Vol 12 No 4, April 2003
1009-1963/2003/12(04)/0408-07

Chinese Physics

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Non-conservation of energy arising from atomic dipole interactions and its effects on light field and coupled atoms*

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(Received 21 August 2002; revised manuscript received 23 December 2002)

The interactions between coupled atoms and a single mode of a quantized electromagnetic field, which involve the terms originating from the dipole interactions, are discussed. In the usual Jaynes–Cummings model for coupled atoms, the terms of non-conservation of energy originating from dipole interactions are neglected, however, we take them into consideration in this paper. The effects of these terms on the evolutions of quantum statistic properties and squeezing of the field, the squeezing of atomic dipole moments and atomic population inversion are investigated. It has been shown that the coupling between atoms modulates these evolutions of fields and atoms. The terms of non-conservation of energy affect these evolutions of fields and atoms slightly. They also have effects on the squeezing of the field, the squeezing of atomic dipole and atomic population inversions. The initial states of atoms also affect these properties.

Keywords: Jaynes–Cummings model, squeezing, collapse and revival
PACC: 4250

1. Introduction

Jaynes–Cummings model (JCM) is a fundamental model in quantum optics, it is exactly solvable. It plays an important role in dealing with interactions of a two-level atom and a single mode of a quantized electromagnetic field. This model has been generalized in various aspects to understand various quantum optics phenomena. Particularly, beyond the energy conservation, the JCM without rotating-wave approximation has been studied.[1−4] Shore and Knight[5] have given a good review for these generalizations. Some of the predictions have been verified in experiments.[6−8] Tavis–Cummings mode (TCM) is used to deal with the interactions of co-operative atoms with radiation field.[9−11] Super-radiance is the co-operative spontaneous emission from multi-atom.[12] In these discussions, N atoms are isolated, the interactions arising from their dipole moments are neglected. More recently, the dipole interactions of coupled atoms are introduced into JCM.[13−15] The effects of dipole interactions on the system consisting of field and coupled atoms are investigated.[16−20] If the spaces between atoms are small, the interactions of the coupled atoms will affect the evolutions of atoms and field. Considering two atoms, A and B, the interaction energy of their dipole moments is V, we have

$$\begin{aligned} V =& \Omega[S_+^{(A)} + S_-^{(A)}][S_+^{(B)} + S_-^{(B)}] \\ =& \Omega[S_+^{(A)}S_-^{(B)} + S_-^{(A)}S_+^{(B)} \\ & + S_+^{(A)}S_+^{(B)} + S_-^{(A)}S_-^{(B)}], \end{aligned} \tag{1}$$

where S_+ and S_- are psudo-spin operators, Ω is the coupling strength of two atoms. In Eq. (1), the first term describes the process in which atom A transits to an excited state and atom B transits to the ground state simultaneously, the second term is for the opposite process. In both processes, the number of photons does not change and the processes conserve the energy. The latter two terms are for the processes in which both atoms transit to the excited states or to the ground states and no photon is absorbed or emitted, so the processes do not conserve the energy. Similar to the rotating-wave approximation in JCM, the latter two terms are usually neglected. From the view of uncertainty principle of energy and time, non-conservation of energy is permitted in a short duration so long as the uncertainty relation is satisfied.

In this paper, we discuss the terms of non-conservation of energy originating from dipole inter-

*Project supported by the Shanghai Leading Academic Discipline Program.

actions, investigate their effects on the evolutions of atoms and field. We give the Hamiltonian of the system consisting of coupled atoms and field to formulate the theory in section 2, discuss the effects originating from the terms of non-conservation of energy on the quantum statistic properties and squeezing for the field in section 3, and study the squeezing for atomic dipole and atomic population inversions in section 4. Finally, some conclusions are given in section 5.

2.Formulation of the theory

According to the uncertainty relation for energy and time, $\Delta E \times \Delta t \geq \hbar$, the energy is permitted to have an uncertainty $\hbar/\Delta t$ in a duration of Δt; in other words, it is not necessary to conserve the energy in a duration Δt with the prerequisite of uncertainty principle. The smaller the degree of deviation from energy conservation is, the longer the duration Δt continues. The former two terms in Eq.(1) are for the processes in which the transition for an atom occurs with another atom transiting in opposite direction. In these processes, the energy variation is so small that the duration Δt is long enough for detecting these processes. The latter two terms describe the processes in which the two atoms transit in the same direction, and the transitions do not conserve the energy in a duration $\Delta t \approx \hbar/\Delta E = 1/(2\pi\nu)$, where ΔE is the energy difference of two transiting levels. The processes occurring in such a short duration ($10^{-11} - 10^{-16}$ second) cannot be detected by usual means. More recently, the pulsed -laser with femto-second or atta-second duration has been developed, thus it provides a method to detect these processes.

The transitions of atoms A and B do not occur simultaneously, so both atoms may be in upper levels or ground states in a period of overlap duration. The atomic transitions go through a period of time. In the period of transition, the atoms can be considered neither in excited states nor in ground states exactly according to the uncertainty principle, and the atomic state is an uncertainty. The processes described by the latter two terms in Eq.(1) can really exist. It is significant to study these processes.

Considering a system consisting of two atoms A and B and a one-mode cavity field, the Hamiltonian can be written as

$$H = H_0 + H', \tag{2}$$

where

$$H_0 = \omega a^+ a + \omega_0 (S_3^{(A)} + S_3^{(B)}); \tag{3}$$

and the interaction H' is

$$\begin{aligned} H' =& g[a^+ (S_-^{(A)} + \mathrm{e}^{-i\delta} S_-^{(B)}) \\ &+ a(S_+^{(A)} + \mathrm{e}^{i\delta} S_+^{(B)})] + V, \end{aligned} \tag{4}$$

where

$$\begin{aligned} V =& \Omega(S_+^{(A)} S_-^{(B)} \mathrm{e}^{-i\delta} + S_-^{(A)} S_+^{(B)} \mathrm{e}^{i\delta}) \\ &+ \Omega'(S_+^{(A)} S_+^{(B)} \mathrm{e}^{i\delta} + S_-^{(A)} S_-^{(B)} \mathrm{e}^{-i\delta}). \end{aligned} \tag{5}$$

Here V is the interaction of dipoles of the two atoms, a^+ and a are creation and annihilation operators of the field with frequency ω, g is the atom-field coupling constant. ω_0 is the atomic transition frequency, $S_+^{(j)}$ and $S_-^{(j)}$ ($j = A$ or B) are pseudo-spin operators of atoms A and B. Since usually the two atoms do not transit synchronously, a relative phase δ is introduced to describe the non-synchronism of transition.[21] The first term in Eq.(5) describes the processes in which the atoms transit in opposite directions and the number of photons does not change, so the energy is conserved. The second term describes the processes in which both atoms transit to the excited or ground state, so the energy is not conserved. The coupling constants of atoms for the conservation and non-conservation of energy are denoted as Ω and Ω' respectively to distinguish the effects arising from the two terms, in fact, we have $\Omega \equiv \Omega'$. If the terms of non-conservation of energy are omitted, $\Omega' = 0$, but $\Omega \neq 0$. Transforming the Hamiltonian into an interaction picture and considering the case of resonance, i.e. $\omega = \omega_0$, we obtain

$$\begin{aligned} H^{(I)} =& g[a^+ (S_-^{(A)} + \mathrm{e}^{-i\delta} S_-^{(B)}) \\ &+ a(S_+^{(A)} + \mathrm{e}^{i\delta} S_+^{(B)})] \\ &+ \Omega(S_+^{(A)} S_-^{(B)} \mathrm{e}^{-i\delta} + S_-^{(A)} S_+^{(B)} \mathrm{e}^{i\delta}) \\ &+ \Omega'[S_+^{(A)} S_+^{(B)} \mathrm{e}^{i(\delta+2\omega t)} \\ &+ S_-^{(A)} S_-^{(B)} \mathrm{e}^{-i(\delta+2\omega t)}]. \end{aligned} \tag{6}$$

We assume that the initial states of both atoms are superposition states of ground states and excited states, denoted as $|1\rangle$ and $|2\rangle$ respectively, the initial state of the one-mode cavity field is a vacuum state, so the initial state of the system is

$$|\Psi(0)\rangle = \cos\theta |0,1,1\rangle + \sin\theta \mathrm{e}^{i\varphi} |0,2,2\rangle, \tag{7}$$

where φ is the superposition phase, $\cos\theta$ and θ are probability amplitudes of the ground and excited

states respectively. At $t > 0$, the state is assumed to be

$$\begin{aligned}|\Psi(t)\rangle =& D(t)\cos\theta|0,1,1\rangle \\ &+ \sin\theta e^{i\varphi}[C(t)|0,2,2\rangle + A_1(t)|1,2,1\rangle \\ &+ A_2(t)|1,1,2\rangle + B(t)|2,1,1\rangle],\end{aligned} \tag{8}$$

where A_1, A_2, B, C and D are coefficients to be determined. The normalizing condition is

$$\begin{aligned}&|D(t)|^2\cos^2\theta + (|A_1(t)|^2 + |A_2(t)|^2 \\ &+ |B(t)|^2 + |C(t)|^2)\sin^2\theta = 1.\end{aligned} \tag{9}$$

Using the interaction Hamiltonian (6) and state vector (8), the Schrödinger equation can be reduced to a set of coupled equations as follows.

$$\begin{cases} i\mathrm{d}D(t)/\mathrm{d}t = \Omega' e^{-i(\delta+2\omega t)}\sin\theta e^{i\varphi}C(t), \\ i\mathrm{d}C(t)/\mathrm{d}t = g e^{i\delta}A_1(t) + gA_2(t) \\ \qquad\qquad + \cot\theta e^{-i\varphi}\Omega' e^{i(\delta+2\omega t)}D(t), \\ i\mathrm{d}A_1(t)/\mathrm{d}t = g e^{-i\delta}C(t) + \sqrt{2}gB(t) + \Omega e^{-i\delta}A_2(t), \\ i\mathrm{d}A_2(t)/\mathrm{d}t = gC(t) + \sqrt{2}g e^{i\delta}B(t) + \Omega e^{i\delta}A_1(t), \\ i\mathrm{d}B(t)/\mathrm{d}t = \sqrt{2}gA_1(t) + \sqrt{2g}e^{-i\delta}A_2(t). \end{cases} \tag{10}$$

If the terms of non-conservation of energy are neglected, i.e. $\Omega' = 0$, Eq.(10) is reduced to

$$\begin{cases} i\mathrm{d}D(t)/\mathrm{d}t = 0, \\ i\mathrm{d}C(t)/\mathrm{d}t = g[e^{i\delta}A_1(t) + A_2(t)], \\ i\mathrm{d}A_1(t)/\mathrm{d}t = g[\sqrt{2}B(t) + e^{-i\delta}C(t)] + \Omega e^{-i\delta}A_2(t), \\ i\mathrm{d}A_2(t)/\mathrm{d}t = g[\sqrt{2}e^{i\delta}B(t) + C(t)] + \Omega e^{i\delta}A_1(t), \\ i\mathrm{d}B(t)/\mathrm{d}t = \sqrt{2}g[A_1(t) + e^{-i\delta}A_2(t)]. \end{cases} \tag{11}$$

Taking the solution of Eq.(11) to be zeroth-order approximation, and substituting it into the right-hand of Eq.(10), the solution of Eq.(10) can be obtained by successive substitution method. The solution of Eq.(11), which is marked with superscript "(0)", is

$$\begin{cases} A_1^{(0)}(t) = -\dfrac{g}{\gamma_1-\gamma_2}e^{-i\delta}(e^{i\gamma_1 t} - e^{i\gamma_2 t}), \\ A_2^{(0)}(t) = -\dfrac{g}{\gamma_1-\gamma_2}(e^{i\gamma_1 t} - e^{i\gamma_2 t}), \\ B^{(0)}(t) = \left[\dfrac{2\sqrt{2}g^2}{\gamma_1-\gamma_2}\left(\dfrac{1}{\gamma_1}e^{i\gamma_1 t} - \dfrac{1}{\gamma_2}e^{i\gamma_2 t}\right) - \dfrac{\sqrt{2}}{3}\right]e^{-i\delta}, \\ C^{(0)}(t) = \dfrac{2g^2}{\gamma_1-\gamma_2}\left(\dfrac{1}{\gamma_1}e^{i\gamma_1 t} - \dfrac{1}{\gamma_2}e^{i\gamma_2 t}\right) + \dfrac{2}{3}, \\ D^{(0)}(t) = 1, \end{cases} \tag{12}$$

where

$$\gamma_1 = 0.5(-\Omega + \sqrt{\Omega^2 + 24g^2}),$$

$$\gamma_2 = 0.5(-\Omega - \sqrt{\Omega^2 + 24g^2}). \tag{13}$$

For $\omega \gg \Omega$ and g, approximating to first-order small quantity of Ω/ω and g/ω, the solution of Eq.(10) is

$$\begin{cases} A_1(t) = A_1^{(0)}(t) - \dfrac{gM}{\mu_1-\mu_2}e^{-i\delta}(e^{i\mu_1 t} - e^{i\mu_2 t}), \\ A_2(t) = A_2^{(0)}(t) - \dfrac{gM}{\mu_1-\mu_2}(e^{i\mu_1 t} - e^{i\mu_2 t}), \\ B(t) = B^{(0)}(t) - 2\sqrt{2}Me^{-i\delta} \\ \qquad \times\left[\dfrac{1}{4} - \dfrac{g^2}{\mu_1-\mu_2}\left(\dfrac{1}{\mu_1}e^{i\mu_1 t} - \dfrac{1}{\mu_2}e^{i\mu_2 t}\right)\right], \\ C(t) = C^{(0)}(t) - M(e^{i2\omega t} - 1), \\ D(t) = D^{(0)}(t) - M\tan\theta\sin\theta(1 - e^{-i2\omega t})e^{i2(\varphi-\delta)}, \end{cases} \tag{14}$$

where

$$M = \frac{1}{2}\frac{\Omega'}{\omega}\cot\theta e^{i(\delta-\varphi)}, \tag{15}$$

$$\mu_1 = 0.5(-\Omega + \sqrt{\Omega^2 + 16g^2}),$$

$$\mu_2 = 0.5(-\Omega - \sqrt{\Omega^2 + 16g^2}). \tag{16}$$

If both atoms are initially in ground states and the initial state of the field is a vacuum, the atomic transition will not occur, so $\theta \neq 0$. If $\theta = \pi/2$ (both atoms are initially in excited states), $M = 0$. This means that the terms of non-conservation of energy have no effect on the atoms and the field in the case of $\theta = \pi/2$ with approximating to first power of Ω/ω and g/ω, but they have effects on the evolutions of atoms and field when taking the higher powers of Ω/ω and g/ω into account.

Since $C(t)$ and $D(t)$ contain the terms $\exp(\pm 2i\omega t)$, the evolutions of probability amplitudes of states $|0,2,2\rangle$ and $|0,1,1\rangle$, which constitute the initial states, will oscillate, but $A_1(t), A_2(t)$ and $B(t)$ do not contain the terms with higher frequency in the case of approximating to first power of Ω/ω and g/ω.

3. Evolutions of the properties of the field

Using Eqs.(9) and (14), the mean number of photons and its fluctuations in the field are

$$\langle N\rangle = \sin^2\theta(|A_1|^2 + |A_2|^2 + 2|B|^2), \tag{17}$$

$$\begin{aligned}\langle(\Delta N)^2\rangle =& \sin^2\theta(|A_1|^2 + |A_2|^2 + 4|B|^2) \\ &- \sin^4\theta(|A_1|^2 + |A_2|^2 + 2|B|^2)^2.\end{aligned} \tag{18}$$

The second-order correlation function is

$$g^{(2)}(0) = \frac{2|B|^2}{\sin^2\theta(|A_1|^2 + |A_2|^2 + 2|B|^2)^2}. \tag{19}$$

The evolutions of $\langle N\rangle$, $\langle(\Delta N)^2\rangle$ and $g^{(2)}(0)$ are shown in Figs.1–3 respectively. In these figures, (a) is for the case of omitting the terms of non-conservation of energy and the coupling between atoms, i.e. $\Omega = \Omega' = 0$, (b), (c) and (e) are for the cases of $\Omega = \Omega' \neq 0$, (d) is for the case of considering the coupling but omitting the terms of non-conservation of energy ($\Omega \neq 0, \Omega' = 0$). For $\Omega = \Omega' = 0$, the evolutions exhibit constant-amplitude oscillations. The coupling between the atoms modulates the amplitudes of these oscillations, but do not change the frequency. The modulating period is shortened with strengthening the coupling. If $\theta = \pi/4$ the field is of super-Poisson distribution and bunching. If $\Omega = \Omega' = 0.5$g and $\theta = \pi/2$, the field exhibits un-bunching periodically [see Fig.3(e)]. The properties of the field are highly sensitive to θ; in other words, the initial states of atoms strongly affect the properties of the field. The terms of non-conservation of energy increases the mean number of photons and affect the second-order correlation functions slightly [see Fig.1(d) and Fig.3(d)].

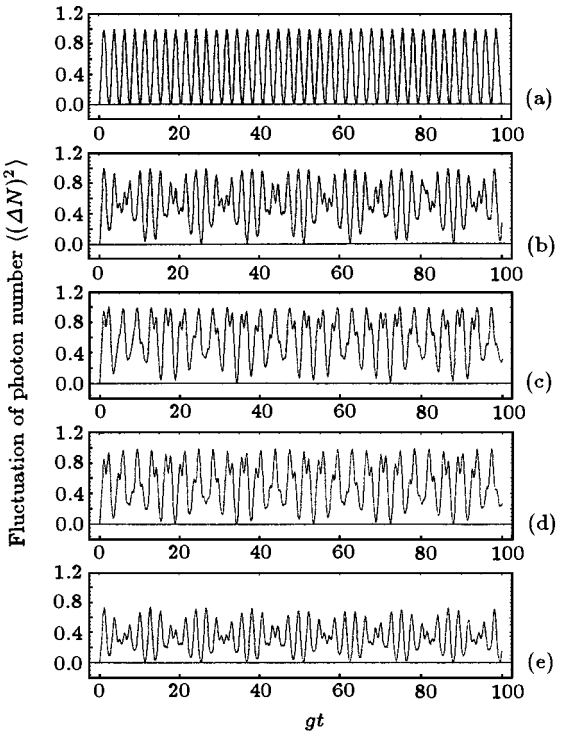


Fig.2. Evolutions of photon number fluctuations. The parameters in Figs.(a)–(e) are same as those in Fig.1.

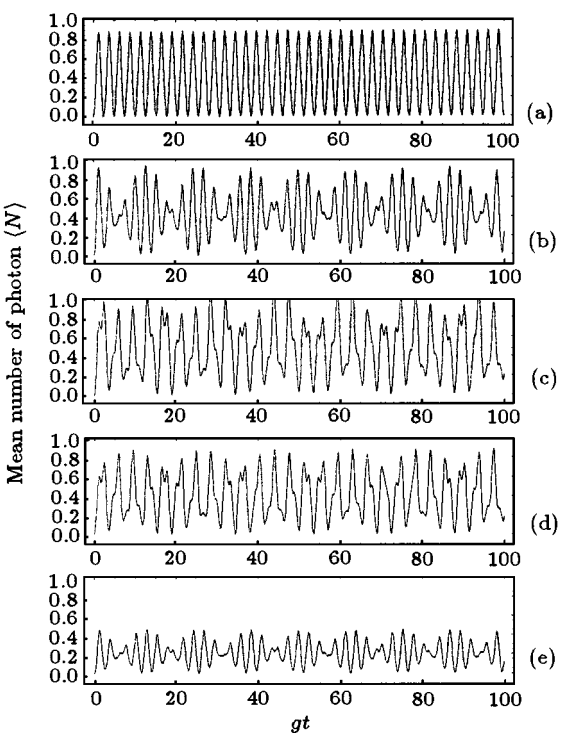


Fig.1. Evolutions of the mean number of photons in the case of ω=10, δ=0, $\varphi = 0$ for (a) $\theta = \pi/4$, $\Omega = \Omega' = 0$, (b) $\theta = \pi/4, \Omega = \Omega' = 0.5$g, (c) $\theta = \pi/4, \Omega = \Omega' = 2$g, (d) $\theta = \pi/4, \Omega = 2$g, Ω'=0, (e) $\theta = \pi/6, \Omega = \Omega' = 0.5$g.

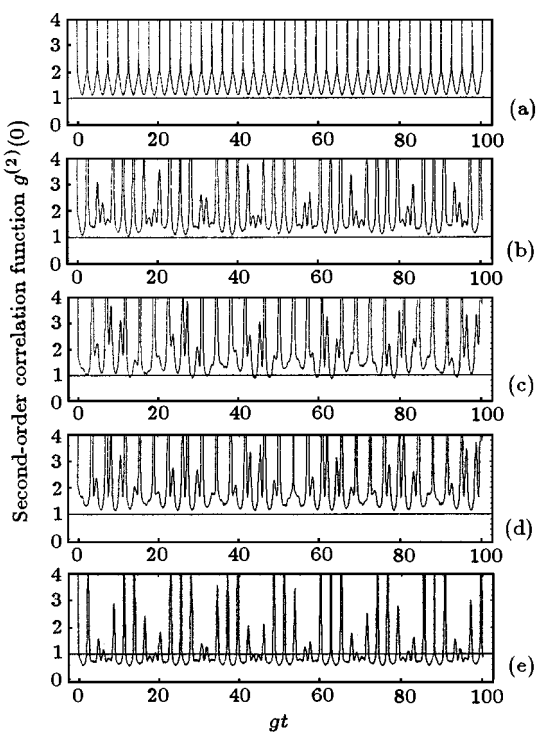


Fig.3. Evolutions of second-order correlation functions in the case of $\omega = 10$, $\delta = 0$, $\varphi = 0$ for (a) $\theta = \pi/4$, $\Omega = \Omega' = 0$, (b) $\theta = \pi/4, \Omega = \Omega' = 0.5$g, (c) $\theta = \pi/4, \Omega = \Omega' = 2$g, (d) $\theta = \pi/4, \Omega = 2$g, Ω'=0, (e) $\theta = \pi/2, \Omega = \Omega' = 0.5$g.

Two slowly varying Hermitian quadrature operators for the amplitude of the field are defined as

$$a_1 = 0.5(a^+ \mathrm{e}^{-i\omega t} + a^{i\omega t}),$$

$$a_2 = 0.5i(a^+ \mathrm{e}^{-i\omega t} - a\mathrm{e}^{i\omega t}). \tag{20}$$

Their expectation values are zero. The squeezing parameters for a_1 and a_2 are defined as

$$S_1 = \langle(\Delta a_1)^2\rangle - \frac{1}{4},$$

$$S_2 = \langle(\Delta a_2)^2\rangle - \frac{1}{4}. \tag{21}$$

If S_1 (or S_2) < 0, the state is said to be squeezed for a_1 (or a_2). Using these definitions, the squeezing parameters are

$$S_1 = \frac{1}{2}\sin^2\theta(|A_1|^2 + |A_2|^2 + 2|B|^2) + \frac{\sqrt{2}}{4}\sin(2\theta)\mathrm{Re}(D^*B\mathrm{e}^{i\varphi}), \tag{22a}$$

$$S_2 = \frac{1}{2}\sin^2\theta(|A_1|^2 + |A_2|^2 + 2|B|^2) - \frac{\sqrt{2}}{4}\sin(2\theta)\mathrm{Re}(D^*B\mathrm{e}^{i\varphi}). \tag{22b}$$

The squeezing of the field are shown in Fig.4, where Figs.4(a), 4(b) and 4(c) are for $\pi/6$. In these cases, $S_1 < 0$, so the fluctuations of quadrature a_1 can be squeezed. The coupling between atoms modulates the evolutions of squeezing parameters. The terms of non-conservation of energy will affect the squeezing [comparing Fig.4(c) with 4(d)]. The squeezing parameters are also sensitive to the initial states of the atoms. For some small values of θ, the field can be squeezed for a_1, but for larger values of θ, the field cannot be squeezed for a_1 [see Fig.4(e)]. It follows that the probability amplitude of the ground state is important for squeezing of field.

4. Evolutions of population inversion and squeezing for atomic dipoles

For a system of two atoms, the population inversion is defined as

$$P = \langle S_3^{(A)} + S_3^{(B)}\rangle \equiv \langle S_3\rangle. \tag{23}$$

Using Eq.(14), we have

$$P = \sin^2\theta(|C|^2 - |B|^2) - \cos^2\theta|D|^2. \tag{24}$$

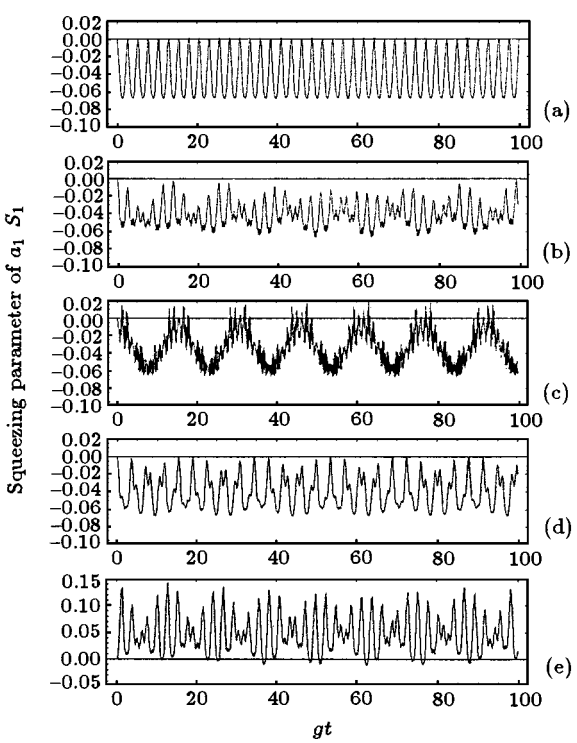


Fig.4. Evolutions of squeezing parameters $S_1$for the quadrature operator a_1 in the case of $\omega = 10$, $\delta = 0$, $\varphi = 0$ for (a) $\theta = \pi/6$, $\Omega = \Omega' = 0$, (b) $\theta = \pi/6, \Omega = \Omega' = 0.5$g, (c) $\theta = \pi/6, \Omega = \Omega' = 2$g, (d) $\theta = \pi/6, \Omega = 2$g, Ω'=0, (e) $\theta = \pi/4, \Omega = \Omega' = 0.5$g.

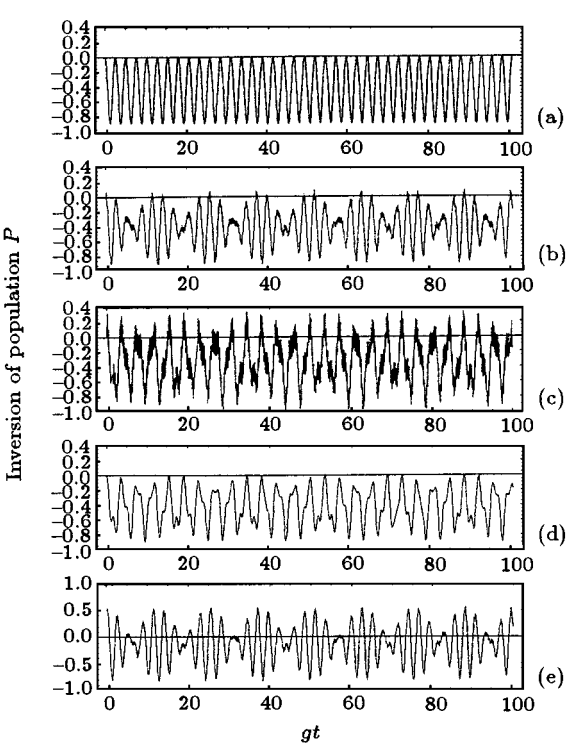


Fig.5. Evolution of population inversions P in the case of $\omega = 10$, $\delta = 0$, $\varphi = 0$ for (a) $\theta = \pi/4$, $\Omega = \Omega' = 0$, (b) $\theta = \pi/4, \Omega = \Omega' = 0.5$g, (c) $\theta = \pi/4, \Omega = \Omega' = 2$g, (d) $\theta = \pi/4, \Omega = 2$g, Ω'=0, (e) $\theta = \pi/3, \Omega = \Omega' = 0.5$g.

The evolutions of population inversion are shown in Fig.5, where 5(a), 5(b), 5(c) and 5(e) are for different values of Ω and $\Omega = \Omega'$, 5(d) is for the case of considering the coupling but omitting the terms of non-conservation of energy. The population is inverted if $P > 0$. The evolutions of population inversion are constant-amplitude oscillations with time in the case of $\Omega = \Omega' = 0$, and are modulated in the case of $\Omega \neq 0$. Whether the population is inverted depends on the initial states of the atoms. If $\theta = \pi/3$, the population inversions occur periodically [see Fig.5(e)].

From Eqs.(24) and (17), the sum of the mean photon number and population inversion is

$$\begin{aligned}\langle N\rangle + \langle S_3\rangle =& \sin^2\theta(|A_1|^2 + |A_2|^2 + |B|^2 + |C|^2) \\ & - \cos^2\theta|D|^2 \\ =& 1 - 2\cos^2\theta|D|^2, \end{aligned} \tag{25}$$

where the normalizing condition (9) has been used. Using Eq.(14) and neglecting the terms of M^2, Eq.(25) can be reduced to

$$\begin{aligned}&\langle N\rangle + \langle S_3\rangle \\ &\approx -[\cos(2\theta) - 2(\Omega'/\omega)\sin(2\theta)\cos\theta\sin^2(\omega t)], \end{aligned} \tag{26}$$

where $\delta = \varphi = 0$. It is shown that $\langle N\rangle + \langle S_3\rangle$ is almost time- independent if the terms of non- conservation of energy are omitted, but exhibits oscillations in small amplitudes (about Ω'/ω) with time in the case of taking the terms of non-conservation of energy into account. This means that the inversion condition is also dependent on Ω', but this dependence is weak in view of ω. The inversion condition now reads

$$\langle N\rangle + \cos(2\theta) - 2(\Omega'/\omega)\sin(2\theta)\sin\theta\sin^2(\omega t) < 0. \tag{27}$$

To study the squeezing of atomic dipoles, two slowly varying Hermitian quadrature operators are defined as

$$d_1 = 0.5[(S_+^{(A)} + S_+^{(B)})\mathrm{e}^{-i\omega_0 t} + (S_-^{(A)} + S_-^{(B)})e^{i\omega_0 t}], \tag{28a}$$

$$d_2 = -0.5i[(S_+^{(A)} + S_+^{(B)})\mathrm{e}^{-i\omega_0 t} - (S_-^{(A)} + S_-^{(B)})e^{i\omega_0 t}]. \tag{28b}$$

They obey the commutation relation

$$[d_1, d_2] = i(S_3^{(A)} + S_3^{(B)}) \equiv iS_3. \tag{29}$$

The Heisenberg uncertainty relation is

$$\langle(\Delta d_1)^2\rangle\langle(\Delta d_2)^2\rangle \geq 0.25\langle S_3\rangle^2, \tag{30}$$

and fundamental quantum fluctuations for the component d_j (j=1 or 2) of the atomic dipole is $0.5|\langle S_3\rangle|$. To investigate the squeezing of the fluctuations in atomic dipoles, two squeezing parameters are defined

$$\begin{aligned} Q_1 &= \langle(\Delta d_1)^2\rangle - 0.5|\langle S_3\rangle|, \\ Q_2 &= \langle(\Delta d_2)^2\rangle - 0.5|\langle S_3\rangle|. \end{aligned} \tag{31}$$

The fluctuations in the component d_j are said to be squeezed if $Q_j < 0$. Using Eq.(14) and normalizing condition (9), we obtain the squeezing parameters

$$\begin{aligned} Q_1 =& 1/2 + \sin^2\theta\mathrm{Re}(A_1A_2^*) \\ & + 1/2\sin(2\theta)\mathrm{Re}(CD^*\mathrm{e}^{-i\varphi}) \\ & - 1/2|\sin^2\theta(|C|^2 - |B|^2) - \cos^2\theta|D|^2|, \end{aligned}$$

$$\begin{aligned} Q_2 =& 1/2 + \sin^2\theta\mathrm{Re}(A_1A_2^*) \\ & - 1/2\sin(2\theta)\mathrm{Re}(CD^*\mathrm{e}^{-i\varphi}) \\ & - 1/2|\sin^2\theta(|C|^2 - |B|^2) - \cos^2\theta|D|^2|. \end{aligned} \tag{32}$$

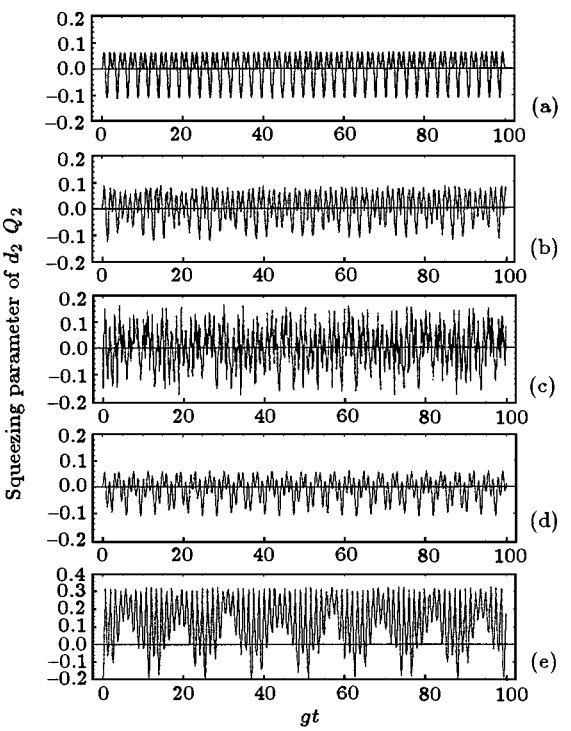


Fig.6. Evolutions of squeezing parameters Q_2 for the quadrature operator d_2, The parameters in Figs.(a)–(e) are same as those in Fig.5.

The evolutions of squeezing for fluctuations in the component d_2 of an atomic dipole are shown in Fig.6, where 6(a), 6(b) and 6(c) are for different values of $\Omega(\Omega = \Omega')$ in the case of $\theta = \pi/4$, 6(d) is for the case of omitting the terms of non-conservation of energy. The evolutions of squeezing for the atomic dipole exhibit constant-amplitude oscillations with time for

$\Omega = \Omega' = 0$. The coupling between atoms modulates the oscillations and the modulating period becomes shorter with strengthening the coupling between two atoms. The squeezing for atomic dipole also depends on the initial states of atoms [see 6(e)]. The terms of non-conservation of energy also affect the squeezing of atomic dipole

5. Conclusion

From the above analysis, three conclusions can be drawn. The first is that the evolutions of properties for the atoms and field exhibit oscillations. The coupling between the atoms modulates the amplitudes of these oscillations, and the modulating period becomes shorter with strengthening the coupling between the atoms. The second is that the properties of the atoms and field are sensitive to the initial states of the atoms. The third is that the terms of non-conservation of energy arising from the atomic dipole interactions affect the evolutions of the properties of the atoms and field, but the effects are weak. We take $\omega = 10$ in order to show these effects obviously. In fact, the transition frequency is much higher than this value. The effects brought about by the terms of non-conservation of energy are not so obvious as those shown in the figures. For the sake of simplicity, the initial state of light is a vacuum in our discussion. If it is not vacuum, enhancing the intensity of light will make the effects arising from the terms of non-conservation of energy more obvious. With high intensity of light, these effects cannot be neglected.

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