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Quantum logic networks for probabilistic teleportation*

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By means of the primitive operations consisting of single-qubit gates, two-qubit controlled-not gates, Von Neuman measurement and classically controlled operations, we construct efficient quantum logic networks for implementing probabilistic teleportation of a single qubit, a two-particle entangled state, and an N-particle entanglement. Based on the quantum networks, we show that after the partially entangled states are concentrated into maximal entanglement, the above three kinds of probabilistic teleportation are the same as the standard teleportation using the corresponding maximally entangled states as the quantum channels.

Keywords: probabilistic teleportation, entanglement, quantum circuit
PACC: 0365, 0367

1. Introduction

Entanglement plays an essential role in quantum information theory. A number of useful applications such as quantum dense coding,[1] quantum secret sharing,[2] certain types of quantum key distribution[3] and Greenberger–Horne–Zeilinger (GHZ) correlations[4] are rooted in the existence of quantum entanglement. One of the extremely striking exhibitions of entanglement is quantum teleportation, originally proposed by Bennett *et al*[5] in the qubit system. Quantum teleportation allows the transmission of an unknown qubit state from a sender to a spatially distant receiver *via* a quantum channel with the aid of some classical communication. The teleportation of a photon polarization state has been realized experimentally with the use of polarization-entangled photons[6] and path-entangled photons,[7] respectively. In virtue of a two-mode squeezed vacuum state, teleportation of a coherent state corresponding to continuous variable system was also demonstrated in the laboratory.[8]

Recently, ever more attention has been paid to the teleportation of an unknown entangled state.[9,10] Lee and Kim[11] investigated how to teleport an entangled pure spin-1/2 state with noisy quantum channels described by Werner states, and found that the purity of the initial state is important in determining the possibility of entanglement transfer. Based on the method of cavity quantum electrodynamics, Ikram *et al*[12] proposed a scheme for teleporting a two-particle entangled state inside high Q cavities. van Enk *et al*[13] discussed a protocol that entangled coherent states can be used to teleport one qubit encoded in Schrödinger cat states, only with the help of simple elements such as beamsplitter and photon counter. In Ref.[14], the teleportation of two entangled pure qubits is achieved by a three-particle entanglement, particularly GHZ state. Due to the influence of environment, the quantum channel composed of a pure entangled state is always taken to be nonmaximal. Therefore, it is of great interest to study the teleportation of an unknown quantum state using a partly entangled state as the quantum channel.[15] By performing a collective unitary transformation, Li *et al*[16] indicated that probabilistic teleportation of a single-particle state can be realized using a nonmaximally entangled quantum channel. Based on the idea of Ref.[16], Shi *et al*[17] proposed a protocol for teleporting a two-particle entangled qubit *via* a three-particle nonmaximally entangled state with a certain probability.

It is generally accepted that quantum computers can solve certain problems more rapidly than classical

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computers.[18] The basic operation in a quantum computer is unitary evolution, which can be accomplished *via* universal quantum logic gates.[19,20] The quantum logic gates are quantum devices that have the same number of input and output qubits and make unitary operation on selected qubits conditioned by the states of control qubits. A gate acting on a single qubit is regarded as a single-qubit gate, while gates acting on qubits are termed as multiqubit gates. These gates are very important in many proposed network constructions of concentrating entanglement,[21] of multiparticle entanglement,[22] of quantum copying,[23,24] and so on. Recently, Barenco *et al*[20] showed that all unitary operations on arbitrarily many bits can be decomposed into the combination of a set of one-bit quantum gates and two-bit Controlled-Not (CNOT) gates.

In this paper we deal with two things. First, in terms of only single-qubit gates, two-qubit CNOT gates and measurements on single qubits, we present quantum logic networks for probabilistic teleportation of a qubit and a two-particle entangled state, using a partially entangled pair and a three-particle nonmaximally entangled state, respectively. Second, we generalize the concept of probabilistic teleportation to the system consisting of many particles, namely, an N-particle entangled state is transmitted *via* a $(N+1)$-particle nonmaximally entangled channel. The quantum circuit of the probabilistic teleportation is then constructed. It is found that if the nonmaximally entangled channels are distilled in advance, the three kinds of probabilistic teleportation are the same as the standard teleportation *via* the corresponding maximally entangled channels.

This paper is organized as follows. In section 2 we briefly review the probabilistic teleportation of a qubit *via* a partly entangled pair. Next a quantum circuit of the collective unitary operation is constructed with the compositions of single-qubit gates and two-qubit CNOT gates. In addition, we obtain an alternative unitary operation, which can also realize the teleportation with the same probability of success, and is much easier to be built from the point of the resource requirement in the quantum circuit. Based on the circuit of the latter unitary operation, quantum logic networks for probabilistically teleporting an unknown qubit by a nonmaximally entangled state are given. In the same way, quantum networks for teleportation of a two-particle entangled state *via* the quantum channel of a pure three-particle entanglement are presented in section 3. In section 4, by means of $N+1$ qubits in a partly entangled state, we consider a generalized case for probabilistic teleportation of an N-particle entangled state. The quantum circuit for this teleportation process is straightforward. Finally, a brief summary of our results is given in section 5.

2. Teleportation of a single qubit

We first review the scheme of probabilistic teleportation discussed in Ref.[16]. Suppose that an unknown state of particle 1, $|\psi\rangle_1 = a|0\rangle_1 + b|1\rangle_1$, $(|a|^2+|b|^2=1)$, needs to be transferred from a sender Alice to a distant receiver Bob *via* a quantum channel of a shared nonmaximally entangled pair, which can be expressed as

$$|\psi\rangle_{23} = \alpha|00\rangle_{23} + \beta|11\rangle_{23}, (|\alpha|^2+|\beta|^2=1), \quad (1)$$

where the coefficients α and β are real, $|\alpha| \geq |\beta|$ (α, β are assumed to be valid in this paper, except otherwise stated). We assume that particle 2 belongs to Alice while particle 3 is at Bob's side. Thus, the total state of the system is $|\Psi\rangle_{123} = |\psi\rangle_1 \otimes |\psi\rangle_{23}$. If a Bell-state measurement between particle 1 and particle 2 is performed by Alice, particle 3 will be collapsed into the following unnormalized states:

$$\langle\phi_{12}^{\pm}|\Psi\rangle_{123} = \frac{1}{\sqrt{2}}(\alpha a|0\rangle_3 \pm \beta b|1\rangle_3), \quad (2)$$

$$\langle\psi_{12}^{\pm}|\Psi\rangle_{123} = \frac{1}{\sqrt{2}}(\beta a|1\rangle_3 \pm \alpha b|0\rangle_3). \quad (3)$$

Here, $|\phi^{\pm}\rangle_{12}$ and $|\psi^{\pm}\rangle_{12}$ are the Bell states of particles 1 and 2,

$$|\phi^{\pm}\rangle_{12} = \frac{1}{\sqrt{2}}(|00\rangle_{12} \pm |11\rangle_{12}), \quad (4)$$

$$|\psi^{\pm}\rangle_{12} = \frac{1}{\sqrt{2}}(|01\rangle_{12} \pm |10\rangle_{12}), \quad (5)$$

which are actually four maximally entangled states forming a complete orthogonal basis in the four-dimensional Hilbert space.

After the Bell measurement, Alice informs Bob of the measurement outcomes through a classical communication, then Bob understands exactly in which one of the four states in Eqs.(2) and (3) is located particle 3. In order to realize the teleportation, Bob needs to reproduce the original state $|\psi\rangle_1$ at his side from the unnormalized states of particle 3. Now an auxiliary qubit with the initial state $|0\rangle_a$ is introduced by Bob. In this case, on the basis

$\{|0\rangle_3|0\rangle_a, |0\rangle_3|1\rangle_a, |1\rangle_3|0\rangle_a, |1\rangle_3|1\rangle_a\}$, a collective unitary operation[16] can be written as

$$U_1 = \begin{pmatrix} \beta/\alpha & \sqrt{1-\beta^2/\alpha^2} & 0 & 0 \\ 0 & 0 & 0 & -1 \\ 0 & 0 & 1 & 0 \\ \sqrt{1-\beta^2/\alpha^2} & -\beta/\alpha & 0 & 0 \end{pmatrix}. \tag{6}$$

This operation transforms the product state $1/\sqrt{2}$ $(\alpha a|0\rangle_3|0\rangle_a \pm \beta b|1\rangle_3|0\rangle_a)$ into the following state

$$\frac{1}{\sqrt{2}}\left[\beta(a|0\rangle_3 \pm b|1\rangle_3)|0\rangle_a + \alpha a\sqrt{1-\frac{\beta^2}{\alpha^2}}|1\rangle_3|1\rangle_a\right]. \tag{7}$$

Likewise, under the operation U_1, the state $\frac{1}{\sqrt{2}}$ $(\beta a|1\rangle_3|0\rangle_a \pm \alpha b|0\rangle_3|0\rangle_a)$ turns into

$$\frac{1}{\sqrt{2}}\left[\beta(a|1\rangle_3 \pm b|0\rangle_3)|0\rangle_a \pm \alpha b\sqrt{1-\frac{\beta^2}{\alpha^2}}|1\rangle_3|1\rangle_a\right]. \tag{8}$$

From Eqs.(7) and (8), if the measurement result on the auxiliary particle is $|0\rangle_a$, the teleportation can be successfully achieved with the classical information from Alice and a corresponding unitary operation (I, R_z, R_x, R_y) on particle 3, where I is a unity operation and R_i is Pauli operation ($i = x, y, z$). If the result is $|1\rangle_a$, the teleportation fails. Hence, the probability of the successful teleportation is $2|\beta|^2$, based on the measurement on the auxiliary particle.

2.1. Gate array for the unitary operation U_1

In general, any unitary operation can be decomposed into universal operations[20] consisting of local unitary operations on a single qubit and two-bit controlled-NOT (CNOT) gates. A two-qubit CNOT gate acts on two quantum bits denoted as the control bit and the target bit respectively. If the control qubit m is in the state $|1\rangle$, the CNOT operator C_{mn} flips the state of the target qubit n, while C_{mn} leaves the qubit n invariant when the qubit m is in the state $|0\rangle$. In terms of single-qubit rotation gates and two-qubit controlled-rotation gates, the operation U_1 can then be expressed as

$$U_1 = C_{a3}(R_{3x} \otimes I_a)U_{3a}(R_{3x} \otimes I_a)(I_3 \otimes R_{az}), \tag{9}$$

where R_{ji} is a Pauli operation R_i on particle j, I_k is a unity operation on particle k $(j, k = 3, a)$, and U_{3a} is a two-bit controlled-rotation operator. If the control qubit 3 is in the state $|1\rangle$, a unitary rotation operation

$$u = \begin{pmatrix} \beta/\alpha & -\sqrt{1-\beta^2/\alpha^2} \\ \sqrt{1-\beta^2/\alpha^2} & \beta/\alpha \end{pmatrix}$$

$$= \begin{pmatrix} \cos\frac{\theta}{2} & -\sin\frac{\theta}{2} \\ \sin\frac{\theta}{2} & \cos\frac{\theta}{2} \end{pmatrix} \tag{10}$$

acts on the auxiliary qubit $|0\rangle_a$, where we assume $\frac{\beta}{\alpha} = \cos\frac{\theta}{2}$. Otherwise, a unity operator I will act on the state of the auxiliary qubit. With the help of the work done by Barenco *et al*[20] regarding how to simulate a general two-bit controlled-rotation operator with only single-bit gates and two-bit CNOT gates, the sequence of the operation U_1 is depicted in Fig.1.

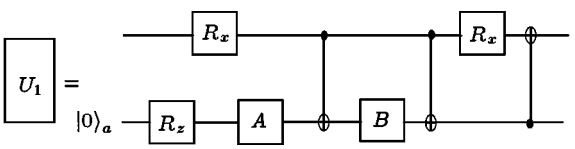


Fig.1. Quantum gate array of the unitary operation U_1 using a set of single-bit gates and CNOT gates.

In Fig. 1, $A = R_y\left(-\frac{\theta}{2}\right)$ and $B = R_y\left(\frac{\theta}{2}\right)$ are the rotations of $-\theta$ and θ around y, respectively, the dot $\bullet$ is the control qubit, and the cross $\oplus$ is the target qubit. In all the circuits given in this paper, we apply the usual convention that time advances from left to right.

2.2. Alternative unitary operation U_2 and its gate array

Clearly, the unitary operation U_1 is important in accomplishing the probabilistic teleportation. In the following, we introduce another unitary transformation U_2, which can also carry out the task with the same probability as U_1. Under the basis $\{|0\rangle_3|0\rangle_a, |0\rangle_3|1\rangle_a, |1\rangle_3|0\rangle_a, |1\rangle_3|1\rangle_a\}$, the transformation U_2 is described by

$$U_2 = \begin{pmatrix} \frac{\beta}{\alpha} & -\sqrt{1-\frac{\beta^2}{\alpha^2}} & 0 & 0 \\ \sqrt{1-\frac{\beta^2}{\alpha^2}} & \frac{\beta}{\alpha} & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}. \tag{11}$$

In terms of the combinations of single-bit gates and two-bit CNOT gates, we obtain the decomposition of U_2,

$$U_2 = (R_{3x} \otimes I_a)U_{3a}(R_{3x} \otimes I_a). \tag{12}$$

Comparing Eq.(12) with Eq.(9), we see that the gate resource composing the operation U_2 is less than that of U_1, since the latter needs two more gates than the former.

The sequence of the gate U_2 is illustrated in Fig.2.

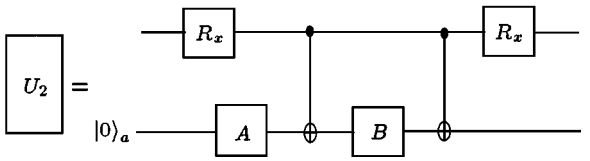


Fig.2. Quantum gate array of the unitary operation U_2 using a set of single-bit gates and CNOT gates.

2.3. Quantum networks for probabilistic teleportation of a qubit

Based on the above description and the unitary operation U_2, the network of probabilistic teleportation of the unknown quantum state $|\psi\rangle_1$ is shown in Fig.3.

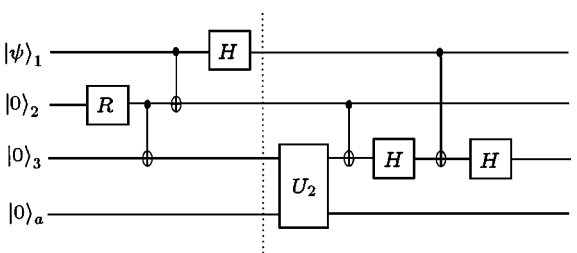


Fig.3. Quantum circuit for probabilistic teleportation of a qubit.

Here $R = \begin{pmatrix} \alpha & -\beta \\ \beta & \alpha \end{pmatrix}$ is a single-qubit rotation transformation, and H is a Hadamard gate defined as $H = \frac{1}{\sqrt{2}} \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}$. If the measurement result on the auxiliary qubit at the output state is $|0\rangle_a$, the teleportation is successful with the final state of particle 3 being reconstructed as the initial state to be teleported. From Fig.3, we see that the probabilistic teleportation can be divided into two steps. The first step is a purification process in which Bob can concentrate the state $|\psi\rangle_{23}$ into a maximally entangled state by introducing an auxiliary particle $|0\rangle_a$ and performing a collective unitary operation U_2 on the basis of $\{|0\rangle_3|0\rangle_a, |0\rangle_3|1\rangle_a, |1\rangle_3|0\rangle_a, |1\rangle_3|1\rangle_a\}$. Just as we have expected, the nonmaximally entangled state $|\psi\rangle_{23} = \alpha|00\rangle_{23}+\beta|11\rangle_{23}$ can be transformed into EPR state with probability $2|\beta|^2$.[25,26] The second step is the standard teleportation using the maximal entanglement as the quantum channel.[5] Apart from the auxiliary qubit and the U_2 gate, the quantum circuit depicted in Fig.3 is equivalent to that in.[27,28]

Note that a controlled unitary operation acting on any number of qubits followed by a measurement of the control qubit can be replaced by the measurement of the control qubit preceding the controlled operation.[29,30]

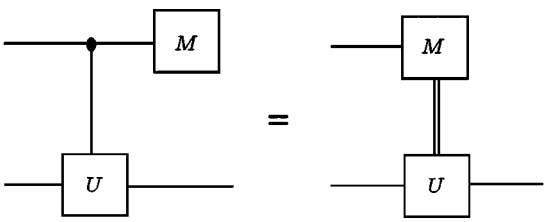


Fig.4. Two equivalent circuits for measurement and controlled-U operation.

Here, U represents a unitary operation depending on the measurement outcomes, the double line coming out of the measurement M denotes the classical information.[31] Therefore, Fig.3 can be re-expressed as Fig.5.

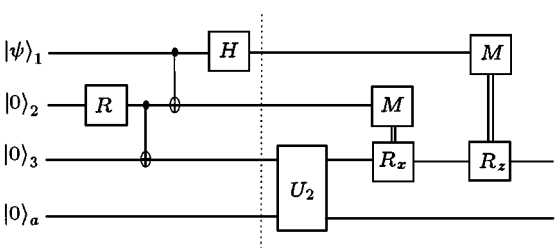


Fig.5. The same as Fig.3, but Von Neumann measurement and classically controlled operation are made at the end instead.

In Fig.5, the controlled operation can be realized locally by Bob depending on the results of the two measurements Alice performed on her own qubit. If and only if the outcome of Alice's measurement is 1, Bob can execute the controlled-R_x or controlled-R_z,[28] where $R_z = HR_xH$. If the measurement result of the auxiliary particle is $|1\rangle_a$, the teleportation fails. If the result is $|0\rangle_a$, the final state of particle 3 at Bob's side will be collapsed into $|\psi\rangle_3 = a|0\rangle_3 + b|1\rangle_3$, which is the desired state. That is to say, if the measurement outcome on the state of the auxiliary qubit is $|0\rangle_a$, perfect teleportation is accomplished.

3. Teleportation of a two-particle entangled state

In this section, we construct the quantum circuit for the teleportation process with reference to Shi *et al*[17] concerning the probabilistic teleportation of a two-particle entangled state *via* a three-particle partial entanglement.

Let us assume that a sender Alice wants to transmit an unknown two-particle entangled pair $|\phi\rangle_{12} = a|00\rangle_{12}+b|11\rangle_{12}$ to a receiver Bob by means of a three-particle nonmaximally entangled state as the quantum channel. The pure three-particle entangled state can be expressed as

$$|\phi\rangle_{345} = \alpha|000\rangle_{345} + \beta|111\rangle_{345}. \tag{13}$$

The particles 1, 2 and 3 are at Alice's side, while particle 4 and 5 belong to Bob. Thus the total state of the system becomes $|\Psi\rangle_{12345} = |\phi\rangle_{12} \otimes |\phi\rangle_{345}$. To achieve the teleportation, Alice performs a Bell state measurement between particles 2 and 3, and subsequently makes a Hadamard transformation on particle 1, which is defined as the following:

$$|0\rangle_1^{'} = H|0\rangle_1 = \frac{1}{\sqrt{2}}(|0\rangle_1 + |1\rangle_1), \tag{14}$$

$$|1\rangle_1^{'} = H|1\rangle_1 = \frac{1}{\sqrt{2}}(|0\rangle_1 - |1\rangle_1). \tag{15}$$

According to the operations performed by Alice, the total state $|\Psi\rangle_{12345}$ can be described as

$$\begin{aligned}
|\Psi\rangle_{12345} =& (a|00\rangle_{12} + b|11\rangle_{12}) \otimes (\alpha|000\rangle_{345} + \beta|111\rangle_{345}) \\
=& \frac{1}{\sqrt{2}}[|\phi^+\rangle_{23}(\alpha a|000\rangle_{145} + \beta b|111\rangle_{145}) \\
&+ |\phi^-\rangle_{23}(\alpha a|000\rangle_{145} - \beta b|111\rangle_{145}) \\
&+ |\psi^+\rangle_{23}(\beta a|011\rangle_{145} + \alpha b|100\rangle_{145}) \\
&+ |\psi^-\rangle_{23}(\beta a|011\rangle_{145} - \alpha b|100\rangle_{145})] \\
=& \frac{1}{2}[|0\rangle_1^{'}|\phi^+\rangle_{23}(\alpha a|00\rangle_{45} + \beta b|11\rangle_{45}) \\
&+ |1\rangle_1^{'}|\phi^+\rangle_{23}(\alpha a|00\rangle_{45} - \beta b|11\rangle_{45}) \\
&+ |0\rangle_1^{'}|\phi^-\rangle_{23}(\alpha a|00\rangle_{45} - \beta b|11\rangle_{45}) \\
&+ |1\rangle_1^{'}|\phi^-\rangle_{23}(\alpha a|00\rangle_{45} + \beta b|11\rangle_{45}) \\
&+ |0\rangle_1^{'}|\psi^+\rangle_{23}(\beta a|11\rangle_{45} + \alpha b|00\rangle_{45}) \\
&+ |1\rangle_1^{'}|\psi^+\rangle_{23}(\beta a|11\rangle_{45} - \alpha b|00\rangle_{45}) \\
&+ |0\rangle_1^{'}|\psi^-\rangle_{23}(\beta a|11\rangle_{45} - \alpha b|00\rangle_{45}) \\
&+ |1\rangle_1^{'}|\psi^-\rangle_{23}(\beta a|11\rangle_{45} + \alpha b|00\rangle_{45})].
\end{aligned} \tag{16}$$

Note that with the introduction of an auxiliary particle $|0\rangle_a$, the unitary operation U_2 under the basis $\{|0\rangle_5|0\rangle_a, |0\rangle_5|1\rangle_a, |1\rangle_5|0\rangle_a, |1\rangle_5|1\rangle_a\}$ can turn the product state $\frac{1}{2}(\alpha a|00\rangle_{45} \pm \beta b|11\rangle_{45}) \otimes |0\rangle_a$ into the following state

$$\begin{aligned}
&\frac{1}{2}\Big[\beta(a|00\rangle_{45} \pm b|11\rangle_{45})|0\rangle_a \\
&+ \alpha a\sqrt{1-\frac{\beta^2}{\alpha^2}}|0\rangle_4|0\rangle_5|1\rangle_a\Big].
\end{aligned} \tag{17}$$

Similarly, using the operation U_2, the state $\frac{1}{2}(\beta a|11\rangle_{45} \pm \alpha b|00\rangle_{45}) \otimes |0\rangle_a$ will be transformed into

$$\begin{aligned}
&\frac{1}{2}\Big[\beta(a|11\rangle_{45} \pm b|00\rangle_{45})|0\rangle_a \\
&\pm \alpha b\sqrt{1-\frac{\beta^2}{\alpha^2}}|0\rangle_4|0\rangle_5|1\rangle_a\Big].
\end{aligned} \tag{18}$$

Hence, the probability of successful teleportation of the quantum state $|\phi\rangle_{12}$ is $2|\beta|^2$. In this way, only a collective unitary operation is required to achieve the probabilistic teleportation, which, with regard to quantum gate array, is simpler than the scheme in Ref.[17] where two different collective unitary operations are needed for the same task, as there is different time sequence between Alice's Hadamard transformation on qubit 1 and the collective unitary operation performed by Bob. Based on the Eqs.(16), (17) and (18), quantum circuit for the teleportation of the two-particle entangled state is presented in Fig.6.

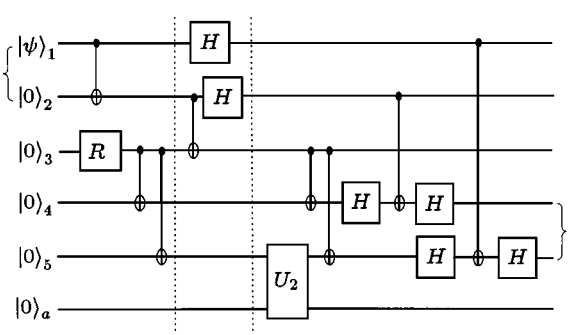


Fig.6. Quantum circuit for probabilistic teleportation of a two-particle entangled state.

If the measurement result of the auxiliary qubit in the final state is $|0\rangle_a$, each of Alice's three qubits in the rightmost of the figure will be in the pure state $\frac{1}{\sqrt{2}}(|0\rangle + |1\rangle)$, completely disentangled from Bob's. As a result, the state of the qubits at Bob's side is entirely unaltered even if Alice measures each of her qubits. Thus, Fig.6 can be redisplayed in Fig.7 with some classically controlled operations.

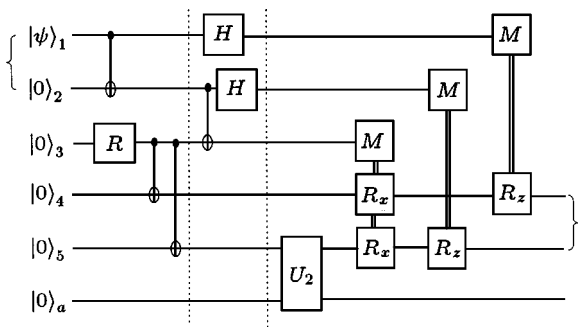


Fig.7. The same as Fig.6, but Von Neumann measurement and classically controlled operation are made at the end instead.

4. Teleportation of an N-particle entangled state

In this section, we generalize the probabilistic teleportation to the case of N particles. Now Alice wants to teleport an unknown N-particle entangled pure state to a distant receiver Bob *via* a nonmaximally entangled state of $N+1$ particles. Suppose that the state to be teleported is

$$|\phi\rangle_N = a|0\rangle^N + b|1\rangle^N. \tag{19}$$

The quantum channel is composed of $N+1$ qubits in the nonmaximally entanglement

$$|\phi\rangle_{N+1} = \alpha|0\rangle^{N+1} + \beta|1\rangle^{N+1}, \tag{20}$$

where $|l\rangle^N = |l\rangle \otimes \ldots |l\rangle \otimes |l\rangle$, $l = 0, 1$. Without loss of generality, we assume that the first $N+1$ particles belong to Alice, the other N particles belong to Bob, and that the sender Alice and the receiver Bob share the entanglement of the form Eq.(20). In this case, the combined state of the total system is $|\Psi\rangle = |\phi\rangle_N \otimes |\phi\rangle_{N+1}$. Now Alice performs a Bell-state measurement between the particles N and $N+1$, and makes a set of Hadamard operations in the particles $(1, 2, \ldots N-1)$ respectively. From Ref.[14], the state vector consisting of the joint measurement performed by Alice about $N+1$ $(1, 2, \ldots N, N+1)$ particles forms a complete and orthogonal basis.

After Alice's joint measurement, the state of N particles $(N+2,\ N+3,\ \ldots,\ 2N,\ 2N+1)$ at Bob's side will be projected into the unnormalized states $\frac{1}{\sqrt{2^N}}\ (\alpha a|0\rangle^N \pm \beta b|1\rangle^N)$ or $\frac{1}{\sqrt{2^N}}\ (\beta a|1\rangle^N \pm \alpha b|0\rangle^N)$. By means of an auxiliary qubit $|0\rangle_a$, the unitary operation U_2 under the basis $\{|0\rangle_{2N+1}|0\rangle_a, |0\rangle_{2N+1}|1\rangle_a, |1\rangle_{2N+1}|0\rangle_a, |1\rangle_{2N+1}|1\rangle_a\}$ can transform the product state $\frac{1}{\sqrt{2^N}}\ (\alpha a|0\rangle^N \pm \beta b|1\rangle^N) \otimes |0\rangle_a$ into the state

$$\frac{1}{\sqrt{2^N}}\left[\beta(a|0\rangle^N \pm b|1\rangle^N)|0\rangle_a + \alpha a\sqrt{1-\frac{\beta^2}{\alpha^2}}|0\rangle^N|1\rangle_a\right]. \tag{21}$$

In the same way, the state $\frac{1}{\sqrt{2^N}}(\beta a|1\rangle^N \pm \alpha b|0\rangle^N) \otimes |0\rangle_a$ will be turned into

$$\frac{1}{\sqrt{2^N}}\left[\beta(a|1\rangle^N \pm b|0\rangle^N)|0\rangle_a \pm \alpha b\sqrt{1-\frac{\beta^2}{\alpha^2}}|0\rangle^N|1\rangle_a\right]. \tag{22}$$

Based on the above description, the quantum network displayed in Fig.8 illustrates the teleportation procedure of the N-particle pure entangled state.

In Fig. 8, the prohibited operation in the dotted box is applied to a known state and can be replaced by the state of Eq.(20). If such a shared entanglement is initially available to Alice and Bob, the probabilistic teleportation can be achieved in terms of unitary transformations and measurements on single qubits, two-qubit CNOT gates and classically controlled operations.

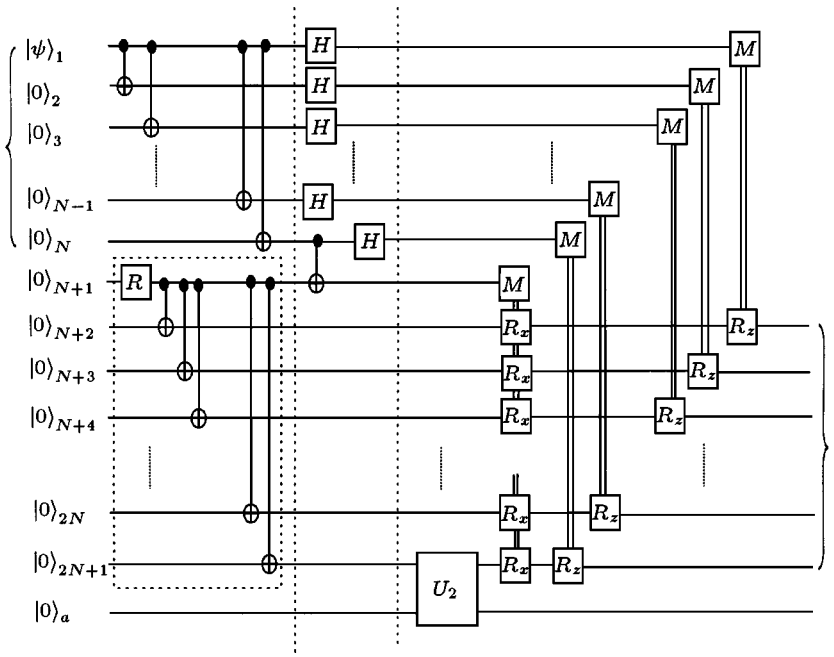


Fig.8. Quantum circuit for probabilistic teleportation of an N-particle entangled state, in which Von Neumann measurement and classically controlled operation are made at the end.

5. Conclusions

With the introduction of an auxiliary particle and the construction of a collective unitary operation, probabilistic teleportation of a qubit and a two-particle entangled pair can be realized through a two-particle[16] and a three-particle[17] nonmaximal entanglement, respectively. In this paper, we have constructed the quantum gate array of the collective unitary operation with the compositions of single-qubit gates and two-qubit CNOT gates. Moreover, we have built an alternative unitary transformation, which can also accomplish the probabilistic teleportation with the same probability. Clearly, the latter collective unitary operation is less than the former one from the point of the number of gates resource requirement. Based on the latter unitary transformation, the quantum logic networks for probabilistic teleporting a qubit and teleporting a two-particle entangled state are presented in terms of only four types of operation: unitary transformations on single qubits, unitary interactions between pairs of qubits, measurements on single qubits, and classically controlled operations. In addition, a scheme about the teleportation of an N-particle entangled state with a certain probability is proposed and the corresponding quantum networks is presented directly. It is shown that after the nonmaximally entangled states are concentrated into the maximal entanglement, the three kinds of probabilistic teleportation can be regarded as standard teleportation[5,14] *via* a maximally entangled quantum channel. It deserves mentioning that if the three types of probabilistic teleportation discussed in this paper are handled firstly according to the technique of the standard teleportation, quantum networks for the teleportation should be complicated and two different collective unitary operations are necessary. It should be noted that as long as the sender and all the receivers initially share the form of the entanglement, as specified in our paper, an unknown quantum state can also be probabilistically teleported to many spatially separated receivers.

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