Vol 12 No 3, March 2003
1009-1963/2003/12(03)/0294-06

Chinese Physics

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Entropy squeezing for a two-level atom in the Jaynes–Cummings model with an intensity-depend coupling*

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(Received 26 August 2002; revised manuscript received 2 December 2002)

We study the squeezing for a two-level atom in the Jaynes–Cummings model with intensity-dependent coupling using quantum information entropy, and examine the influences of the initial state of the system on the squeezed component number and direction of the information entropy squeezing. Our results show that, the squeezed component number depends on the atomic initial distribution angle, while the squeezed direction is determined by both the phases of the atom and the field for the information entropy squeezing. Quantum information entropy is shown to be a remarkable precision measure for atomic squeezing.

Keywords: information entropy squeezing, variance squeezing, information entropy uncertainty relation

PACC: 4250

1. Introduction

Over the previous few years, atomic squeezing has attracted considerable attention owning to its potential application in high-resolution spectroscopy,[1] the high-precision atomic fountain clock,[2] high-precision spin polarization measurements,[3] generation of squeezed light,[4] etc. In Ref.[1–5], Wodkiewcz and other authors established the conception of squeezed states of an atom,[1−5] Walls and Zoller[6] showed that a squeezed atom emits squeezed fluorescence in a coherently driven two-level system. Much work on atomic squeezing in the Jaynes–Cummings model (JCM) has been reported.[7−11]

On the other hand, considerable attention has been paid to the study of JCM with an intensity-dependent coupling due to the periodical and symmetrical dynamics properties in this model. Many authors have studied the squeezing of the field,[12] the phase,[13,14] the entropy of the field[15] and the dynamics properties[16] in this model. However, less attention has been paid to the study of the atomic squeezing by using the quantum information entropy theory for the considered model, though the entropy of the field in other similar atom-field system has been studied.[17] In this paper, we study the atomic squeezing in terms of quantum information entropy theory, and compare its results with the conclusion of atomic squeezing based on the Heisenberg uncertainty relation (HUR) for the system considered. We find that information entropy is a sensitive measure for the atomic squeezing. This paper is organized as follows. In section 2 we introduce our Hamiltonian model and give an exact expression for the reduced density matrix $\rho_a(t)$ of the atom and employ the reduced density matrix $\rho_a(t)$ to investigate the squeezing properties of the atom, based on the entropy uncertainty relation (EUR) and HUR, respectively. Finally, we give our numerical result and theoretical analysis in section 3.

2. The model and calculation formula of the entropy squeezing of an atom

We consider a single-mode field interacting with an effective two-level atom depending on intensity-coupling. The Hamiltonian of the system under the

*Project supported by the National Natural Science Foundation of China (Grant No 19874020), by the Natural Science Foundation of Hunan Province, China (Grant No 01JJY3030) and by the Ministry of Education, China.

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rotating wave approximation is

$$H = \omega_0 S_z + \omega a^+ a + \lambda[S_+ R + S_- R^+] \quad (\hbar = 1), \quad (1)$$

where a^+ and a are the creation and annihilation operators of the radiation field, S_+ and S_- are pseudospin operators of the atom, λ is the coupling constant, ω and ω_0 are the frequencies of the field and the atomic transition, respectively. The operators R^+ and R are defined in the following way[18]

$$R = a\sqrt{N}, \qquad R^+ = \sqrt{N}a^+, \quad (2)$$

where $N = a^+ a$. We can say that the Hamiltonian (1) describes effectively the intensity-dependent coupling between the atom and the radiation field. Owing to the fact that excitation number operator $C = N + S_+ S_-$ is a constant of motion, the time-dependent Schrödinger equation for the state vector $|\psi(t)\rangle$ can be solved exactly. Here, we assume that at the initial moment the radiation field is in the coherent state

$$|\alpha\rangle = \sum_{n=0}^{\infty} f_n |n\rangle, \qquad f_n = \exp\left(-\frac{\overline{n}}{2}\right)\frac{\alpha^n}{\sqrt{n!}}, \quad (3)$$

and the atom is in the coherent superposition state $|\xi\rangle$

$$|\xi\rangle = \cos\left(\frac{\theta}{2}\right)|+\rangle + \exp(-\mathrm{i}\varphi)\sin\left(\frac{\theta}{2}\right)|-\rangle, \quad (4)$$

where $\alpha = \sqrt{\overline{n}}\exp(\mathrm{i}\beta)$; $|+\rangle$ (or $|-\rangle$) denotes the excited (ground) state of the atom; $\overline{n}$ is the initial average photon number, and β is the phase of the coherent field; $0 \le \theta \le \pi$ denotes the atomic distribution; and $0 \le \varphi \le 2\pi$ is the phase of the atomic dipole. In the case of resonance ($\omega = \omega_0$), by solving Schrödinger equation of the system in the interaction picture, we can obtain the system vector at time t

$$|\psi(t)\rangle = \sum_n C_n(t)|+, n\rangle + \sum_n D_n(t)|-, n\rangle, \quad (5)$$

where

$$\begin{aligned} C_n(t) =& f_n \cos\left(\frac{\theta}{2}\right)\cos((n+1)\lambda t)) \\ & - \mathrm{i} f_{n+1}\exp(-\mathrm{i}\varphi)\sin\left(\frac{\theta}{2}\right)\sin((n+1)\lambda t), \\ D_n(t) =& f_n \exp(-\mathrm{i}\varphi)\sin\left(\frac{\theta}{2}\right)\cos(n\lambda t) \\ & - \mathrm{i} f_{n-1}\cos\left(\frac{\theta}{2}\right)\sin(n\lambda t), \end{aligned} \quad (6)$$

and $|+, n\rangle = |+\rangle \otimes |n\rangle$, $\quad |-, n\rangle = |-\rangle \otimes |n\rangle$.

By using Eq.(5), the reduced density matrix of the atom may be obtained

$$\rho_a(t) = \mathrm{Tr}_f |\psi(t)\rangle\langle\psi(t)| = \begin{bmatrix} \rho_{22} & \rho_{21} \\ \rho_{12} & \rho_{11} \end{bmatrix}, \quad (7)$$

where

$$\begin{aligned} \rho_{22} =& \sum_n |f_n|^2 \cos^2(\theta/2)\cos^2((n+1)\lambda t) \\ & + \sum_n |f_{n+1}|^2 \sin^2(\theta/2)\sin^2((n+1)\lambda t) \\ & + \mathrm{i}/4\left[\sum_n f_n f_{n+1}^* \sin(2(n+1)\lambda t)\exp(\mathrm{i}\varphi) - \mathrm{c.c.}\right], \\ \rho_{21} =& 1/2\left[\sum_n |f_n|^2 \exp(\mathrm{i}\varphi)\sin\theta\cos(n\lambda t)\cos((n+1)\lambda t)\right. \\ & \left. + \sum_n f_{n+1} f_{n-1}^* \exp(\mathrm{i}\varphi)\sin\theta\sin((n+1)\lambda t)\right] \\ & + \mathrm{i}\left[\sum_n f_n f_{n-1}^* \sin^2(\theta/2)\sin(n\lambda t)\cos((n+1)\lambda t)\right. \\ & \left. - \sum_n f_n^* f_{n+1}\sin^2(\theta/2)\cos(n\lambda t)\sin((n+1)\lambda t)\right], \\ \rho_{11} =& 1 - \rho_{22}, \qquad \rho_{12} = \rho_{21}^*. \end{aligned} \quad (8)$$

Next, we investigate the information entropy squeezing of the atom for the system considered by using Eq.(7). Several authors[19,20] have studied quantum uncertainty by using the quantum information entropy and obtained EUR for position and moment. One of the authors of this paper has obtained EUR of the two-level atom and defined the entropy squeezing for the two-level atom using the quantum information entropy theory.[21] The information entropy of the Pauli operators $S_\alpha (\alpha = x, y, z)$ for a two-level atom system is

$$H(S_\alpha) = -\sum_{i=1}^{2} P_i(S_\alpha)\ln P_i(S_\alpha), \qquad \alpha = x, y, z \quad (9)$$

where $P_i(S_\alpha) = \langle\psi_{\alpha i}|\rho|\psi_{\alpha i}\rangle, (i = 1, 2)$ is the probability distribution for two possible outcomes of measurements of an operator S_α. $H(S_x), H(S_y)$ and $H(S_z)$ satisfy

$$H(S_x) + H(S_y) \ge 2\ln 2 - H(S_z). \quad (10)$$

The inequality (10) may also be rewritten as:

$$\delta H(S_x)\delta H(S_y) \ge \frac{4}{\delta H(S_z)}, \quad (11)$$

where $\delta H(S_\alpha) = \exp[H(S_\alpha)]$. We define the squeezing of the atom by EUR (11), named as entropy squeezing,

which has received little attention in previous discussions. The fluctuation in the component $S(\alpha)(\alpha \equiv x$ or $y)$ of the atomic dipole is said to be squeezed in entropy if the information entropy $H(S_\alpha)$ satisfies the condition:

$$E(S_\alpha) = \delta H(S_\alpha) - \frac{2}{\sqrt{|\delta H(S_z)|}} < 0, \quad (\alpha \equiv x \ \ or \ \ y). \tag{12}$$

While the atomic variance squeezing based on HUR may be defined as the fluctuation in the component S_α of the atomic dipole and is said to be squeezed if $S(\alpha)$ satisfies the condition

$$V(S_\alpha) = \Delta S(\alpha) - |\langle S_Z \rangle|/2 < 0, \quad (\alpha \equiv x \ \text{or} \ y), \tag{13}$$

where $\Delta S_\alpha = [\langle S_\alpha^2 \rangle - \langle S_\alpha \rangle^2]^{\frac{1}{2}}$.

It is obvious that Eq.(13) is trivial when $\langle S_z \rangle = 0$ (as $\Delta S_\alpha \geq 0$) and fails to provide any useful information. For example, when the atom is in the state $|\psi_{y+}\rangle = (|+\rangle + \mathrm{i}|-\rangle)/\sqrt{2}$, which is the eigenstate of the Pauli operator S_y, $\langle S_z \rangle$ is always zero. We cannot obtain any squeezing information from Eq.(13). However, from Eq.(12), the state $|\psi_{y+}\rangle$ may be considered as the maximum squeezed state. This is due to the fact that, when $\langle S_z \rangle = 0$, Eq.(12) based on the EUR is not trivial and can provide full information on the atomic squeezing. Next, we calculate the atomic entropy squeezing in the system considered. By using the atomic reduced density operator $\rho_a(t)$, given by Eqs.(7) and (9), we obtain the information entropies of the atomic operators S_x, S_y and S_z:

$$\begin{aligned} H(S_x) = & -\left[\frac{1}{2} + \mathrm{Re}(\rho_{21}(t))\right] \ln\left[\frac{1}{2} + \mathrm{Re}(\rho_{21}(t))\right] \\ & -\left[\frac{1}{2} - \mathrm{Re}(\rho_{21}(t))\right] \ln\left[\frac{1}{2} - \mathrm{Re}(\rho_{21}(t))\right], \\ H(S_y) = & -\left[\frac{1}{2} + \mathrm{Im}(\rho_{21}(t))\right] \ln\left[\frac{1}{2} + \mathrm{Im}(\rho_{21}(t))\right] \\ & -\left[\frac{1}{2} - \mathrm{Im}(\rho_{21}(t))\right] \ln\left[\frac{1}{2} - \mathrm{Im}(\rho_{21}(t))\right], \\ H(S_z) = & -\rho_{22}(t)\ln\rho_{22}(t) - \rho_{11}\ln\rho_{11}(t), \end{aligned} \tag{14}$$

where $\rho_{ij}(t)$ is given by the Eq.(8).

3. Numerical results and discussion

In this section we will numerically discuss the properties of entropy squeezing for the atom in the considered system in the different conditions of the atomic and field initial states.

3.1. The influence of the atomic distribution angle

The atomic distribution angle θ reflects the distribution of the atom in the excited state and in the ground state. When $\theta = 0$ or π (excited or ground state), the atom is in an incoherent state; and $\theta \neq 0$ or π, the atom is in the coherent superposition state. Here we investigate the influence of the atomic distribution angle on the information entropy squeezing.

3.1.1. The atom initially in an incoherent state

The time evolutions of squeezing factors $E(S_x)$, $E(S_y)$, $V(S_x)$ and $V(S_y)$ are shown in Fig.1, while the atomic inversion is in Fig.3(a), for the atom initially in an excited state ($\theta = 0$) and the field in the coherent state, with the average photon number $\overline{n} = 20$ and the phase $\beta = \pi/2$. When $t \in ((2k+1)\pi/4\lambda, (2k+3)\pi/4\lambda, \ k = 0, 1, 2, \ldots)$, we can see from Figs.1(a) and 1(c) that there are great differences between $E(S_x)$ and $V(S_x)$, where the former shows the atomic squeezing while the latter predicts no squeezing. This stems from the fact that atomic inversion $W(t) = \frac{1}{2}\langle S_z \rangle = 0$ (plotted in Fig.3(a)) at these time stages; the definition of variance squeezing is trivial and we cannot obtain any information on atomic squeezing from Eq.(13), while the definition of the information entropy squeezing (Eq.(12)) is nontrivial and can provide sufficient information. We finds that when the atom is initially in the ground state, the evolutions of these squeezing factors are exactly the same as those when the atom is initially in the excited state with the same β, φ, $\overline{n}$.

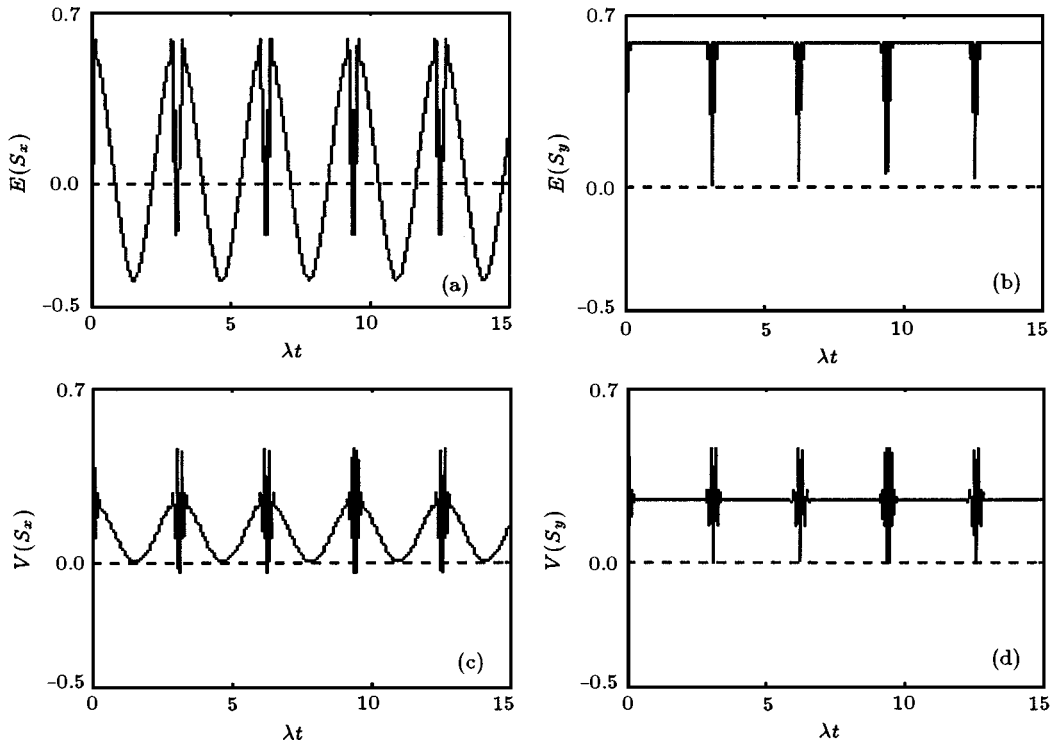


Fig.1. The time evolution of the squeezing factors, information entropy squeezing factors $E(S_x)$(a) and $E(S_y)$ (b). Variance squeezing factors $V(S_x)$(c) and $V(S_y)$ (d). The atom is initially in the excited state and the field in the coherent state, with the initial average photon number $\overline{n} = 20$ and the phase $\beta = \pi/2$.

3.1.2. The atom initially in a coherent superposition state

Fig. 2 displays the squeezing factors for the atom initially in the coherent superposition state with $\theta = \pi/2$ and $\varphi = \pi/4$, and the coherent field with $\overline{n} = 20$, the phase $\beta = \pi/4$. It is obvious from Figs.2(a) and 2(b) that there is an alternative information entropy squeezing in the atomic polarization components S_x and S_y, whereas neither $V(S_x)$ nor $V(S_y)$ displays variance squeezing, as illustrated in Figs.2(c) and 2(d). As a typical example, when $t = t_R(2k+1)/2 = (2k+1)\pi/2\lambda$ $(k = 0, 1, \ldots)$, one half of the time of the atomic inversion, the atom has achieved a pure state

$$|\psi_A(t_R/2)\rangle \simeq (|+\rangle + \mathrm{i}|-\rangle)/\sqrt{2}, \qquad (15)$$

which is the eigenstate of the atomic Pauli operator S_y. In terms of information entropy squeezing, we obtain information entropies of atomic Pauli operators S_x, S_y, and S_z at this time : $H(S_x) = 0.0$, $H(S_y) = H(S_z) = \ln 2$, correspondingly, $\delta H(S_y) = 1$, $\delta H(S_x) = \delta H(S_z) = 2$. Using these results, we have calculated entropy squeezing factors: $E(S_y) \simeq -0.414 < 0$, and $E(S_x) = 0.586 > 0$. This shows that the operator S_y exhibits optimal entropy squeezing when no entropy squeezing occurs in the operator S_x at the half time of the atomic inversion. This analytical result is in accord with the result of numerical calculation shown in Fig.2(b). As another typical example, when $t = k\pi/\lambda$, the atom has also achieved a pure state

$$|\psi_{\mathrm{A}}(t_{\mathrm{R}}/2)\rangle \simeq (|+\rangle + |-\rangle)/\sqrt{2}, \qquad (16)$$

which is the eigenstate of the atomic Pauli operator S_x. $E(S_x)$ has its smallest value -0.414 (shown in Fig.2(a)). The difference between the results of variance squeezing and that of information squeezing can be interpreted as follows: owing to the influence of the initial atomic coherence, the amplitude of the atomic inversion $W(t) = \frac{1}{2}\langle S_z\rangle$ evolves to certain very tiny values, close to zero (shown in Fig.3(b)), the definition of the variance based on the HUR is almost no longer valid.

We find that, whether or not there is information entropy squeezing and which component is squeezed depend on the atomic coherence, i.e. the parameters θ, β and φ. Further study may find that the atomic initial distribution angle θ decides how many components are to be squeezed. If $\theta = 0$ or $\theta = \pi$, there is no more than one component to be squeezed, but there

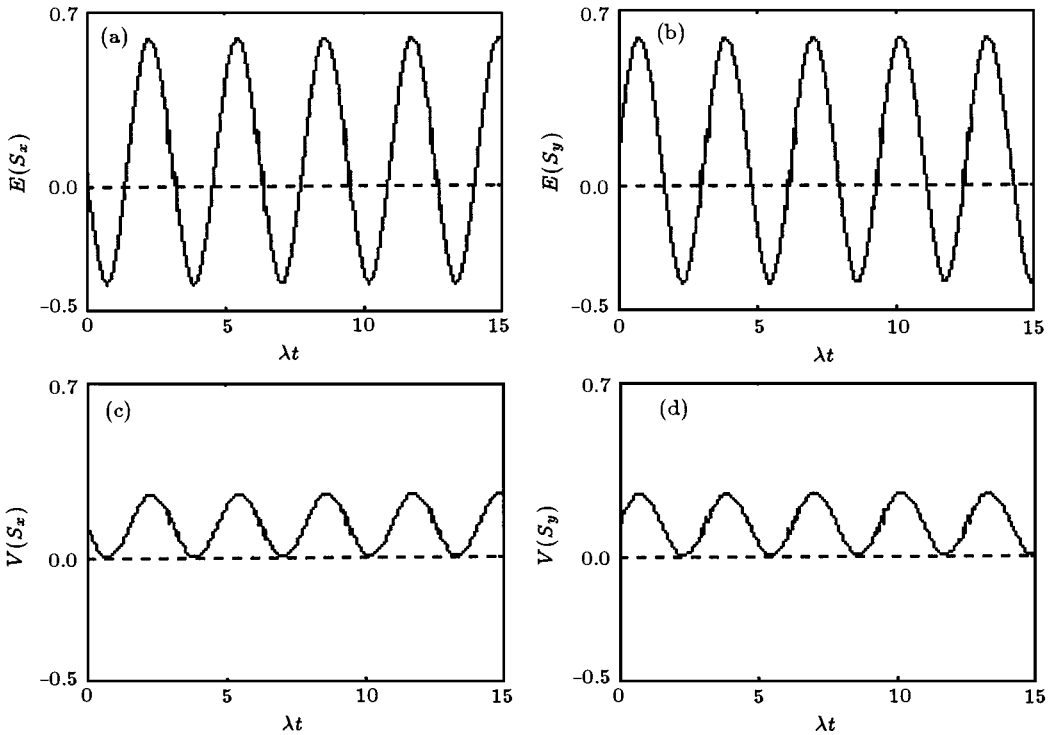


Fig.2. The time evolution of the squeezing factors, information entropy squeezing factors $E(S_x)$ (a) and $E(S_y)$(b). Variance squeezing factors $V(S_x)$ (c) and $V(S_y)$ (d). The atom is initial in the coherent superposition state with $\theta = \pi/2$, the phase $\varphi = \pi/4$, and the field in the coherent state with the initially the average photon number $\overline{n} = 20$, the phase $\beta = \pi/4$.

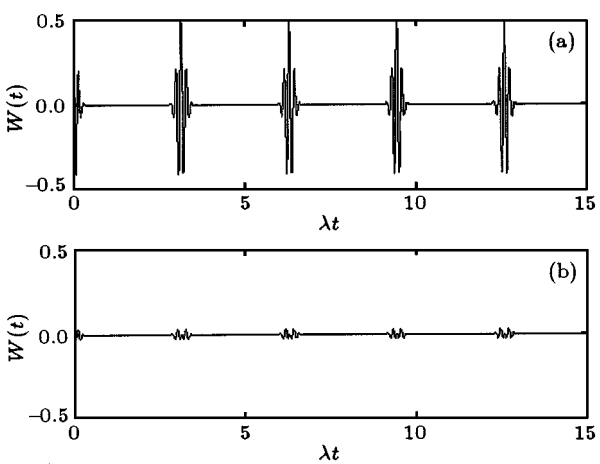


Fig.3. The time evolution of the atomic inversion. (a) The atom is initial in the excited state and the field in the coherent state with the initial average photon number $\overline{n} = 20$, the phase $\beta = \pi/2$. (b) The atom is initial in the coherent superposition state with $\theta = \pi/2$, the phase $\varphi = \pi/4$ and the field in the coherent state with the initial the average photon number $\overline{n} = 20$, the phase $\beta = \pi/4$.

may be two components to be alternatively squeezed at different times, when the atom is initially in the coherent superposition state. In particular, when $\theta = \pi/2$, $\beta = \pi/4$, $\varphi = \pi/4$, there is optimal periodic information entropy squeezing in both S_x and S_y at different times.

3.2. The influence of the phase of the atom and the field on the information entropy squeezing

Here we discuss the influences of the phases of the atom and the field on the information entropy squeezing. The results show that the direction of the atomic information entropy squeezing depends on the phases of the field and the atom.

We discuss the influence of the phase β of the field for $\theta = \pi/2$, $\varphi = \pi/4$ (see Figs.4). We find that if β tends to 0 (or π), there is the information entropy squeezing in S_y but almost no information entropy squeezing in S_x. When β tends to $\pi/2$, there is the information entropy squeezing in S_x but no squeezing in S_y. This means that the phase of the field decides the direction of information entropy squeezing. It is very interesting that the characters of the evolutions of $E(S_x)$ and $E(S_y)$ exchange completely in the cases of $\beta = 0$ and $\beta = \pi/4$ (see Fig.4). We also find that the evolutions of $E(S_x)$ and $E(S_y)$ are approximately symmetric on $\beta = \pi/4$.

Letting $\theta = \pi/2$, $\beta = \pi/4$, we find that all the results obtained previously in subsection (3.1.2) are valid for the phase φ of the atom, indicating that the

influence of the atomic phase on the atomic information entropy squeezing is the same as that of the phase of the field.

In conclusion, we have studied the entropy squeezing for a two-level atom in JCM with an intensity-dependent coupling by using quantum information entropy. We can conclude that: (1) The number of the information entropy squeezed components depends on the atomic initial distribution angle. (2) The direction of the information entropy squeezing is determined by both the phases of the atom and the field. (3) Quantum information entropy is a remarkable precision measure for atomic squeezing, it overcomes the triviality of the variance squeezing based on HUR. Especially, if the atom is in the eigenstate of the operator S_x or S_y, the entropy squeezing is quite a good measure.

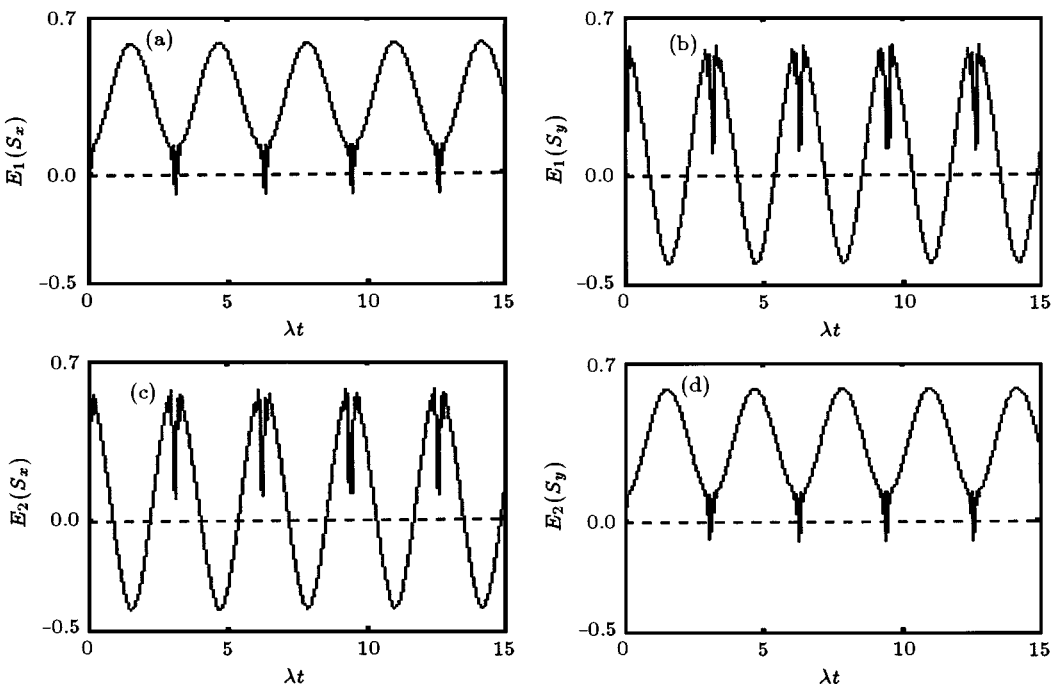


Fig.4. The time evolution of the information entropy squeezing factors, (a) $E(S_x)$ with $\beta = 0$; (b) $E(S_y)$ with $\beta = 0$; (c) $E(S_x)$ with $\beta = \pi/4$; (d) $E(S_y)$ with $\beta = \pi/4$. The atom is initial in the coherent superposition with $\theta = \pi/2$, $\varphi = \pi/4$, the field in the coherent state with the initial the average photon number $\overline{n} = 20$.

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