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Influence of group-velocity mismatch on soliton switching in a nonlinear fibre coupler*

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In this work, the influence of group-velocity mismatch on soliton self-routing pulse switching in a nonlinear fibre coupler is discussed in detail by the use of both variational approach and numerical simulation. The results obtained show that the group-velocity mismatch leads to the relative displacement between the two orthogonal polarization modes, increase of the critical power, and reduction of the elimination-light ratio. For sub-ps pulse, the influence cannot be neglected.

Keywords: self-routing pulse switching, group-velocity mismatch, fibre coupler, soliton
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1. Introduction

Optical switching has been an intense research topic in recent years because of its potential to eliminate the bottleneck caused by the conversion between optical and electrical signals.[1−4] It can be achieved by the use of a nonlinear fibre coupler exhibiting birefringence between its two channels. When normal optical pulses propagate in the birefringent fibre, they may be split into many pulses due to nonlinear interaction between the two orthogonal polarization channels, and hence deteriorate the switching operation of the fibre coupler. However, such an effect can be avoided when the propagating signal pulses consist of solitons.[5−10]

Since soliton is formed by the balance between group velocity dispersion and self-phase modulation, the switching speed of a nonlinear fibre coupler is also limited by their interaction. In this work, the influence of group-velocity mismatch on soliton pulse switching in a nonlinear fibre coupler is discussed in detail by the use of both variational approach and numerical simulation, some novel results are obtained.

2. Theoretical analysis

Nonlinear optical pulse propagation in the fibre exhibiting linear birefringence can be described as [5]

$$\begin{aligned} &i\left(\frac{\partial q_x}{\partial \xi}+\delta\frac{\partial q_x}{\partial \tau}\right)+\frac{1}{2}\frac{\partial^2 q_x}{\partial \tau^2} \\ &+\left(|q_x|^2+\frac{2}{3}|q_y|^2\right)q_x+kq_x+\frac{2}{3}q_x^*q_y^2=0, \\ &i\left(\frac{\partial q_y}{\partial \xi}-\delta\frac{\partial q_y}{\partial \tau}\right)+\frac{1}{2}\frac{\partial^2 q_y}{\partial \tau^2} \\ &+\left(|q_y|^2+\frac{2}{3}|q_x|^2\right)q_y+kq_y+\frac{2}{3}q_y^*q_x^2=0, \end{aligned} \tag{1}$$

where q_x and q_y represent the slowly varying envelopes of the two orthogonal polarization components in the birefringent fibre respectively, δ denotes the group-velocity mismatch between the two orthogonal polarization components, k is the modal birefringence coefficient, and[1,8]

$$\delta=\frac{\Delta n\tau_0}{2c|\beta_2^2|}\approx\frac{k}{\omega\tau_0}. \tag{2}$$

Thus

$$k\approx\frac{\omega\Delta n\tau_0^2}{2c|\beta_2^2|}, \tag{3}$$

where β_2 is the second-order dispersion, τ_0 is the pulse width, c and ω represent the velocity and frequency of optical wave, respectively, and $\Delta n=n_x-n_y$ is the difference of refractive index. It can be seen that k is proportional to the square of pulse-width.

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When the new components

$$u = \sqrt{\frac{1}{3}}(q_x + iq_y),$$
$$v = \sqrt{\frac{1}{3}}(q_x - iq_y), \tag{4}$$

are introduced, the Eqs.(1) can be rewritten as

$$i\frac{\partial u}{\partial \xi} + i\delta\frac{\partial u}{\partial v} + \frac{1}{2}\frac{\partial^2 u}{\partial \tau^2} + (|u|^2 + 2|v|^2)u + kv = 0,$$
$$i\frac{\partial v}{\partial \xi} - i\delta\frac{\partial v}{\partial \tau} + \frac{1}{2}\frac{\partial^2 v}{\partial \tau^2} + (|v|^2 + 2|u|^2)v + ku = 0, \tag{5}$$

where u and v represent the two eigenmodes of the equations.

In the variational approach, the Lagrangian density of the Eqs.(5) can be defined as

$$\begin{aligned} L(\xi,\tau) =& \frac{i}{2}u^*\frac{\partial u}{\partial \xi} + \frac{i}{2}\delta \times u^*\frac{\partial u}{\partial \tau} + \frac{1}{4}|u|^4 - \frac{1}{4}\left|\frac{\partial u}{\partial \tau}\right|^2 \\ &+ \frac{i}{2}v^*\frac{\partial v}{\partial \xi} - \frac{i}{2}\delta \times v^*\frac{\partial v}{\partial \tau} + \frac{1}{4}|v|^2 - \frac{1}{4}\left|\frac{\partial v}{\partial \tau}\right|^2 \\ &+ \frac{1}{3}|u|^2|v|^2 + \frac{1}{2}k(u^*v + uv^*), \end{aligned} \tag{6}$$

then the trial function can be chosen as

$$\begin{aligned} u(\xi,\tau) =& a\sqrt{\eta}\sec h[\eta(\tau - \Delta)]\cos\theta \\ &\times \exp[i\omega(\tau - \Delta) + i\phi + i\psi + iC(\tau - \Delta)^2], \\ v(\xi,\tau) =& a\sqrt{\eta}\sec h[\eta(\tau + \Delta)]\sin\theta \\ &\times \exp[i\omega(\tau + \Delta) + i\phi - i\psi + iC(\tau + \Delta)^2], \end{aligned} \tag{7}$$

where ω, C and ϕ are the frequency, chirp coefficient and common phase of soliton pulses, 2Δ and 2ψ are relative displacement (separation) and relative phase of the solitons between the two channels, respectively, θ is the coupling angle governing the power exchange between the two channels. All these parameters are functions of propagation distance ξ.

The averaged Lagrangian is defined as

$$\overline{L} = \int_{-\infty}^{\infty} L(\xi,\tau)\mathrm{d}\tau. \tag{8}$$

Substituting Eqs.(6) and (7) into Eq.(8) gives

$$\begin{aligned} \overline{L} =& a^2\left[\frac{\mathrm{d}\Delta}{\mathrm{d}\xi}\omega\cos 2\theta - 2\delta\omega\cos 2\theta - \omega^2\right. \\ &+ k\sin 2\theta\cos(2\psi - 2\omega\Delta)\exp\left(-\frac{\pi}{2}\eta^2\Delta^2\right) \\ &+ \frac{1}{3}a^2\eta\sin^2 2\theta\exp\left(-\frac{\pi}{2}\eta^2\Delta^2\right) \\ &- \frac{1}{6}a^2\eta\sin^2 2\theta - \frac{1}{2}\eta(\eta - 2a^2) \\ &\left.- \cos 2\theta\frac{\mathrm{d}\psi}{\mathrm{d}\xi}\frac{\pi^2}{6\eta^2}\left(C^2 + \frac{1}{2}\frac{\mathrm{d}C}{\mathrm{d}\xi}\right) - \frac{\mathrm{d}\phi}{\mathrm{d}\xi}\right]. \end{aligned} \tag{9}$$

Varying Eq.(9) with respect to ψ leads to the equation

$$\sin 2\theta\left[k\sin(2\psi - 2\omega\Delta)\exp\left(-\frac{\pi}{2}\eta^2\Delta^2\right) + \frac{\mathrm{d}\theta}{\mathrm{d}\xi}\right] = 0 \tag{10}$$

Also varying Eq.(9) with respect to ω leads to the evolution equation for Δ

$$\begin{aligned} &\frac{\mathrm{d}\Delta}{\mathrm{d}\xi}\cos 2\theta - 2\delta\cos 2\theta - 2\omega \\ &+ 2k\Delta\sin 2\theta\sin(2\psi - 2\omega\Delta) \\ &\times \exp\left(-\frac{\pi}{2}\eta^2\Delta^2\right) = 0. \end{aligned} \tag{11}$$

The critical power can be defined as the minimum input peak power necessary to maintain about half of the energy in the initially excited mode (u), and is approximately given by

$$P_{\mathrm{th}} \approx \left[\frac{a^2\eta}{k}\exp\left(\frac{\pi}{2}\eta^2\Delta^2\right)\right]_{\mathrm{th}}. \tag{12}$$

Analysing the critical case as the reference [4], we can obtain

$$P_{\mathrm{th}}/P_c \approx \exp\left(\frac{\pi}{2}\eta^2\Delta^2\right), \tag{13}$$

where P_c is the critical power without group-velocity mismatch.

The evolution of Δ is approximately given by

$$\frac{\mathrm{d}\Delta}{\mathrm{d}\xi} \approx 2\delta. \tag{14}$$

From Eq.(10)–(13), by the use of variational approach, we can describe the behaviours of the soliton self-routing pulse switching in the fibre coupler.

3. Self-routing pulse switching

For a half-beat coupler, the length of the coupler is[5]

$$\xi_H = \frac{\pi}{2k}. \tag{15}$$

It can be seen that the length varies with the pulse width.

For a nonlinear fibre coupler, the relative displacement between the two eigenmodes in terms of orthogonal polarization caused by group-velocity mismatch is given by

$$\Delta = 2\xi_H \times \delta. \tag{16}$$

From Eq.(2), we can obtain

$$\Delta \approx \frac{\pi}{\omega \tau_0}. \tag{17}$$

It becomes clear that for a half-beat coupler, the shorter the pulse-width is, the stronger the affect of group-velocity mismatch on soliton switching will be. The relative displacement is also inversely proportional to the pulse width. These phenomena can further be verified by the use of numerical simulations.

Based on Eq.(5), the soliton switching behaviour in a nonlinear fibre coupler with or without group-velocity mismatch is simulated by the use of split-step fast Fourier transform (SSFFT). The initial input pulse can always be considered as

$$u = \sqrt{P}\,\mathrm{sec}\,h(\tau),$$

$$v = 0, \tag{18}$$

where P is the intensity of the input soliton.

For simplicity, we take the normalized coupling coefficient as 0.5[5] when the pulse width is 1ps in a nonlinear fibre coupler. The corresponding parameters adopted in the simulations are: the second-order dispersion $\beta_2 = -20\mathrm{ps}^2/\mathrm{km}$, light wavelength λ=1550nm, and the refractive index difference $\Delta n = 2.4 \times 10^{-5}$. For example, for a 10ps optical pulse, its normalized group-velocity mismatch (time delay) is 1.0, which agrees with reference,[8] namely the normalized group-velocity mismatch is from 0.3 to 3 in a single mode fibre. In our simulations, it is always assumed that the effect of group-velocity mismatch on coupling coefficient is much smaller than that of the pulse width.

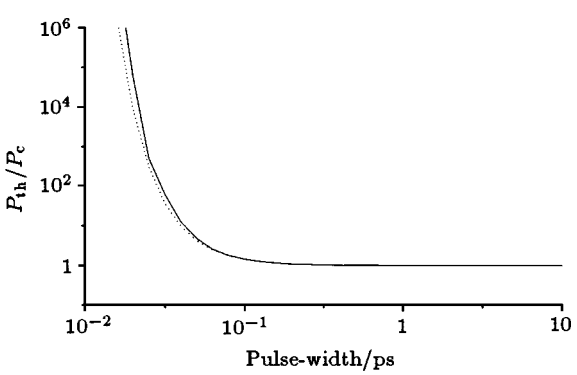


Fig.1. The evolution of ratio P_{th}/P_c versus pulse width for the propagation distance $\xi = \xi_H$. The solid line is obtained by variational approach, and the dashed line is by direct numerical simulation.

Figure 1 shows the evolution of ratio P_{th}/P_c versus pulse width for the propagation distance $\xi = \xi_H$, where the solid line is obtained by the use of Eq.(2) and (3) in a variational approach, and the dotted line is the direct numerical simulation result. It can be seen from these results that the critical power with relative displacement is different from the one without relative displacement. When the pulse width is short (0.1 ps, say), the critical power with relative displacement is far greater than that without relative displacement. The results obtained by both methods are in good agreement when the pulse width is greater than 0.1 ps. However, obvious difference can be found when the pulse width is less than 0.05ps. Thus, the variational approach can be used to effectively investigate the soliton self-routing switching behaviours of the fibre coupler when the pulse width is not too short.

The transmission coefficient can be defined as

$$T = \frac{\int_{-\infty}^{\infty} |v|^2 \mathrm{d}\tau}{\int_{-\infty}^{\infty} (|u|^2 + |v|^2)\mathrm{d}\tau} = \cos^2\theta. \tag{19}$$

From Eq.(10), the evolution of coupling angle is given by

$$\frac{\mathrm{d}\theta}{\mathrm{d}\xi} = -k\sin(2\psi - 2\omega\Delta)\exp\left(-\frac{\pi}{2}\eta^2\Delta^2\right). \tag{20}$$

Figure 2 demonstrates the evolution of transmission coefficient versus soliton pulse width for the propagation distance $\xi = \xi_H$. The solid line in the figure is obtained by the use of both Eq.(19) and Eq.(20), whereas the dotted line is the direct numerical simulation result. Significant difference can be found at different values of the pulse width. If the pulse width is relatively large (1 ps, say), the transmission coefficient is close to 1. When the pulse-width is short (e.g. 0.1 ps), the transmission coefficient becomes slightly smaller. When the pulse-width is still shorter (e.g. 0.05 ps), the transmission coefficient becomes obviously small. The effect of relative displacement on transmission coefficient is also obvious, the shorter the pulse-width is, the greater the influence of group-velocity mismatch on soliton switching. For example, the transmission coefficient is nearly zero when the pulse-width is shorter than 0.04ps, namely the energy of the pulse is maintained in the initial excited mode (u).

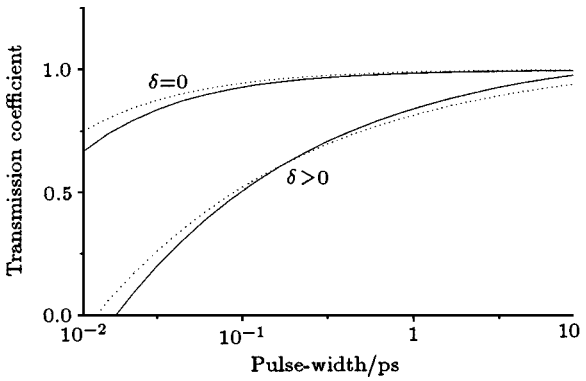


Fig.2. The evolution of transmission coefficient versus pulse width of soliton for the propagation distance $\xi = \xi_H$. The solid line is obtained by variational approach, and the dashed line is by direct numerical simulation.

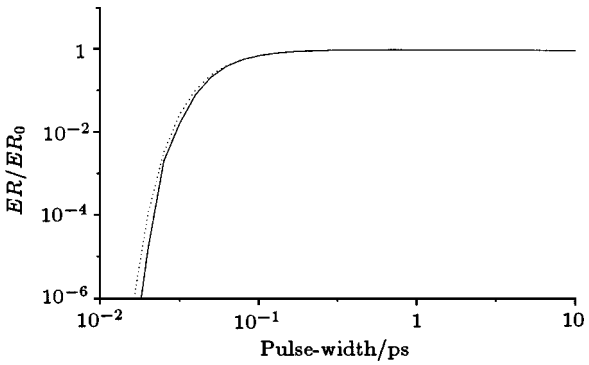


Fig.3. The ratio of the elimination light ratio versus pulse width of soliton for the propagation distance $\xi = \xi_H$. The solid line is obtained by variational approach, and the dashed line is by direct numerical simulation.

We may define the elimination light ratio (ER) as

$$ER = \left(\frac{|v|^2}{|u|^2}\right)_{\max}. \tag{21}$$

From Eqs.(7), we have

$$\begin{aligned} ER \approx & (\mathrm{ctg}^2\theta)_{\max} \mathrm{e}^{-\frac{\pi}{2}\eta^2\Delta^2} \\ = & ER_0 \mathrm{e}^{-\frac{\pi}{2}\eta^2\Delta^2}, \end{aligned} \tag{22}$$

where ER_0 is the elimination light ratio without relative displace (namely, Δ=0).

Figure 3 shows the normalized elimination light ratio versus pulse width of the soliton for the propagation distance $\xi = \xi_H$. The solid line and dotted line in Fig.3 are obtained by using Eq.(16) and Eq.(22), and direct numerical simulations, respectively. When ER is smaller than a special value, the fibre coupler cannot induce switching operation. When the pulse width is shorter (e.g. 0.1ps), the elimination light ratio with relative displacement is much smaller than that without relative displacement. The results obtained by the use of variational approach agree well with those by numerical simulations when the pulse width is greater than 0.1ps. However, clear difference can be found between the two approaches when the pulse width becomes smaller than 0.05ps.

In fact, when the pulse width is too short, the high-order perturbations in a nonlinear fibre coupler, such as intra-pulse Raman scattering, third-order dispersion and self-steepening, cannot be neglected. However, for simplicity, we did not take into consideration their effects on the fibre coupler.

4. Conclusion

A relative displacement is introduced to describe the effect of group-velocity mismatch on the nonlinear fibre coupler. The influence of the group-velocity mismatch on soliton self-routing pulse switching is discussed in detail by using both the variational approach and numerical simulation. It shows that the group-velocity mismatch leads to increasing the critical power for switching and the reduction of the elimination light ratio. The switching behaviour is critically dependent on the pulse width, and for sub-ps pulses, the influence cannot be neglected. When comparing the results obtained by the use of variational approach and direct numerical simulation respectively, a good agreement can be found when the pulse width is not too short.

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