Vol 12 No 4, April 2003
1009-1963/2003/12(04)/0371-06

Chinese Physics

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Quantization of the space–time with topological defect*

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(Received 27 November 2002; revised manuscript received 2 January 2003)

We present the classical solution of Lagrange equations for the black hole with a global monopole or with a cosmic string. Then we obtain the wavefunction of the space–time by solving the Wheeler–De Witt equation. De Broglie–Bohm interpretation applied to the wavefunction gives the quantum solution of the space–time. In the end, the quantum effect on Hawking radiation is studied.

Keywords: monopole black hole, cosmic string black hole, quantization
PACC: 0470, 0460

1. Introduction

As is known, phase transitions in the early Universe can give rise to topological defects of various kinds, cosmic string, global monopole, domain wall, texture and so on.[1,2] The type of defects depends on the topology of the vacuum manifold μ. Barriola and Vilenkin[3] obtained an approximate solution of Einstein equations for the space–time outside a global monopole. The metric describes a black hole that carries a global monopole charge. Such a black hole can be formed if a global monopole is swallowed by an ordinary black hole in the early Universe. Mukunda and Vilenkin[4] also obtained a vacuum solution for the space–time outside a cosmic string. The metric has a topology of a cone. Both of the two space–times have topological defects. In this paper, we give the quantization of these two kinds of space–time.

In the fields of gravity theory and cosmology, quantum gravity is one of the most attractive subjects. People mainly bring about two methods for the quantization of space–time: path integral method and canonical quantization method. Here we focus on the canonical quantization. This method is presented by Dirac[5] and by Arnowitt *et al.*[6] The dynamics of geometry in this form is given as a totally constraint system. The constraints are used as conditions for the physical state. The quantum condition gives the Wheeler–De Witt(WD) equation, which is a functional differential equation with respect to the metric on a three-dimensional manifold. It is very difficult to give an exact solution for WD equation. However, by taking account of the symmetry of space–time,WD equation may become a solvable one. This is called the mini-superspace method.

Kuchar[7] studied the geometrodynamics of spherical Schwarzschild space–time. The black hole mass can be defined as a dynamical variable instead of an integration constant. In Refs.[8]–[11] the canonical quantization was applied to the spherical geometry inside the black hole horizon. They obtained an exact solution of WD equation on the condition that it must be an eigenstate of the mass function. In order to obtain information from the wavefunction, Kenmoku *et al.*[8,9] introduced de Broglie–Bohm (dBB) interpretation.[12,13] They estimated the quantum fluctuations of the space–time quantitatively.

In this paper, we study two kinds of black holes: the global monopole black hole and the cosmic string black hole. The corresponding space–times are quantized, respectively. Using dBB interpretation on wavefunction, we obtain the quantum form of the metrics. We find that the quantum form is very different from the classical one, and the quantum effect is very remarkable in the vicinity of event horizon. The quantum effect on Hawking radiation is also studied.

*Project supported by the National Natural Science Foundation of China (Grant Nos 10273017 and 10073006), and the Foundation of Shanghai Development for Science and Technology, China(Grant No 01-JC14035).

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This paper is organized as follows. In Sec.2, we give the classical solution for a global monopole black hole or cosmic string black hole. In Sec.3, we derive the wavefunction for the space–time of the monopole black hole or cosmic string black hole. In Sec.4, using the dBB interpretation, we obtain the quantum solution for the metrics. In Sec.5, we discuss the thermal effect of the quantum black holes. Finally, a brief conclusion and discussion is given. Throughout the paper, the units $G = c = \hbar = k = 1$ are used.

2.Classical solution

The metric of a global monopole black hole can be written as

$$\begin{aligned} \mathrm{d}s^2 =& -\left(1-8\pi\eta^2-\frac{2M^{'}}{r^{'}}\right)\mathrm{d}t^{'2} \\ &+\left(1-8\pi\eta^2-\frac{2M^{'}}{r^{'}}\right)^{-1}\mathrm{d}r^{'2}+r^{'2}\mathrm{d}\Omega_2^2, \end{aligned} \tag{1}$$

where $M^{'}$ is the mass of the black hole, η is the scale of a gauge-symmetry breaking. For a typical grand unification scale $\eta \sim 10^{16}\mathrm{GeV}$, we have $8\pi\eta^2 \ll 1$. If we make the change of variables

$$\begin{aligned} t^{'} &= \left(1-8\pi\eta^2\right)^{-\frac{1}{2}} t, \\ r^{'} &= \left(1-8\pi\eta^2\right)^{\frac{1}{2}} r, \\ M^{'} &= \left(1-8\pi\eta^2\right)^{\frac{3}{2}} m, \end{aligned} \tag{2}$$

Eq.(1) becomes

$$\begin{aligned} \mathrm{d}s^2 =& -\left(1-\frac{2m}{r}\right)\mathrm{d}t^2+\left(1-\frac{2m}{r}\right)^{-1}\mathrm{d}r^2 \\ &+\left(1-8\pi\eta^2\right)r^2\mathrm{d}\Omega_2^2. \end{aligned} \tag{3}$$

It is easy to find from Eq.(3) that the space around the monopole has a deficit solid angle. Thus the space has a topological defect.

Another space–time with topological defects is described by

$$\begin{aligned} \mathrm{d}s^2 =& -\left(1-\frac{2m}{r}\right)\mathrm{d}t^2+\left(1-\frac{2m}{r}\right)^{-1}\mathrm{d}r^2 \\ &+r^2\left(\mathrm{d}\theta^2+b^2\sin^2\theta\mathrm{d}\varphi^2\right). \end{aligned} \tag{4}$$

This is a black hole of mass m with a cosmic string extending along the $\theta = 0$ and $\theta = \pi$ axes. It is also a static vacuum solution but has a topology of cone, which is different from the Schwarzschild black hole. Here $b = 1 - 4\mu$, μ being the mass per unit length of the string.

Now we rewrite Eqs.(3) and (4) in the form of

$$\begin{aligned} \mathrm{d}s^2 =& -\left(1-\frac{2m}{r}\right)\mathrm{d}t^2+\left(1-\frac{2m}{r}\right)^{-1}\mathrm{d}r^2 \\ &+a^2r^2\left(\mathrm{d}\theta^2+b^2\sin^2\theta\mathrm{d}\varphi^2\right), \end{aligned} \tag{5}$$

where $a = \sqrt{1-8\pi\eta^2}, b = 1$ and $a = 1, b = 1-4\mu$ correspond to Eqs.(3) and (4), respectively.

In order to quantize the space–time, we make transformations below

$$t \to R, r \to T. \tag{6}$$

Then Eq.(5) can be written as

$$\begin{aligned} \mathrm{d}s^2 =& -\left(1-\frac{2m}{T}\right)\mathrm{d}R^2+\left(1-\frac{2m}{T}\right)^{-1}\mathrm{d}T^2 \\ &+a^2T^2\left(\mathrm{d}\theta^2+b^2\sin^2\theta\mathrm{d}\varphi^2\right). \end{aligned} \tag{7}$$

Assume the quantized metric has the form

$$\begin{aligned} \mathrm{d}s^2 =& -\frac{\alpha^2(T)}{U(T)}\mathrm{d}T^2+U(T)\,\mathrm{d}R^2 \\ &+V(T)\left(\mathrm{d}\theta^2+b^2\sin^2\theta\mathrm{d}\varphi^2\right), \end{aligned} \tag{8}$$

then the corresponding action for the gravitational field is obtained

$$\begin{aligned} S_g =& \frac{1}{16\pi}\int \mathrm{d}^4x\sqrt{-g}^{(4)}R \\ =& \frac{1}{16\pi}\int \mathrm{d}T\,\mathrm{d}R\mathrm{d}\theta\mathrm{d}\varphi\sqrt{-g}\frac{1}{2\alpha^3V^2} \\ &\times\left(2V^2\ddot{U}\alpha-2V^2\dot{U}\dot{\alpha}+4\ddot{V}U\alpha V-4\dot{V}\dot{\alpha}UV\right. \\ &\left.+4\dot{V}\dot{U}V\alpha-\dot{V}^2U\alpha+4\alpha^3V\right) \\ =& \frac{v_0}{4}\int \mathrm{d}T\left[-\frac{1}{\alpha}\left(\dot{U}\dot{V}+\frac{U\dot{V}^2}{2V}\right)+2\alpha\right], \end{aligned} \tag{9}$$

where the dot denotes the derivative with respect to T. The volume of the system $v_0 \equiv \int_0^\infty \mathrm{d}R$ is treated to be finite.

The simplest model that gives rise to global monopoles is described by the action

$$\begin{aligned} S_m =& \frac{1}{16\pi}\int \mathrm{d}^4x\sqrt{-g}\left[\frac{1}{2}\partial_\mu\phi^a\partial^\mu\phi^a\right. \\ &\left.-\frac{1}{4}\lambda(\phi^a\phi^a-\eta^2)^2\right], \end{aligned} \tag{10}$$

where ϕ^a is a triplet of scalar fields, a=1, 2, 3. This model has a global $O(3)$ symmetry, which is spontaneously broken to $U(1)$. The field configuration describing a monopole is $\phi^a = \eta f(r^{'})x^a/r^{'}$, where

$x^a x^a = r'^2$. x^a is the 'Cartesian' coordinates. Similarly, by making variable changes $r' \leftarrow r \rightarrow T, t \leftarrow R$, Eq.(10) becomes

$$S_m = \frac{v_0}{4}\int \mathrm{d}T\alpha V\left[\frac{1}{2U}\dot{f}^2 + \frac{4f^2}{V} - \frac{1}{4}\lambda(f^2-1)^2\eta^2\right]\eta^2. \quad (11)$$

Eqs. (9) and (11) give the Lagrangian for the space–time

$$L = \frac{v_0}{4}\left[-\frac{1}{\alpha}\left(\dot{U}\dot{V} + \frac{U\dot{V}^2}{2V}\right) + 2\alpha + 4\alpha f^2\eta^2 + \frac{\alpha V}{2U}\dot{f}^2\eta^2 - \frac{\alpha V}{4}\lambda(f^2-1)^2\eta^4\right]. \quad (12)$$

After solving the Lagrange equations

$$\frac{\mathrm{d}}{\mathrm{d}T}\left(\frac{\partial L}{\partial \dot{U}}\right) - \frac{\partial L}{\partial U} = 0,$$
$$\frac{\mathrm{d}}{\mathrm{d}T}\left(\frac{\partial L}{\partial \dot{V}}\right) - \frac{\partial L}{\partial V} = 0,$$
$$\frac{\mathrm{d}}{\mathrm{d}T}\left(\frac{\partial L}{\partial \dot{f}}\right) - \frac{\partial L}{\partial f} = 0,$$
$$\frac{\mathrm{d}}{\mathrm{d}T}\left(\frac{\partial L}{\partial \dot{\alpha}}\right) - \frac{\partial L}{\partial \alpha} = 0, \quad (13)$$

we obtain the classical solution for the metric

$$\alpha = 1, U = -\left(1 - \frac{2m}{T}\right), \quad V = aT^2, \quad f \approx 1. \quad (14)$$

It should be noted that the relation $8\pi\eta^2 \ll 1$ has been considered in Eq.(13).

3. Wavefunction

Equation (14) shows that the field configuration for a monopole is $\phi^a = \eta x^a/r$. Thus the Lagrangian becomes the simple form

$$L = \frac{v_0}{4}\left[-\frac{1}{\alpha}\left(\dot{U}\dot{V} + \frac{U\dot{V}^2}{2V}\right) + 2\alpha\right]. \quad (15)$$

The relation $4\eta^2 \ll 2$ is considered in Eq.(15). Change the variables from U and V to z_+ and z_- as follows[8]

$$z_+ = U\sqrt{V}, z_- = \sqrt{V}, \quad (16)$$

then Lagrangian becomes the symmetric form

$$L = \frac{v_0}{4}\left[-\frac{2}{\alpha}\dot{z}_+\dot{z}_- + 2\alpha\right], \quad (17)$$

where z_+ and z_- appear as cyclic coordinates. The canonical momenta conjugate to z_+, z_- and α are derived from this Lagrangian

$$\pi_+ \equiv \frac{\partial L}{\partial \dot{z}_+} = -\frac{v_0}{2\alpha}\dot{z}_-,$$
$$\pi_- \equiv \frac{\partial L}{\partial \dot{z}_-} = -\frac{v_0}{2\alpha}\dot{z}_+,$$
$$\pi_\alpha \equiv \frac{\partial L}{\partial \dot{\alpha}} = 0. \quad (18)$$

The Hamiltonian for this system can be written as

$$H = -\alpha\left(\frac{2}{v_0}\pi_+\pi_- + \frac{v_0}{2}\right). \quad (19)$$

From the primary constraint ($\pi_\alpha = 0$) we have the Hamiltonian constraint $H = 0$. Now we introduce the mass function as follows

$$M = \frac{2}{v_0^2}\pi_+ z_+ \pi_+ + \frac{z_-}{2}. \quad (20)$$

The Poisson bracket of this mass function with H vanishes

$$[H, M] = -i\frac{2}{v_0^2}\pi_+ H = 0. \quad (21)$$

Therefore, H and M have the same eigenstate.

In the Schrödinger representation, the canonical momenta are quantized as

$$\hat{\pi}_+ = -i\frac{\partial}{\partial z_+}, \hat{\pi}_- = -i\frac{\partial}{\partial z_-}. \quad (22)$$

Thus the Hamiltonian constraint gives the WD equation, and the mass function gives another one.

$$\hat{H}\psi\left(z_+, z_-\right) = \left(-\frac{2}{v_0}\hat{\pi}_+\hat{\pi}_- - \frac{v_0}{2}\right)\psi = 0, \quad (23)$$

$$\hat{M}\psi\left(z_+, z_-\right) = \left(\frac{2}{v_0^2}\hat{\pi}_+ z_+ \hat{\pi}_+ + \frac{z_-}{2}\right)\psi = am\psi. \quad (24)$$

It should be noted that the mass eigenvalue is not m but am. This is different from Ref.(8).

Instead of solving WD equation and the mass eigenvalue equation, we consider firstly the linear combination of the Hamiltonian and the mass operator

$$\hat{L} = -2i\left[\frac{1}{v_0}\hat{\pi}_+ z_+ \hat{H} + \hat{\pi}_-(\hat{M} - am)\right] = i[z_+\hat{\pi}_+ - (z_- - 2am)\hat{\pi}_-]. \quad (25)$$

It is obviously $\hat{H}, \hat{M}-am$ and $\hat{L}$ have the same eigenstate. Make variables transformation as follows

$$\begin{aligned} x =& v_0\sqrt{-z_+/(z_- - 2am)}, \\ y =& v_0\sqrt{-z_+(z_- - 2am)}, \end{aligned} \tag{26}$$

we obtain the eigenstate of operator $\hat{L}$ from its eigenequation $\hat{L}\psi(x,y)=0$

$$\psi(x,y)=\psi(y). \tag{27}$$

Then both Eqs.(21) and (22) are reduced to

$$\frac{\mathrm{d}^2\psi(y)}{\mathrm{d}y^2}+\frac{1}{y}\frac{\mathrm{d}\psi}{\mathrm{d}y}+\psi=0. \tag{28}$$

This is the Bessel differential equation of zeroth order. Thus we obtain the eigenfunction of Hamiltonian and mass operator

$$\psi(y)=cH_0^{(2)}(y), \tag{29}$$

where c is an integration constant and $H_0^{(2)}$ is the second kind Hankel function of zeroth order. The combination of $H_0^{(1)}$ and $H_0^{(2)}$ is shown not to approach the classical solution. Therefore, we only choose $H_0^{(2)}$.

4. Quantum solution

According to de Broglie–Bohm interpretation,we decompose the wavefunction as follows:

$$\psi(z_+,z_-)=R(z_+,z_-)\mathrm{e}^{iS(z_+,z_-,)}. \tag{30}$$

After substituting Eq.(30) into Eqs.(23) and (24), we obtain

$$\begin{aligned} &\frac{2}{v_0}\frac{\partial S}{\partial z_+}\cdot\frac{\partial S}{\partial z_-}+\frac{v_0}{2}+V_q=0, \\ &\frac{\partial}{\partial z_-}\left(R^2\frac{\partial S}{\partial z_+}\right)+\frac{\partial}{\partial z_+}\left(R^2\frac{\partial S}{\partial z_-}\right)=0, \\ &\frac{2}{v_0^2}z_+\left(\frac{\partial S}{\partial z_+}\right)^2+\frac{1}{2}(z_- - 2am)+M_q=0, \\ &\frac{\partial}{\partial z_+}\left(R^2 z_+\frac{\partial S}{\partial z_+}\right)=0, \end{aligned} \tag{31}$$

where V_q denotes the quantum potential and M_q denotes the quantum effect to the mass function. They are defined by

$$\begin{aligned} V_q =& -\frac{2}{v_0 R}\frac{\partial^2 R}{\partial z_+\partial z_-}, \\ M_q =& -\frac{2}{v_0^2 R}\frac{\partial}{\partial z_+}\left(z_+\frac{\partial R}{\partial z_+}\right). \end{aligned} \tag{32}$$

Similarly, the eigenvalue equation of operator $\hat{L}$ can be written as

$$\begin{aligned} &z_+\frac{\partial R}{\partial z_+}-(z_- - 2am)\frac{\partial R}{\partial z_-}=0, \\ &z_+\frac{\partial S}{\partial z_+}-(z_- - 2am)\frac{\partial S}{\partial z_-}=0. \end{aligned} \tag{33}$$

The dBB interpretation assumes that the quantum trajectory is described by $Z_+(T)=z_+(T)$ and $Z_-(T)=z_-(T)$. The momenta of the quantum geometry of the black hole are assumed to be given by

$$\begin{aligned} \pi_+ =& -\frac{v_0}{2}\dot{Z}_- = \left.\frac{\partial S}{\partial z_+}\right|_{\dot{z}_+=Z_+,z_-=Z_-}, \\ \pi_- =& -\frac{v_0}{2}\dot{Z}_+ = \left.\frac{\partial S}{\partial z_-}\right|_{\dot{z}_+=Z_+,z_-=Z_-}, \end{aligned} \tag{34}$$

where the velocities are determined by the classical relations, i.e. Eqs.(18) with $\alpha=1$. Substitute Eq.(29) into Eqs.(32) and (34), we obtain

$$\begin{aligned} \dot{Z}_- =& \frac{2}{\pi v_0}\cdot\frac{a}{Z_+|H_0^{(2)}(y)|^2}, \\ \dot{Z}_+ =& \frac{2}{\pi v_0}\cdot\frac{a}{(Z_- - 2am)Z_+|H_0^{(2)}(y)|^2}, \\ V_q =& -\frac{v_0}{2}\left[1-\frac{4}{\pi^2}\cdot\frac{a^2}{y^2|H_0^{(2)}(y)|^4}\right], \\ M_q =& \frac{1}{v_0}(Z_- - 2am)V_q. \end{aligned} \tag{35}$$

From the first two equations in Eqs.(35), we have

$$U=-\left(1-\frac{2am}{\sqrt{V}}\right). \tag{36}$$

So we obtain the quantum solution for the space–time

$$\begin{aligned} \mathrm{d}s^2 =& -\left(1-\frac{2am}{\sqrt{V(T)}}\right)^{-1}\mathrm{d}T^2 \\ &+\left(1-\frac{2am}{\sqrt{V(T)}}\right)\mathrm{d}R^2 \\ &+V(T)(\mathrm{d}\theta^2+b^2\sin^2\theta\mathrm{d}\varphi^2). \end{aligned} \tag{37}$$

Going back to the former variables, i.e. $T\to r, R\to t$, we have

$$\begin{aligned} \mathrm{d}s^2 =& -\left(1-\frac{2am}{\sqrt{V(r)}}\right)\mathrm{d}t^2 \\ &+\left(1-\frac{2am}{\sqrt{V(r)}}\right)^{-1}\mathrm{d}r^2 \\ &+V(r)(\mathrm{d}\theta^2+b^2\sin^2\theta\mathrm{d}\varphi^2), \end{aligned} \tag{38}$$

where $V(r)$ is related to r as follows

$$r=\int\frac{\pi}{2a}Z|H_0^{(2)}(Z)|^2\mathrm{d}\sqrt{V}, \tag{39}$$

Z being defined by $Z = v_0(\sqrt{V} - 2am)$.

When Z approaches 0, i.e. the vicinity of event horizon, we have $V_q \to \infty$ and $\frac{\mathrm{d}\sqrt{V}}{\mathrm{d}r} \to \infty$. It is obvious that the quantum effect in the vicinity of event horizon is very remarkable. However, when Z approaches infinity, i.e. the flat region, we have $V_q = -\frac{v_0}{2}(1-a^2) \neq 0$ and $\frac{\mathrm{d}\sqrt{V}}{\mathrm{d}r} = a \neq 1$ for monopole black hole.This is different from the Schwarzschild black hole. In fact, Eqs.(35) tell us that the quantum potential of the monopole black hole is smaller than that of the Schwarzschild or cosmic string black hole.

5. Thermal effect

The most general static and spherical space–time is given by

$$\mathrm{d}s^2 = -u(r)\mathrm{d}t^2 + u(r)^{-1}\mathrm{d}r^2 + v(r)\mathrm{d}\Omega_2^2. \tag{40}$$

Klein–Gordon equation in such a space–time can be written as

$$\begin{aligned} &-\frac{1}{u}\frac{\partial^2\psi}{\partial t^2} + \frac{1}{v}\frac{\partial}{\partial r}\left(vu\frac{\partial\psi}{\partial r}\right) \\ &+ \frac{1}{v\sin\theta}\frac{\partial}{\partial\theta}\left(\sin\theta\frac{\partial\psi}{\partial\theta}\right) \\ &+ \frac{1}{v\sin^2\theta}\frac{\partial^2\psi}{\partial\varphi^2} = 0. \end{aligned} \tag{41}$$

We introduce the tortoise coordinate, i.e.

$$r_* = r + \frac{1}{2\kappa}\ln(r - r_H), \tag{42}$$

where κ and r_H is the surface gravity and the radial of the black hole. r_H is defined by the event horizon equation $u(r) = 0$. In the vicinity of event horizon, Eq.(41) should be reduced to the standard wavefunction[14]

$$\frac{\partial^2 R}{\partial r_*^2} - \frac{\partial^2 R}{\partial t^2} = 0, \tag{43}$$

therefore we have

$$\lim_{r\to r_H} u^2\left[1 + \frac{1}{2\kappa(r - r_H)}\right] = 1. \tag{44}$$

Thus from Eq.(44) we obtain the surface gravity

$$\kappa = \frac{1}{2}\cdot\frac{\mathrm{d}u}{\mathrm{d}r}|_{r\to r_H} \tag{45}$$

and the Hawking temperature

$$T = \frac{1}{4\pi}\cdot\frac{\mathrm{d}u}{\mathrm{d}r}|_{r\to r_H}. \tag{46}$$

For the classical solution Eq.(5), we have

$$T_c = \frac{1}{8\pi m}. \tag{47}$$

For the quantum solution, Eq.(38), we have

$$T_q = \frac{1}{8\pi m}\cdot\frac{2}{\pi}\cdot\frac{1}{Z|H_0^{(2)}(Z)|^2}|_{Z\to\pi/4} = \frac{1}{8\pi m}. \tag{48}$$

It should be noted that we propose to make a cutoff $\varepsilon = \frac{\pi}{4v_0}$ in the vicinity of event horizon $Z = 0$ for the reason that Hawking temperature, Eq.(48), will be divergent if we set $Z = 0$. Here $v_0 \equiv \int_0^\infty \mathrm{d}R$ is the volume of the system and is treated to be finite. The cutoff ε is of the order the Planck length if the volume v_0 is large enough. In particular, ε is only one Planck length when $v_0 = \pi/4$. Furthermore, we note that the Hawking temperature of the cosmic black hole is same as that of the Schwarzschild case. While the temperature of the monopole black hole is lower than that of the Schwarzschild.

6. Conclusion and discussion

In conclusion, we have obtained the classical form for the metric by solving Lagrange equations. By using the dBB interpretation on wavefunction, we obtain the quantum form.The classical one is very different from that of the quantum. The quantum potential and Hawking temperature of the monopole black hole are smaller than that of the Schwarzschild one. The quantum potential and Hawking temperature of the cosmic string are same as those of the Schwarzschild. Through the analysis of quantum potential, we find that the quantum effect is very remarkable in the vicinity of event horizon. In order to avoid the divergence of Hawking temperature, we propose to introduce a cutoff in the vicinity of event horizon. We think that the event horizon is steadily located at $Z = 0$ in the classical case. For a quantum black hole, uncertainty principle tells us we cannot determine the position of event horizon. Because of the quantum fluctuation in the vicinity of event horizon, the position of event horizon becomes fluctuated and we cannot say where it is. Therefore, we should introduce a cut-off in order to avoid the divergence of Hawking radiation.

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