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Synchronization of the time-varying parameter chaotic system and its application to secure communication*

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In this paper we propose and investigate the synchronization of a new chaotic model with time-varying parameters and apply it to improve the security of chaotic communication. In this model, the chaotic system is modulated by both the message and the varying parameters. The varying parameters distort the phase space so heavily that they prevent the carrier from being broken by nonlinear dynamic forecasting method. Theory and simulation experiments with speech signal communication indicate that the receiver can gain a perfect synchronization with the transmitter, and the intruder cannot break down this communication system. We also discuss the robustness of the new communication system.

Keywords: time-varying parameter, chaotic synchronization, nonlinear dynamic forecasting
PACC: 0545, 0100

1. Introduction

Since Pecora and Carroll[1] reported in 1990 that certain chaotic systems can possess a self-synchronization property, synchronization in chaotic systems and its applications, particularly in communication, have been extensively investigated and considerable progress has been made.[2−11] Because a chaotic signal is wide-band and noise-like, it is considered that the use of chaotic carriers can provide a high secure communication. Therefore, many works have been done to mask messages by using chaotic systems.[2−6,12−17] But, before a scheme can be used, it must undergo different attacks. In fact, most investigations of secure communication that focus on low-dimensional chaotic systems did not make the best use of antideciphering.[18] Short[19−21] has applied the nonlinear dynamic (NLD) forecasting to extract the hidden signal in the chaotic transmitting signal from the reconstructed phase space. Zhou and Lai[22] have proved that the communication scheme using a simple time-delay chaotic system is also ineffective. Moreover, recent work[23] shows that even the hyperchaotic carrier is probably not secure enough. Accordingly, many schemes[13−17] have been proposed to improve the security of chaotic communication. Even some very complicated combined chaotic systems have been proposed for chaotic communication, such as the cascaded heterogeneous chaotic system.[23]

Reviewing the NLD forecasting method, we can see that it is based on the local geometry of the phase space reconstructed from the intercepted carrier. It predicts the actual chaotic carrier behaviour. Then the hidden message can be detected once the NLD-predicted carrier behaviour is subtracted from the actual chaotic carrier. Thus, if the local geometry of phase space can be disturbed, the NLD forecasting method may not be applied successfully. Based on this idea, we propose a new model to improve the security of chaotic communication. The message is used to modulate the parameter of a chaotic dynamical system at regular intervals. The transmitted chaotic carrier contains both the hidden message and the varying-parameter message. The receiver will obtain the varying parameter and then have synchronization with the transmitter by active passive dynamic (APD) synchronization. The behaviour of a chaotic carrier becomes so complex that the phase space is disturbed seriously and hard to predict. Therefore, it increases the security of the chaotic communication. In this paper we investigate the synchronization principle of time-varying parameter (TVP) chaotic systems and derive the robustness of synchronization to parameter mis-

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match. We perform a speech communication experiment by the APD technique. The experimental results confirm our analysis.

2.General theory of a new chaotic communication model

In the following, we give a general chaotic model to prove our idea. The system is described as:

$$\dot{\boldsymbol{x}} = \boldsymbol{F}(x, \mu). \tag{1}$$

It can be divided into two subsystems.

$$\boldsymbol{\mu}_2 = \begin{cases} \mu(\boldsymbol{x}_1(t) \circ \delta(t - (nt_0 - \tau))) & t \in [nt_0, (n+1)t_0), n = 1, 2, 3, \cdots, \tau \in (0, t_0), \\ \boldsymbol{\mu}_{20} & t \in [0, t_0). \end{cases} \tag{3}$$

Here t is the time variable, t_0 is the time interval of parameter variation and $\boldsymbol{\mu}_{20}$ is the initial parameter vector. $\circ$ denotes convolution. The receiver is described as:

$$\dot{\tilde{\boldsymbol{x}}}_1 = \boldsymbol{F}_1(\tilde{\boldsymbol{x}}_1, g, \boldsymbol{\mu}_1, \tilde{\boldsymbol{\mu}}_2), \tag{4a}$$

The transmitter is described as:

$$\dot{\boldsymbol{x}}_1 = \boldsymbol{F}_1(\boldsymbol{x}_1, g, \boldsymbol{\mu}_1, \boldsymbol{\mu}_2), \tag{2a}$$

$$\dot{\boldsymbol{x}}_2 = \boldsymbol{F}_2(\boldsymbol{x}_1, \boldsymbol{x}_2, \boldsymbol{\mu}_1, \boldsymbol{\mu}_2), \tag{2b}$$

$$g = f(\nu) + I, \tag{2c}$$

where $\boldsymbol{x}_1, \boldsymbol{x}_2$ are state vectors, $\boldsymbol{\mu}_1$ and $\boldsymbol{\mu}_2$ are parameter vectors, ν is a variable of $\boldsymbol{x}_2$ and g is the transmitted signal carrying a hidden message I. The amplitude of I is much smaller than that of $f(\nu)$. The parameter vector $\boldsymbol{\mu}_2$ is time varying,

$$\dot{\tilde{\boldsymbol{x}}}_2 = \boldsymbol{F}_2(\tilde{\boldsymbol{x}}_1, \tilde{\boldsymbol{x}}_2, \boldsymbol{\mu}_1, \tilde{\boldsymbol{\mu}}_2), \tag{4b}$$

$$I = g - f(\nu). \tag{4c}$$

The parameter vector varies according to that of the transmitter:

$$\tilde{\boldsymbol{\mu}}_2 = \begin{cases} \mu(\tilde{\boldsymbol{x}}_1(t) \cdot \delta(t - (nt_0 - \tau))) & t \in [nt_0, (n+1)t_0), n = 1, 2, 3, \cdots, \tau \in (0, t_0), \\ \boldsymbol{\mu}_{20} & t \in [0, t_0). \end{cases} \tag{5}$$

Set the state error vectors as:

$$\begin{aligned} \boldsymbol{e}_1 &= \tilde{\boldsymbol{x}}_1 - \boldsymbol{x}_1, \\ \boldsymbol{e}_2 &= \tilde{\boldsymbol{x}}_2 - \boldsymbol{x}_2, \\ \Delta\boldsymbol{\mu}_2 &= \tilde{\boldsymbol{\mu}}_2 - \boldsymbol{\mu}_2. \end{aligned} \tag{6}$$

The error equations can be obtained:

$$\dot{\boldsymbol{e}_1} = D_{\vec{x}_1}\boldsymbol{F}_1 \cdot \boldsymbol{e}_1 + D_{\vec{\mu}_2}\boldsymbol{F}_1 \cdot \Delta\boldsymbol{\mu}_2, \tag{7a}$$

$$\dot{\boldsymbol{e}_2} = D_{\vec{x}_1}\boldsymbol{F}_2 \cdot \boldsymbol{e}_1 + D_{\vec{x}_2}\boldsymbol{F}_2 \cdot \boldsymbol{e}_2 + D_{\vec{\mu}_2}\boldsymbol{F}_2 \cdot \Delta\boldsymbol{\mu}_2, \tag{7b}$$

where $D_{\vec{x}_1}\boldsymbol{F}_1$ and $D_{\vec{\mu}_2}\boldsymbol{F}_1$ are the Jacobians of the $\boldsymbol{x}_1$ subsystem vector field with respect to $\boldsymbol{x}_1$ and $\boldsymbol{\mu}_2$. $D_{\vec{x}_1}\boldsymbol{F}_2$, $D_{\vec{x}_2}\boldsymbol{F}_2$ and $D_{\vec{\mu}_2}\boldsymbol{F}_2$ are the Jacobians of the $\boldsymbol{x}_2$ subsystem vector field with respect to $\boldsymbol{x}_1$, $\boldsymbol{x}_2$ and $\boldsymbol{\mu}_2$. When Eqs.(2a) and (4a) do not have parameter vector $\boldsymbol{\mu}_2$, that is to say

$$\begin{aligned} \dot{\boldsymbol{x}}_1 &= \boldsymbol{F}_1(\boldsymbol{x}_1, g, \boldsymbol{\mu}_1) \\ \dot{\tilde{\boldsymbol{x}}}_1 &= \boldsymbol{F}_1(\tilde{\boldsymbol{x}}_1, g, \boldsymbol{\mu}_1) \end{aligned} \quad \text{and} \quad D_{\vec{\mu}_2}\boldsymbol{F}_1 = 0,$$

Equations (7a) can be simplified to $\dot{\boldsymbol{e}}_1 = D_{\vec{x}_1}\boldsymbol{F}_1 \cdot \boldsymbol{e}_1$. If we choose an equation properly to make $D_{\vec{x}_1}\boldsymbol{F}_1 \leq 0$, we are going to obtain $t \to \infty$, $\boldsymbol{e}_1 \to 0$. So when $t \to \infty$, we have $\tilde{\boldsymbol{x}}_1 \to \boldsymbol{x}_1$. Because vector $\boldsymbol{\mu}_2$ is determined only by state vector $\boldsymbol{x}_1$, when $t \to \infty$, we have $\tilde{\boldsymbol{\mu}}_2 \to \boldsymbol{\mu}_2$, $\Delta\boldsymbol{\mu}_2 \to 0$. Then the error equations can be written as follows:

$$\boldsymbol{e}_1 = 0, \tag{8a}$$

$$\Delta\boldsymbol{\mu}_2 = 0, \tag{8b}$$

$$\dot{\boldsymbol{e}}_2 = D_{\vec{x}_2}\boldsymbol{F}_2 \cdot \boldsymbol{e}_2. \tag{8c}$$

From the above equations we can see that the error equations do not relate to parameter vector $\boldsymbol{\mu}_2$. In fact, it is just the same as error equations from normal chaotic systems (the chaotic systems with constant parameters). Because it has been proved that if the Lyapunov function (L) for the error dynamics of the normal chaotic systems has $\dot{L} < 0$, the systems will synchronize, the time-varying parameter chaotic systems can also satisfy this rule and achieve synchronization.

Since the perturbation of parameter mismatch on chaos synchronization inevitably exists in any realistic environment, the investigation of perturbation is important to synchronization. To discuss the robustness of our TVP chaotic communication model, we add the perturbation terms $D_{\vec{\mu}_1}\boldsymbol{F}_1 \cdot \Delta\boldsymbol{\mu}_1$ and $D_{\vec{\mu}_1}\boldsymbol{F}_2 \cdot \Delta\boldsymbol{\mu}_1$ to

the right-hand sides of Eqs.(7a) and (7b) respectively. Then the perturbed error equations can be written as

$$\dot{\boldsymbol{e}}_1 = D_{\vec{x}_1}\boldsymbol{F}_1 \cdot \boldsymbol{e}_1 + D_{\vec{\mu}_1}\boldsymbol{F}_1 \cdot \Delta\boldsymbol{\mu}_1, \tag{9a}$$

$$\begin{aligned}\dot{\boldsymbol{e}}_2 =& D_{\vec{x}_1}\boldsymbol{F}_2 \cdot \boldsymbol{e}_1 + D_{\vec{x}_2}\boldsymbol{F}_2 \cdot \boldsymbol{e}_2 \\ &+ D_{\vec{\mu}_1}\boldsymbol{F}_2 \cdot \Delta\boldsymbol{\mu}_1 + D_{\vec{\mu}_2}\boldsymbol{F}_2 \cdot \Delta\boldsymbol{\mu}_2.\end{aligned} \tag{9b}$$

We can solve Eq.(9),

$$\boldsymbol{e}_1 = \mathrm{e}^{D_{\vec{x}_1}\vec{F}_1 \cdot t}\boldsymbol{e}_1(0) + \int_0^t \mathrm{e}^{D_{\vec{x}_1}\vec{F}_1(t-\tau)} D_{\vec{\mu}_1}\boldsymbol{F}_1 \cdot \Delta\boldsymbol{\mu}_1 \mathrm{d}\tau, \tag{10a}$$

$$\begin{aligned}\boldsymbol{e}_2 =& \mathrm{e}^{D_{\vec{x}_2}\vec{F}_2 \cdot t}\boldsymbol{e}_2(0) + \int_0^t \mathrm{e}^{D_{\vec{x}_2}\vec{F}_2(t-\tau)} \\ &\times (D_{\vec{\mu}_1}\boldsymbol{F}_2 \cdot \Delta\boldsymbol{\mu}_1 + D_{\vec{\mu}_2}\boldsymbol{F}_2 \cdot \Delta\boldsymbol{\mu}_2 + D_{\vec{x}_1}\boldsymbol{F}_2 \cdot \boldsymbol{e}_1)\mathrm{d}\tau.\end{aligned} \tag{10b}$$

The root mean-square values of $\boldsymbol{e}_1$ and $\boldsymbol{e}_2$ are

$$\sqrt{\langle(\boldsymbol{e}_1)^2\rangle} = \sqrt{\left\langle \left[\int_0^t \mathrm{e}^{D_{\vec{x}_1}\vec{F}_1(t-\tau)} D_{\vec{\mu}_1}\boldsymbol{F}_1 \mathrm{d}\tau\right]^2 \right\rangle} \cdot \Delta\boldsymbol{\mu}_1, \tag{11a}$$

$$\begin{aligned}&\sqrt{\langle(\boldsymbol{e}_2)^2\rangle} \\ =& \sqrt{\left\langle \left[\int_0^t \mathrm{e}^{D_{\vec{x}_2}\vec{F}_2(t-\tau)}(A \cdot \Delta\boldsymbol{\mu}_1 + D_{\vec{\mu}_2}\boldsymbol{F}_2 \cdot \Delta\boldsymbol{\mu}_2)\mathrm{d}\tau\right]^2 \right\rangle},\end{aligned} \tag{11b}$$

where

$$A = D_{\vec{\mu}_1}\boldsymbol{F}_2 + D_{\vec{x}_1}\boldsymbol{F}_2 \cdot \int_0^t \mathrm{e}^{D_{\vec{x}_1}\vec{F}_1(t-\tau)} D_{\vec{\mu}_1}\boldsymbol{F}_1 \mathrm{d}\tau,$$

$$\begin{aligned}\Delta\boldsymbol{\mu}_2 =& \mu(\tilde{\boldsymbol{x}}_1(t) \circ \delta(t-(nt_0-\tau)) \\ &- \mu(\boldsymbol{x}_1(t) \circ \delta(t-(nt_0-\tau)), \\ \Delta\boldsymbol{\mu}_2 =& \mu'\boldsymbol{e}_1(t) \circ \delta(t-(nt_0-\tau)),\end{aligned} \tag{12}$$

with μ' being the derivative of μ. Using Eq.(10a) to replace Eq.(12), we obtain

$$\begin{aligned}\Delta\boldsymbol{\mu}_2 =& \left(\mu' \int_0^t \mathrm{e}^{D_{\vec{x}_1}\vec{F}_1(t-\tau)} \right. \\ &\left. \times D_{\vec{\mu}_1}\boldsymbol{F}_1 \mathrm{d}\tau \cdot \delta(t-(nt_0-\tau)\right) \cdot \Delta\boldsymbol{\mu}_1.\end{aligned} \tag{13}$$

Setting

$$B = D_{\vec{\mu}_2}\boldsymbol{F}_2\left(\mu' \int_0^t \mathrm{e}^{D_{\vec{x}_1}\vec{F}_1(t-\tau)} D_{\vec{\mu}_1}\boldsymbol{F}_1 \mathrm{d}\tau \circ \delta(t-(nt_0-\tau))\right),$$

we can simplify Eq.(11b) as

$$\sqrt{\langle(\boldsymbol{e}_2)^2\rangle} = \sqrt{\left\langle \left[\int_0^t \mathrm{e}^{D_{\vec{x}_2}\vec{F}_2(t-\tau)}(A+B)\mathrm{d}\tau\right]^2 \right\rangle} \cdot \Delta\boldsymbol{\mu}_1. \tag{14}$$

As average values, $\sqrt{\left\langle \left[\int_0^t \mathrm{e}^{D_{\vec{x}_1}\vec{F}_1(t-\tau)} D_{\vec{\mu}_1}\boldsymbol{F}_1 \mathrm{d}\tau\right]^2 \right\rangle}$ and $\sqrt{\left\langle \left[\int_0^t \mathrm{e}^{D_{\vec{x}_2}\vec{F}_2(t-\tau)}(A+B)\mathrm{d}\tau\right]^2 \right\rangle}$ are nearly unchanged with time, thus $\sqrt{\langle(\boldsymbol{e}_1)^2\rangle}$ and $\sqrt{\langle(\boldsymbol{e}_2)^2\rangle}$ are approximately linear with $\Delta\boldsymbol{\mu}_1$. When $\Delta\boldsymbol{\mu}_1 = 0$, $\Delta\boldsymbol{\mu}_2 \neq 0$ we can still obtain: $t \to \infty$, $\boldsymbol{e}_1 \to 0$. Therefore, when $t \to \infty$, we have $\tilde{\boldsymbol{x}}_1 \to \boldsymbol{x}_1$. Because vector $\boldsymbol{\mu}_2$ is determined only by state vector $\boldsymbol{x}_1$, when $t \to \infty$, we have $\tilde{\boldsymbol{\mu}}_2 \to \boldsymbol{\mu}_2$, $\Delta\boldsymbol{\mu}_2 \to 0$. Thus the state errors caused by $\Delta\boldsymbol{\mu}_2$ can be reduced by the TVP chaotic system. This means that the TVP chaotic communication systems are not sensitive to the mismatch of $\boldsymbol{\mu}_2$.

3. Simulation experiments

We use a Lorenz system as an example to illustrate our idea. The transmitter equations are

$$\frac{\mathrm{d}x}{\mathrm{d}t} = \sigma(-x) + g(t), \quad \frac{\mathrm{d}y}{\mathrm{d}t} = \mu x - y - xz,$$

$$\frac{\mathrm{d}z}{\mathrm{d}t} = xy - bz, \quad g(t) = s(t) + \sigma y;$$

The receiver equations are

$$\frac{\mathrm{d}\tilde{x}}{\mathrm{d}t} = \sigma(-\tilde{x}) + g(t), \quad \frac{\mathrm{d}\tilde{y}}{\mathrm{d}t} = \tilde{\mu}\tilde{x} - \tilde{y} - \tilde{x}\tilde{z},$$

$$\frac{\mathrm{d}\tilde{z}}{\mathrm{d}t} = \tilde{x}\tilde{y} - b\tilde{z}, \quad \tilde{s}(t) = g(t) - \sigma\tilde{y},$$

where x, y, z represent the transmitter variables and $\tilde{x}, \tilde{y}, \tilde{z}$ are the receiver variables. The parameters are $\boldsymbol{\mu}_1 = (\sigma, b)'$, σ=10, b=2.666. In fact, μ is not sensitive to the initial value. Because the typical value of parameter μ of the Lorenz system is 28, we set the initial parameter values of our system: μ=28, $\tilde{\mu}$=28. The parameter variation function can be chosen freely. Here we choose a sine function.

For the transmitter equations, we set

$$\mu = \begin{cases} 10 \times \sin(x(t) \circ \delta(t - (0.1k - 0.05))) + 38 & t \in [0.1k, 0.1(k+1)), k = 1, 2, 3, \cdots \\ 28 & t \in [0, 0.1). \end{cases}$$

For the receiver equations we have

$$\tilde{\mu} = \begin{cases} 10 \times \sin(\tilde{x}(t) \circ \delta(t - (0.1k - 0.05))) + 38 & t \in [0.1k, 0.1(k+1)), k = 1, 2, 3, \cdots \\ 28 & t \in [0, 0.1). \end{cases}$$

It must be mentioned that the varying range of μ is [28,48]. In this range, the Lorenz system maintains chaotic.

Setting $\boldsymbol{e}_1 = \tilde{x} - x, \boldsymbol{e}_2 = \tilde{y} - y, e_3 = \tilde{z} - z$, we obtain the error equations

$$\begin{aligned} \dot{e}_x &= -10e_x, \\ \dot{e}_y &= -e_y - xe_z + \mu(x)e_x + x\Delta\mu(x), \\ \dot{e}_z &= xe_y - 2.666e_z. \end{aligned}$$

When $t \to \infty$, $e_x \to 0$ we can obtain $\mu(x)e_x \to 0$. Since μ is determined only by x, then for $e_x \to 0$, $\Delta\mu \to 0$, we have $x\Delta\mu \to 0$. The error equations can be simplified as follows:

$$\begin{aligned} \dot{e}_x &= -10e_x, \\ \dot{e}_y &= -e_y - xe_z, \\ \dot{e}_z &= xe_y - 2.666e_z. \end{aligned}$$

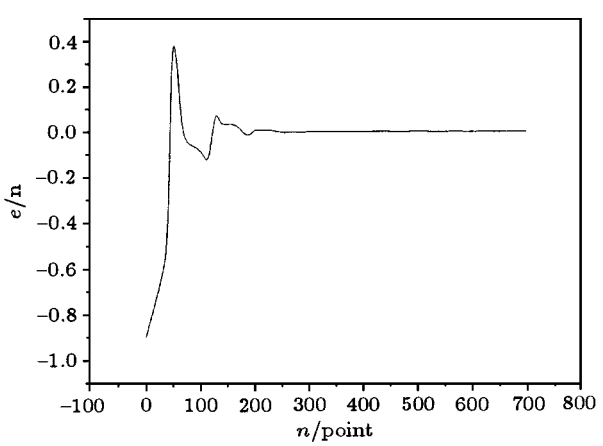


Fig.1. the state synchronization error versus time.

The Lyapunov function for the error dynamics is given by $L = e_y^2 + e_z^2$. It is easy to prove that $\dot{L} = -2(e_y^2 + 2.666e_z^2) < 0$. Once synchronization is achieved, the parameters of the receiver change identically with the transmitter. The origin (0,0,0) is a globally asymptotically stable fixed point, which simply means that the error between the transmitter and the receiver becomes to zero. Figure 1 presents the curve of the synchronization error versus time: $\tilde{e}(t) = g(t) - \sigma\tilde{y}$. Figure 2 gives the parameter synchronization curve $\Delta\mu = \tilde{\mu} - \mu$. The two curves indicate that the receiver is synchronized fast with the transmitter.

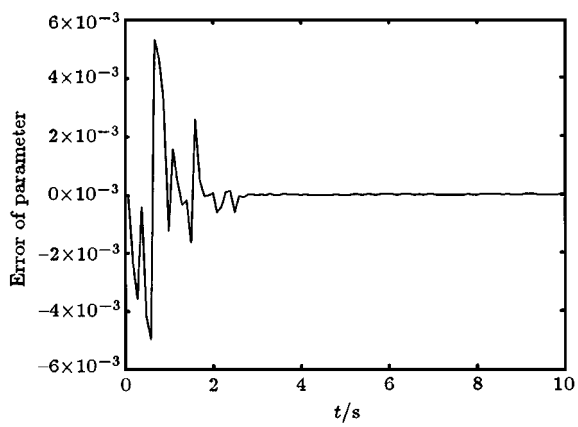


Fig.2. the synchronization error of μ.

In the following, we simulate the above system with numerical calculation. The Lorenz system is generated by the fourth-order Runge–Kutta method with a step-size of 0.01. A sine signal $[s(t) = \sin(20\pi \times t)]$ is masked by the above system [Fig.3(a)] and the driving signal $g(t)$ is transmitted through the channel [Fig.3(b)]. Finally, the hidden sine message is recovered by $\tilde{s}(t) = g(t) - \sigma\tilde{y}$ [Fig.3 (c)]. Figure 3 shows that the system can synchronize and recover the signal precisely.

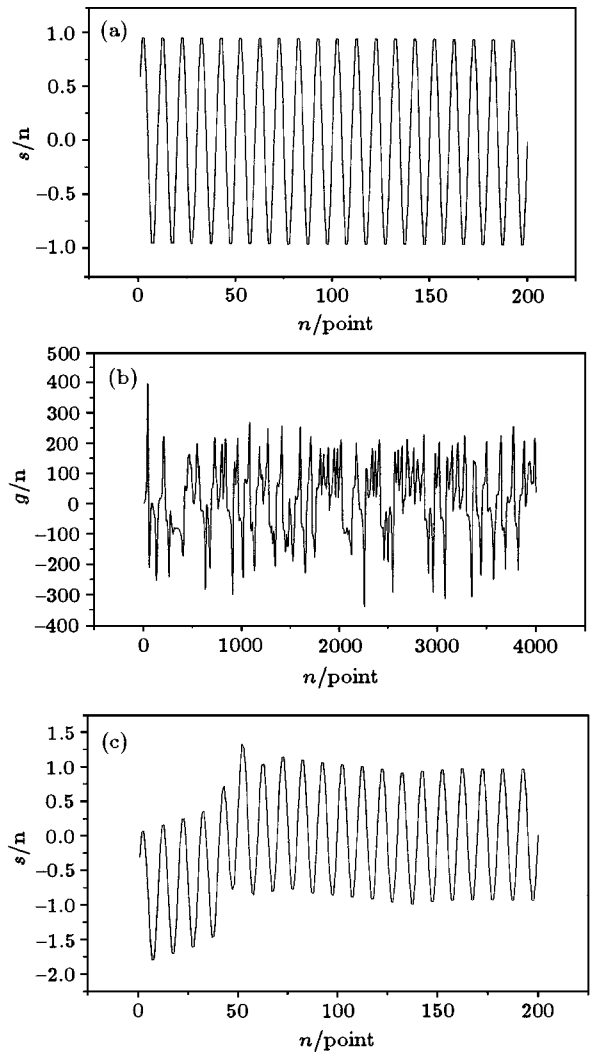


Fig.3. (a) the original sine message; (b) the transmitted signal with a sine message; (c) the resumed sine signal by the receiver.

The most important for a secure communication system is that it must pass through different attacks. Therefore, we use the NLD forecasting method to test the security of the TVP chaotic system.[20,21] For the convenience of comparison, we also illustrate the unmasking result of the Lorenz system with constant parameters.

The NLD forecasting method is based on the phase space reconstruction with time-delay method. The phase spaces of a constant-parameter chaotic system and TVP chaotic system are illustrated in Fig.4, respectively. Comparing Fig.4(a) with Fig.4(b), we can see that there is an obvious geometric structure in the phase space of the constant-parameter system. However, this structure is destroyed heavily even in any local phase space of the TVP chaotic system. These prevent the hidden message from being extracted by NLD forecasting.

Figure 5 shows the NLD forecasting results of the TVP chaotic carrier. We still use the sine wave $s(t) = \sin(20\pi \times t)$ in Fig.3(a) as the hidden message. Figure 5 (a) is the message extracted by the NLD forecasting method and Fig.5 (b) is the frequency spectrum of the waveform shown in Fig.5(a). Clearly, the sine message cannot be recognized from the message extracted by NLD forecasting, either in time-domain or in frequency-domain. This proves that the TVP chaotic system is secure enough to prevent from being broken down by the NLD forecasting method.

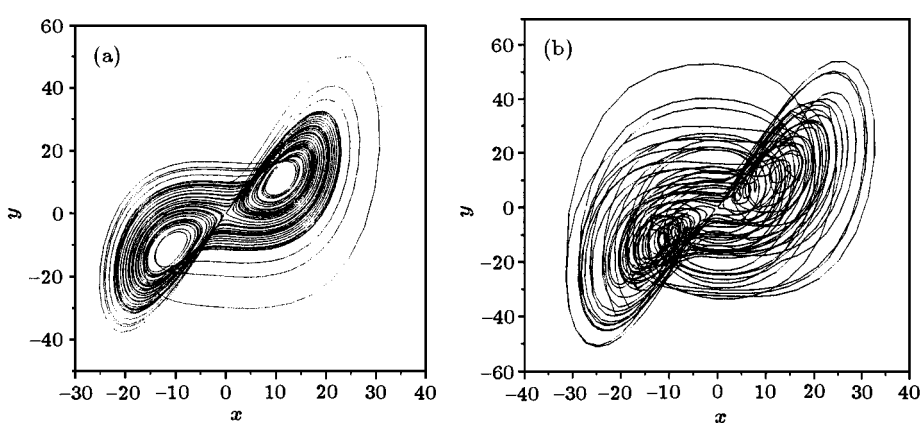


Fig.4. (a) the phase space of Lorenz system with constant parameter; (b) the phase space of Lorenz system with time-varying parameter.

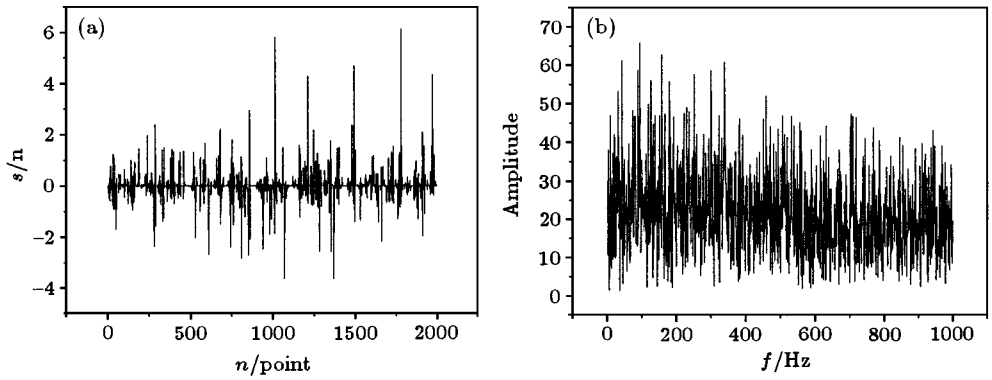


Fig.5. (a) the result using NLD prediction; (b) the frequency spectrum.

We also give the NLD forecasting results of the Lorenz system with constant parameters. Figure 6(a) shows the message extracted by NLD forecasting. Figure 6(b) is the frequency spectrum of the waveform in Fig.6(a). The sine wave is revealed in the frequency-domain obviously. The maximal amplitude in Fig.6(b) occurs at about f=10Hz, indicating the presence of the sine message.

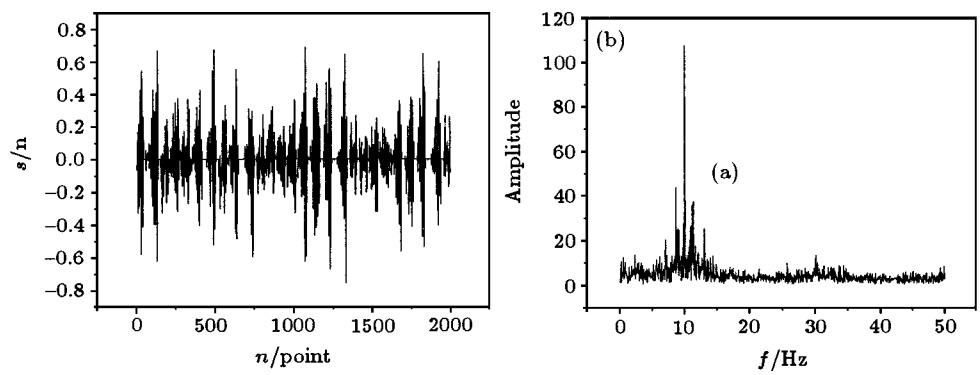


Fig.6. (a)the result of NLD forecasting; (b)the frequency spectrum.

The largest Lyapunov exponent[24,25] of this varying-parameter chaotic system and the constant-parameter Lorenz system are also computed. The largest Lyapunov exponent of our system is 2.94. And the largest Lyapunov exponent of the constant-parameter system is 1.42. This means that the chaotic system with varying parameter is much more sensitive to initial values than the one with constant parameters.

According to the above analysis, we give the numerical computation result of the TVP system. Figure 7(a) shows the relation between the slight perturbation of the parameters σ, b and $\sqrt{\langle[\tilde{x}-x]^2\rangle}$, where the lines L1 and L2 correspond to the parameter mismatches $\Delta\sigma$ and Δb respectively. We find that the sensitivity to parameter mismatch increases linearly with increasing the mismatch of parameters σ and b. And the state flow is not sensitive to parameter μ. As a whole, when the parameter mismatch is not very large ($\Delta\sigma < 1, \Delta b < 1$), the state error of TVP chaotic system will be small, so the system tends to be robust. Figures 7(b) and 7(c) show $\sqrt{\langle[\tilde{y}-y]^2\rangle}$ and $\sqrt{\langle[\tilde{z}-z]^2\rangle}$ respectively. The numerical computation confirms our analysis.

Finally, a speech communication experiment is given. The speech message ['yes'] is used in this experiment. Figure 8 (a) is the waveform of the speech message masked and transmitted by transmitter. Figure 8 (b) is the speech message resumed by the receiver. This result shows the hidden speech message can be recovered successfully.

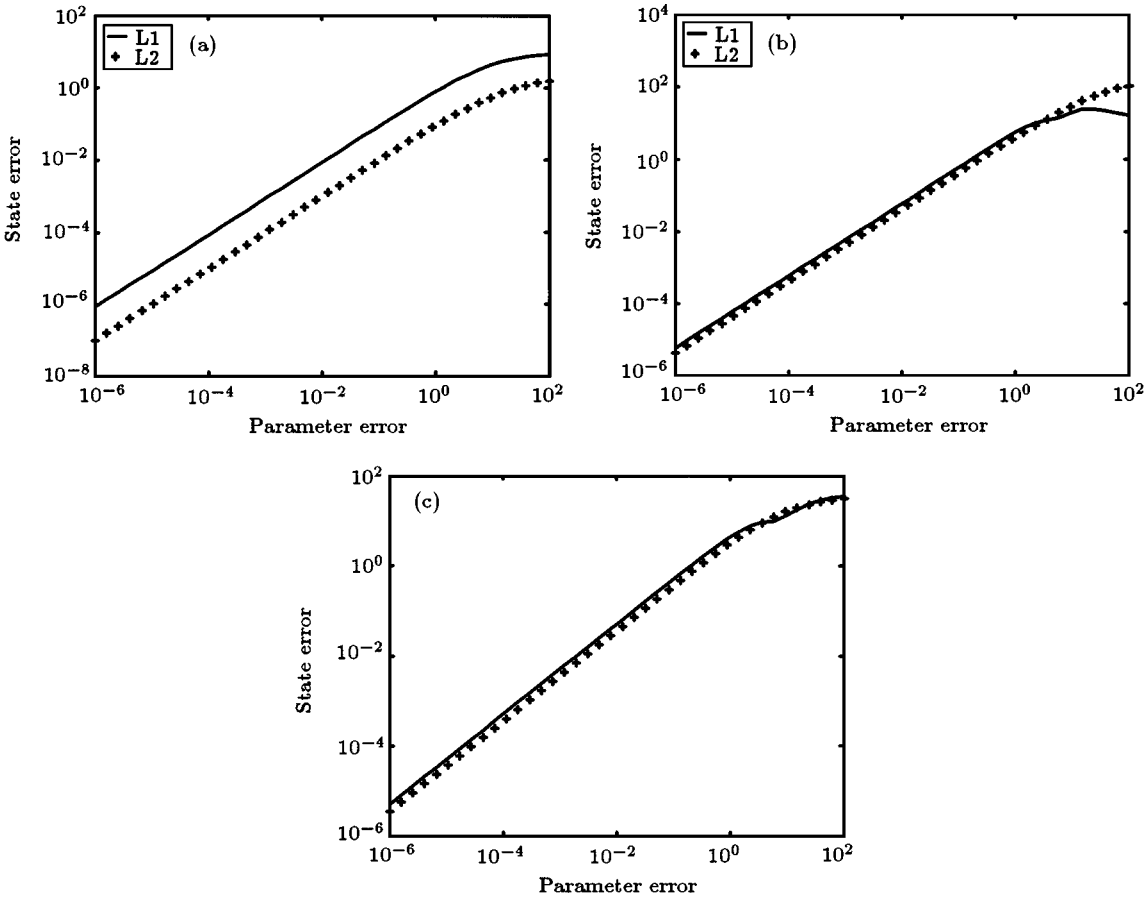


Fig.7. (a) Dependence of $\sqrt{\langle[\tilde{x}-x]^2\rangle}$ on the deviation of the parameters; (b) Dependence of $\sqrt{\langle[\tilde{y}-y]^2\rangle}$ on the deviation of the parameters; (c) Dependence of $\sqrt{\langle[\tilde{z}-z]^2\rangle}$ on the deviation of the parameters.

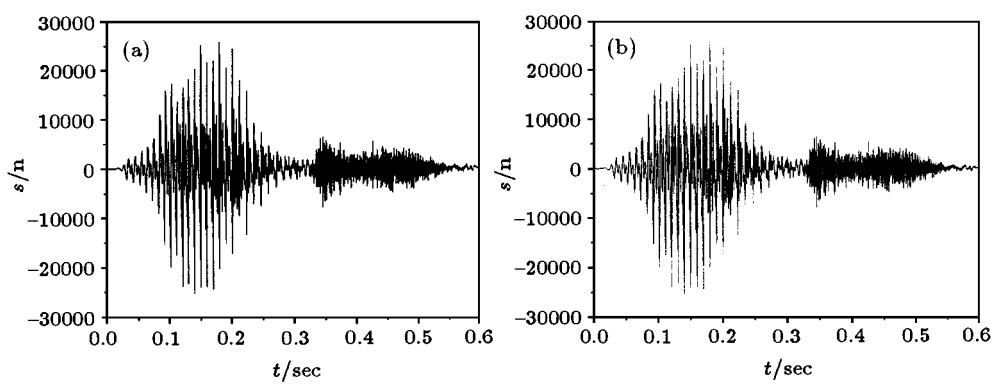


Fig.8. (a) the signal message "yes"; (b) the resumed message "yes".

4. Conclusion

We have proposed a new chaotic communication model with time-varying parameters instead of constant parameters. We present the synchronous principle and robustness of synchronization to parameter mismatch. Speech communication experiment has been carried out. Simulation results show that the phase space structure of the chaotic systems is broken successfully. This prevents the hidden message from being extracted from an intercepted carrier. Moreover, we find that the function that describes the variation of parameters needs to not be very complicated. Even a simple parameter variation function such as a sine function can prevent the communication system from being broken down by NLD forecasting. Chaotic

systems which accord to our model can realize perfect synchronization and secure communication. Our simulation experiments prove that the time-varying-parameter chaotic communications are much more secure than the chaotic communication with constant parameters.

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