^{\gamma_2}, \tag{15b}$$

where

$$\gamma_1 = (I_{1e}I_{2e} - 2I_{1e}J)/(I_{1e}I_{2e} - 4J^2),$$

$$\gamma_2 = (I_{1e}I_{2e} - 2I_{2e}J)/(I_{1e}I_{2e} - 4J^2),$$

$$C_5 = \gamma_1^{-\gamma_1} C_1^{-2I_{1e}J/(I_{1e}I_{2e}-4J^2)},$$

and

$$C_6 = \gamma_2^{-\gamma_2} C_3^{-2I_{2e}J/(I_{1e}I_{2e}-4J^2)}.$$

Thus we obtain the asymptotic solutions of the cluster-mass distributions

$$\begin{aligned} a_{mk}(t) =& A_m(t) a_m^{k-1}(t) \\ \simeq& \frac{2\gamma_1}{I_{1m}C_5} t^{-1-\gamma_1} \left(1 - C_5^{-1} t^{-\gamma_1}\right)^{k-1}, \\ & m = 1, 2, \ldots, M; \end{aligned} \tag{16a}$$

$$\begin{aligned} b_{nk}(t) =& B_n(t) b_n^{k-1}(t) \\ \simeq& \frac{2\gamma_2}{I_{2n}C_6} t^{-1-\gamma_2} \left(1 - C_6^{-1} t^{-\gamma_2}\right)^{k-1}, \\ & n = 1, 2, \ldots, N. \end{aligned} \tag{16b}$$

Eqs. (16) can be further rewritten as follows:

$$\begin{aligned} a_{mk}(t) \simeq& \frac{2\gamma_1}{I_{1m}C_5} t^{-1-\gamma_1} \exp(-x_m), \\ & m = 1, 2, \ldots, M; \end{aligned} \tag{17a}$$

$$\begin{aligned} b_{nk}(t) \simeq& \frac{2\gamma_2}{I_{2n}C_6} t^{-1-\gamma_2} \exp(-y_n), \\ & n = 1, 2, \ldots, N, \end{aligned} \tag{17b}$$

which are valid in the region

$$k \gg 1, \quad t \gg 1, \quad x_m = (k/C_5) t^{-\gamma_1} = \text{finite},$$

$$y_n = (k/C_6) t^{-\gamma_2} = \text{finite}. \tag{18}$$

Krapivsky[6] used a function $S(t)$ to denote the characteristic cluster-mass of such an aggregation system, and the long-time concentration $c_k(t)$ of k-mer aggregates of any type can then be described in the following scaling form:

$$c_k(t) \simeq t^{-w} \Phi[k/S(t)], \quad S(t) \propto t^z. \tag{19}$$

One can also obtain the power-law expressions for the total number $N(t)$ and the total mass $M(t)$ of the clusters as follows:[6]

$$N(t) = \sum_{k=1}^{\infty} c_k(t) \propto t^{-\lambda},$$

$$M(t) = \sum_{k=1}^{\infty} k c_k(t) \propto t^{-\mu}. \tag{20}$$

The four exponents (w, z, λ, μ) are used to describe the universal scaling nature of the aggregation-annihilation processes in the long-time limit. From Eqs.(19) and (20), one can find the scaling exponent relations

$$\lambda = w - z, \quad \mu = w - 2z. \tag{21}$$

In this case, the asymptotic solutions (17) show that all the cluster-mass distributions satisfy the above conventional scaling law in the long-time limit, and their exponents are easily obtained from Eqs.(17) and (18),

$$\begin{aligned} w &= \frac{2I_{1e}I_{2e} - 2I_{1e}J - 4J^2}{I_{1e}I_{2e} - 4J^2}, \\ z &= \frac{I_{1e}I_{2e} - 2I_{1e}J}{I_{1e}I_{2e} - 4J^2}, \ \lambda = 1, \\ \mu &= \frac{2I_{1e}J - 4J^2}{I_{1e}I_{2e} - 4J^2} \end{aligned}$$

for A^m clusters, $\quad m = 1, 2, \ldots, M;$ (22a)

$$\begin{aligned} w &= \frac{2I_{1e}I_{2e} - 2I_{2e}J - 4J^2}{I_{1e}I_{2e} - 4J^2}, \\ z &= \frac{I_{1e}I_{2e} - 2I_{2e}J}{I_{1e}I_{2e} - 4J^2}, \ \lambda = 1, \\ \mu &= \frac{2I_{2e}J - 4J^2}{I_{1e}I_{2e} - 4J^2} \end{aligned}$$

for B^n clusters, $\quad n = 1, 2, \ldots, N.$ (22b)

The results show that the exponents w, z and μ (except λ) depend on the values of reaction rates I_{1e}, I_{2e}, and J. In this case, both the total number and the total mass of each species decrease with time. Moreover, neither A group nor B group remains at the end, which is independent of initial data A_0 and B_0.

Case 2: $I_{1e} > 2J$ **and** $I_{2e} = 2J$

We obtain two integrals of Eqs.(11) in this case,

$$\frac{\mathrm{d}\alpha}{\mathrm{d}t} = A_0 \exp\left[\frac{2JI_{1e}B_0}{I_{2e}(I_{1e} - 2J)A_0} \left(1 - \alpha^{1-2J/I_{1e}}\right)\right], \tag{23a}$$

$$\begin{aligned} \frac{\mathrm{d}\beta}{\mathrm{d}t} =& \left[\frac{I_{1e} - 2J}{I_{1e}} A_0 B_0^{-I_{1e}/2J} \ln\beta \right. \\ & \left. + B_0^{-(I_{1e}-2J)/2J}\right]^{-2J/(I_{1e}-2J)}. \end{aligned} \tag{23b}$$

From Eq.(23a) we determine the solution of $\alpha(t)$ in an implicit form

$$\begin{aligned} \sum_{i=1}^{\infty} \Bigg\{ & (-C_8)^{-i} \prod_{j=0}^{i-1} \left(\frac{I_{1e}}{I_{1e} - 2J} - j\right) \\ & \times \left[\exp(C_8) - \exp(C_8 \alpha^{1-2J/I_{1e}}) \right. \\ & \left. \times \alpha^{2iJ/I_{1e}-i+1}\right] \Bigg\} = C_7 t, \end{aligned} \tag{24}$$

where $C_7 = A_0\exp[2JI_{1e}B_0/I_{2e}(I_{1e}-2J)A_0]$ and $C_8 = 2JI_{1e}B_0/I_{2e}(I_{1e}-2J)A_0$. If there exists an integral L satisfying the equation $2LJ/I_{1e} - L + 1 = 0$, the infinite terms in Eq.(24) will reduce to the finite terms of $i = 1 \sim L$. It can be concluded from Eqs.(23) and (13) that $\alpha, \beta \to \infty$ at $t \to \infty$. In the long-time limit, the value of the first term of the summation is much greater than the sum of the others, and Eq.(24) can then be approximately rewritten as

$$\frac{I_{1e}C_8^{-1}}{I_{1e}-2J}\alpha^{2J/I_{1e}}\exp\left(C_8\alpha^{1-2J/I_{1e}}\right) \simeq C_7 t. \tag{25}$$

From Eq.(25) we obtain

$$\alpha(t) \simeq C_8^{-I_{1e}/(I_{1e}-2J)}(\ln t)^{I_{1e}/(I_{1e}-2J)}. \tag{26}$$

Substituting Eq.(26) into Eq.(10a), one can determine the asymptotic solution

$$\beta(t) \simeq C_9 t(\ln t)^{-2J/(I_{1e}-2J)}, \tag{27}$$

where $C_9 = C_8^{I_{1e}/(I_{1e}-2J)}(I_{1e}-2J)A_0/I_{1e}$. In the long-time limit, the asymptotic solutions of the mass distributions are obtained as follows:

$$a_{mk}(t) \simeq \frac{2I_{1e}}{I_{1m}(I_{1e}-2J)}C_8^{I_{1e}/(I_{1e}-2J)}t^{-1} \times (\ln t)^{-(2I_{1e}-2J)/(I_{1e}-2J)}\exp(-x_m),$$

$$m = 1, 2, \ldots, M; \tag{28a}$$

$$b_{nk}(t) \simeq \frac{2}{I_{2n}C_9}t^{-2}(\ln t)^{2J/(I_{1e}-2J)}\exp(-y_n),$$

$$n = 1, 2, \ldots, N, \tag{28b}$$

which are valid in the scaling region

$$k \gg 1, \quad t \gg 1,$$

$$x_m = C_8^{I_{1e}/(I_{1e}-2J)}k(\ln t)^{-I_{1e}/(I_{1e}-2J)} = \text{finite},$$

$$y_n = C_9^{-1}kt^{-1}(\ln t)^{2J/(I_{1e}-2J)} = \text{finite}. \tag{29}$$

These indicate that the conventional scaling description (19) of the cluster-mass distribution breaks down for each species, and both the mass distribution of A^m clusters and that of B^n clusters obey a rather peculiar scaling regime. We may further modify the above scaling description (19) into[21]

$$c_k(t) \simeq h^k[g(t)]^{-w_1}[f(t)]^{-w_2}\Phi[k/S(t)],$$

$$S(t) \propto [g(t)]^{z_1}[f(t)]^{z_2}, \; g'(t), f'(t) > 0, \tag{30}$$

where h denotes a constant ($0 < h \leq 1$), and $g(t)$ and $f(t)$ are two unusual functions of time such as e^t, $\log t$, 2^t, and so on. The total number and the total mass of the clusters can be rewritten as

$$N(t) = \sum_{k=1}^{\infty} c_k(t) \propto [g(t)]^{-\lambda_1}[f(t)]^{-\lambda_2},$$

$$M(t) = \sum_{k=1}^{\infty} kc_k(t) \propto [g(t)]^{-\mu_1}[f(t)]^{-\mu_2}, \tag{31}$$

and the corresponding exponent relations are

$$\lambda_1 = w_1 - z_1, \quad \mu_1 = w_1 - 2z_1,$$

$$\lambda_2 = w_2 - z_2, \quad \mu_2 = w_2 - 2z_2 \quad \text{for } h = 1, \tag{32a}$$

$$\lambda_1 = \mu_1 = w_1, \quad \lambda_2 = \mu_2 = w_2 \quad \text{for } 0 < h < 1. \tag{32b}$$

When $f(t) = g(t) \equiv t$, one can find the relations between these exponents (w_1, w_2, z_1, z_2, λ_1, λ_2, μ_1, μ_2) and those (w, z, λ, μ) of the conventional scaling law (19)

$$w = w_1 + w_2, \quad z = z_1 + z_2,$$

$$\lambda = \lambda_1 + \lambda_2, \quad \mu = \mu_1 + \mu_2. \tag{33}$$

In this case, by letting $g(t) \equiv t$ and $f(t) \equiv \ln t$, we obtain the scaling exponents

$$w_1 = 1, \quad w_2 = \frac{2I_{1e}-2J}{I_{1e}-2J},$$

$$z_1 = 0, \quad z_2 = \frac{I_{1e}}{I_{1e}-2J},$$

$$\lambda_1 = 1, \quad \lambda_2 = 1,$$

$$\mu_1 = 1, \quad \mu_2 = -\frac{2J}{I_{1e}-2J}$$

$$\text{for } A^m \text{ clusters}, \quad m = 1, 2, \ldots, M; \tag{34a}$$

$$w_1 = 2, \quad w_2 = -\frac{2J}{I_{1e}-2J},$$

$$z_1 = 1, \quad z_2 = -\frac{2J}{I_{1e}-2J},$$

$$\lambda_1 = 1, \quad \lambda_2 = 0,$$

$$\mu_1 = 0, \quad \mu_2 = \frac{2J}{I_{1e}-2J}$$

$$\text{for } B^n \text{ clusters}, \quad n = 1, 2, \ldots, N. \tag{34b}$$

These indicate that the exponents are only dependent on the equivalent aggregation rate I_{1e} of A group and the annihilation rate J. Comparison between the total mass $M_A(t) \propto t^{-1}(\ln t)^{2J/(I_{1e}-2J)}$ of A^m species and $M_B(t) \propto (\ln t)^{-2J/(I_{1e}-2J)}$ of B^n species shows that B^n clusters dominate over A^m clusters in the long-time limit, which is independent of the initial concentrations A_0 and B_0. It is not surprising because $I_{1e} > I_{2e}$. The results also show that both the total number and total mass of each species decrease with

time. Hence, two species-groups annihilate each other completely and no species can survive at the end.

Case 3: $I_{1e} = 2J$ **and** $I_{2e} > 2J$

From Eqs.(11) we derive two integrals as follows:

$$\frac{\mathrm{d}\alpha}{\mathrm{d}t} = \left[\frac{I_{2e}-2J}{I_{2e}} B_0 A_0^{-I_{2e}/2J} \ln\alpha + A_0^{-(I_{2e}-2J)/2J}\right]^{-2J/(I_{2e}-2J)}, \quad (35a)$$

$$\frac{\mathrm{d}\beta}{\mathrm{d}t} = B_0 \exp\left[\frac{2JI_{2e}A_0}{I_{1e}(I_{2e}-2J)B_0}\left(1-\beta^{1-2J/I_{2e}}\right)\right]. \quad (35b)$$

This case is just contrary to the above case 2. By employing the above method, we obtain the asymptotic solutions of $\alpha(t)$ and $\beta(t)$ as follows:

$$\alpha(t) \simeq C_{10} t (\ln t)^{-2J/(I_{2e}-2J)}, \quad (36a)$$

$$\beta(t) \simeq C_{11} (\ln t)^{I_{2e}/(I_{2e}-2J)}, \quad (36b)$$

where $C_{10} = [(I_{2e}-2J)B_0/I_{2e}][2JI_{2e}A_0/I_{1e}(I_{2e}-2J)B_0]^{I_{2e}/(I_{2e}-2J)}$ and $C_{11} = [2JI_{2e}A_0/I_{1e}(I_{2e}-2J)B_0]^{-I_{2e}/(I_{2e}-2J)}$. In this case, the conventional scaling description of the cluster-mass distribution also breaks down for all species, and we obtain the modified scaling descriptions for the cluster-mass distributions as follows:

$$a_{mk}(t) \simeq \frac{2}{I_{1m}C_{10}} t^{-2} (\ln t)^{2J/(I_{2e}-2J)} \exp(-x_m),$$
$$x_m = C_{10}^{-1} k t^{-1} (\ln t)^{2J/(I_{2e}-2J)},$$
$$m = 1, 2, \ldots, M, \quad (37a)$$

$$b_{nk}(t) \simeq \frac{2I_{2e}}{I_{2n}(I_{2e}-2J)C_{11}} t^{-1} \times (\ln t)^{-(2I_{2e}-2J)/(I_{2e}-2J)} \exp(-y_n),$$
$$y_n = C_{11}^{-1} k (\ln t)^{-I_{2e}/(I_{2e}-2J)}, \quad n = 1, 2, \ldots, N. \quad (37b)$$

Eqs. (37) indicate that the clusters of B group dominate over those of A group in the long-time limit. By letting $g(t) \equiv t$ and $f(t) \equiv \ln t$ in Eq.(30), we obtain the scaling exponents

$$w_1 = 2, \quad w_2 = -\frac{2J}{I_{2e}-2J},$$
$$z_1 = 1, \quad z_2 = -\frac{2J}{I_{2e}-2J},$$
$$\lambda_1 = 1, \quad \lambda_2 = 0,$$
$$\mu_1 = 0, \quad \mu_2 = \frac{2J}{I_{2e}-2J}$$
for A^m clusters, $\quad m = 1, 2, \ldots, M;$ (38a)

$$w_1 = 1, \quad w_2 = \frac{2I_{2e}-2J}{I_{2e}-2J},$$
$$z_1 = 0, \quad z_2 = \frac{I_{2e}}{I_{2e}-2J},$$
$$\lambda_1 = 1, \quad \lambda_2 = 1,$$
$$\mu_1 = 1, \quad \mu_2 = -\frac{2J}{I_{2e}-2J}$$
for B^n clusters, $\quad n = 1, 2, \ldots, N.$ (38b)

These show that the exponents are only dependent on the equivalent aggregation rate I_{2e} of B group and the annihilation rate J. Moreover, from Eqs.(38) one can conclude that all species will annihilate completely and no clusters are retained at the end.

Case 4: $I_{1e} \geq 2J$ **and** $I_{2e} < 2J$

In this case, we have the same Eqs. (12) of $\mathrm{d}\alpha/\mathrm{d}t$ and $\mathrm{d}\beta/\mathrm{d}t$ for $I_{1e} > 2J$, while the expressions of $\mathrm{d}\alpha/\mathrm{d}t$ and $\mathrm{d}\beta/\mathrm{d}t$ are the same as Eqs.(35) for $I_{1e} = 2J$. Meanwhile, we find that for this case, $\beta \to \infty$ at $t \to \infty$. Hence, $\beta^{1-2J/I_{2e}} \to 0$ at $t \gg 1$. In the long-time limit, we obtain the asymptotic solution of $\beta(t)$ from Eqs.(12b) and (35b),

$$\beta(t) \simeq C_{12} t, \quad (39)$$

where $C_{12} = C_4^{-2J/(I_{1e}-2J)}$ for $I_{1e} > 2J$ or $C_{12} = B_0 \exp[2JI_{2e}A_0/I_{1e}(I_{2e}-2J)B_0]$ for $I_{1e} = 2J$. Substituting Eq.(39) into Eqs.(10), one can determine the asymptotic solution of $\alpha(t)$,

$$\alpha(t) \simeq C_{13} - C_{14} t^{-(2J-I_{2e})/I_{2e}}, \quad (40)$$

where $C_{13} = (B_0/C_{12})^{I_{1e}/2J}$ and $C_{14} = [I_{2e}A_0/(2J-I_{2e})]C_{12}^{-2J/I_{2e}}$. We can again obtain the scaling solutions of the cluster-mass distributions

$$a_{mk}(t) \simeq \frac{2(2J-I_{2e})C_{14}}{I_{1m}I_{2e}C_{13}^2} \times \left(\frac{C_{13}-1}{C_{13}}\right)^k t^{-2J/I_{2e}} \exp(-x_m),$$
$$x_m = \frac{C_{14}}{C_{13}(C_{13}-1)} k t^{-(2J-I_{2e})/I_{2e}},$$
$$m = 1, 2, \ldots, M; \quad (41a)$$

$$b_{nk}(t) \simeq \frac{2}{I_{2n}C_{12}} t^{-2} \exp(-y_n),$$
$$y_n = C_{12}^{-1} k t^{-1}, \quad n = 1, 2, \ldots, N. \quad (41b)$$

The results indicate that B^n species satisfies the conventional scaling form (19) with the exponents

$$w = 2, \quad z = 1, \quad \lambda = 1, \quad \mu = 0; \quad (42a)$$

while A^m species obeys the modified scaling form (30) with the exponents

$$w = \lambda = \mu = \frac{2J}{I_{2e}}, \quad z = \frac{2J - I_{2e}}{I_{2e}}. \tag{42b}$$

It is not difficult to obtain the total masses of A^m and B^n clusters as follows:

$$M_{A^m}(t) \simeq \frac{2(2J - I_{2e})C_{14}}{I_{1m}I_{2e}} t^{-2J/I_{2e}},$$

$$M_{B^n}(t) \simeq \frac{2C_{12}}{I_{2n}}, \tag{43}$$

which indicate that the total mass of A^m species decreases with time, while that of B^n species is a time-independent constant at $t \gg 1$. In this case, B group dominates over A group at a very long time and only the clusters of B group remain finally, which are independent of initial concentrations A_0 and B_0.

Case 5: $I_{1e} < 2J$ **and** $I_{2e} \geq 2J$

In this case, the expressions of $\mathrm{d}\alpha/\mathrm{d}t$ and $\mathrm{d}\beta/\mathrm{d}t$ are the same as Eqs.(12) for $I_{2e} > 2J$ or same as Eqs.(23) for $I_{2e} = 2J$. It is found that for this case $\alpha \to \infty$ at $t \to \infty$. Thus we obtain the asymptotic solutions of $\alpha(t)$ and $\beta(t)$ in the long-time limit,

$$\alpha(t) \simeq C_{15}t, \tag{44a}$$

$$\beta(t) \simeq C_{16} - C_{17}t^{-(2J-I_{1e})/I_{1e}}, \tag{44b}$$

where $C_{15} = C_2^{-2J/(I_{2e}-2J)}$ for $I_{2e} > 2J$ or $C_{15} = A_0\exp[2JI_{1e}B_0/I_{2e}(I_{1e} - 2J)A_0]$ for $I_{2e} = 2J$, $C_{16} = (A_0/C_{15})^{I_{2e}/2J}$, and $C_{17} = [I_{1e}B_0/(2J-I_{1e})]C_{15}^{-2J/I_{1e}}$. Thus, we find that the mass distribution of A^m clusters approaches the conventional scaling form

$$a_{mk}(t) \simeq \frac{2}{I_{1m}C_{15}} t^{-2}\exp(-x_m),$$

$$x_m = C_{15}^{-1}kt^{-1}, \quad m = 1, 2, \ldots, M, \tag{45}$$

with the exponents (42a). Meanwhile, the conventional scaling description breaks down for B^n species, and it satisfies the modified scaling description

$$b_{nk}(t) \simeq \frac{2(2J - I_{1e})C_{17}}{I_{2n}I_{1e}C_{16}^2} \times \left(\frac{C_{16} - 1}{C_{16}}\right)^k t^{-2J/I_{1e}}\exp(-y_n),$$

$$y_n = \frac{C_{17}}{C_{16}(C_{16} - 1)} kt^{-(2J-I_{1e})/I_{1e}},$$

$$n = 1, 2, \ldots, N, \tag{46a}$$

with the exponents

$$w = \lambda = \mu = \frac{2J}{I_{1e}}, \quad z = \frac{2J - I_{1e}}{I_{1e}}. \tag{46b}$$

The results show that A group dominates over B group in the long-time limit, which is just contrary to that in the above case 4. It is obvious that only the clusters of A group can survive finally.

Case 6: $I_{1e} = I_{2e} = 2J$

Now we turn to the special case, $I_{1e} = I_{2e} = 2J$. In this case, Eqs.(11) reduce to the following equations:

$$\left(\frac{\mathrm{d}\alpha}{\mathrm{d}t}\right)^{-1} \frac{\mathrm{d}^2\alpha}{\mathrm{d}t^2} = -\frac{B_0}{A_0}\alpha^{-1}\frac{\mathrm{d}\alpha}{\mathrm{d}t}, \tag{47a}$$

$$\left(\frac{\mathrm{d}\beta}{\mathrm{d}t}\right)^{-1} \frac{\mathrm{d}^2\beta}{\mathrm{d}t^2} = -\frac{A_0}{B_0}\beta^{-1}\frac{\mathrm{d}\alpha}{\mathrm{d}t}. \tag{47b}$$

Eqs. (47) can be directly solved to yield

$$\alpha(t) = [(A_0 + B_0)t + 1]^{A_0/(A_0+B_0)}, \tag{48a}$$

$$\beta(t) = [(A_0 + B_0)t + 1]^{B_0/(A_0+B_0)}. \tag{48b}$$

Then we obtain the scaling solutions of $a_{mk}(t)$ and $b_{nk}(t)$ in the long-time limit,

$$a_{mk}(t) \simeq \frac{2A_0}{I_{1m}}[(A_0+B_0)t]^{-(2A_0+B_0)/(A_0+B_0)}\exp(-x_m),$$

$$x_m = k[(A_0 + B_0)t)]^{-A_0/(A_0+B_0)},$$

$$m = 1, 2, \ldots, M, \tag{49a}$$

$$b_{nk}(t) \simeq \frac{2B_0}{I_{2n}}[(A_0+B_0)t]^{-(A_0+2B_0)/(A_0+B_0)}\exp(-y_n),$$

$$y_n = k[(A_0 + B_0)t]^{-B_0/(A_0+B_0)},$$

$$n = 1, 2, \ldots, N. \tag{49b}$$

These show that the mass distribution of each species obeys the conventional scaling law (19), and the scaling exponents are

$$w = \frac{2A_0 + B_0}{A_0 + B_0}, \quad z = \frac{A_0}{A_0 + B_0}, \quad \lambda = 1,$$

$$\mu = \frac{B_0}{A_0 + B_0} \quad \text{for } A^m \text{ clusters}, \tag{50a}$$

$$w = \frac{A_0 + 2B_0}{A_0 + B_0}, \quad z = \frac{B_0}{A_0 + B_0}, \quad \lambda = 1,$$

$$\mu = \frac{A_0}{A_0 + B_0} \quad \text{for } B^n \text{ clusters}, \tag{50b}$$

which show that the exponents w, z and μ have relations with the initial data A_0 and B_0. The results also imply that the heavy group with larger initial concentration dominates over the light one in the long-time limit. Moreover, all the species annihilate completely and no species remains at the end.

Case 7: $I_1 < 2J$ **and** $I_2 < 2J$

We also obtain the differential Eqs. (12) for this case. It is obvious that $C_1 > 0$ and $C_3 > 0$. The solutions of $\alpha(t)$ and $\beta(t)$ are complicated and dependent on the values of C_2 and C_4.

When $(2J - I_{1e})I_{2e}A_0 > (2J - I_{2e})I_{1e}B_0$, we find that $C_2 > 0$ and $C_4 < 0$. Thus it can be concluded that $\alpha \to \infty$ at $t \to \infty$. So, $\alpha^{1-2J/I_{1e}} \to 0$ in the long-time limit. From Eqs.(12) one can determine the same asymptotic solutions (44) of $\alpha(t)$ and $\beta(t)$ for this case. Hence, this case is equivalent to the above Case 5.

When $(2J - I_{1e})I_{2e}A_0 = (2J - I_{2e})I_{1e}B_0$, it is found that $C_2 = C_4 = 0$. In this case, Eqs.(12) reduce to Eqs. (14). One can obtain the exact solutions of $a_{mk}(t)$ and $b_{nk}(t)$ similar to Eqs.(16). So, the results of this case are identical with those in Case 1.

When $(2J - I_{1e})I_{2e}A_0 < (2J - I_{2e})I_{1e}B_0$, $C_2 < 0$ and $C_4 > 0$. So $\beta \to \infty$ at $t \to \infty$. One can obtain the same asymptotic solutions of $a_{mk}(t)$ and $b_{nk}(t)$ as Eqs.(41). In this case, the behaviours of the system are similar to those in Case 4.

3. Summary

We have studied an aggregation-annihilation process of two-species-group system on the basis of mean-field theory. Considering the constant-reaction-rate model, we have analysed the kinetic behaviours of the aggregation system with joint annihilation between two species groups. The results show that for the symmetrical initial conditions the evolution behaviour of the system depends crucially on the ratios of equivalent aggregation rates (I_{1e}, I_{2e}) to the annihilation rate J. In the first case of $I_{1e} > 2J$ and $I_{2e} > 2J$, we have found that the cluster-mass distribution of each species obeys the conventional scaling law in the long-time limit. The governing exponents are dependent on the equivalent aggregation rates as well as the annihilation rate. In the second and third cases in which one equivalent aggregation is greater than $2J$ while the other equals $2J$, the conventional scaling description breaks down for all species; meanwhile, all species in either group scale according to a modified peculiar scaling form. In the fourth and fifth cases in which one equivalent aggregation rate is greater than or equal to $2J$ while the other being less than $2J$, each species in the group with less equivalent rate obeys the conventional scaling law while all the species in the other group evolve according to the modified scaling regime, and the exponents depend only on the less equivalent aggregation rate as well as the annihilation rate. In the sixth case of $I_{1e} = I_{2e} = 2J$, each species also possesses the conventional scaling description for its mass distribution, and the scaling exponents depend only on the initial concentrations. In the last case of $I_{1e} < 2J$ and $I_{2e} < 2J$, it is found that the evolution behaviour of the system depends on both the reaction rates and the initial concentrations. We have obtained the distinct evolution behaviours of the system for different ratios of $(2J - I_{1e})I_{2e}A_0/(2J - I_{2e})I_{1e}B_0$.

In short, the evolution of the two-species-group aggregation process with complete annihilation always satisfies a conventional or modified scaling law, and the scaling exponents depend strongly on the reaction rates. Meanwhile, the initial concentrations also play an important role in the kinetic behaviour of the system. The evolution behaviour of the total mass of each species also depends strongly on the reaction rates. The results indicate that all the species annihilate each other completely except for the cases in which at least one equivalent aggregation rate is less than $2J$. Moreover, at most one species-group can be retained by the dynamics of the system at the end.

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