Vol 12 No 2, February 2003
1009-1963/2003/12(02)/0184-05

Chinese Physics

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Patterns in passive optical systems with boundary effects

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(Received 9 July 2002; revised manuscript received 14 October 2002)

Pattern formation is closely related to system boundary conditions in nonlinear dynamic systems, apart from the system control parameters. To avoid complexity, system boundary conditions are usually considered to be infinite or periodic, and the initial conditions spatially homogeneous. But it is not always the case in real situations, or sometimes periodic boundary conditions are not exact. To show the important and interesting boundary effects in real pattern formation, we suggest a simple universal boundary condition in a typical optical pattern formation system. Numerical simulations of the passive optical system show that pattern characteristics such as distribution symmetry, peak number, structure strength, evolution course and stability are all greatly influenced by the system boundary conditions.

Keywords: boundary effects, symmetry, stability
PACC: 4265, 2410H

1. Introduction

Much theoretical and experimental research has been reported on pattern formation in nonlinear dynamic systems,[1−7] such as those described by Kuramoto–Sivashinsky equation, reaction-diffusion equation, complex Ginzburg–Landau equation, drift-wave equation, complex Lorenz form of the Maxwell–Bloch equation, coupled map lattice equation, etc. And it has been reported that there are many similar dynamic characteristics among these temporal–spatial dynamic systems.[1−3] Pattern formation refers to that in extended nonlinear medium, above a threshold, system uniform amplitude distribution becomes unstable and splits into correlated domains.[2] In optics, the interactions between electromagnetic field and polarizable medium with losses induced by absorption, diffraction, dispersion, etc. constitutes a mechanism of pattern formation, which is similar to instability in chemical reaction-diffusion dynamics known as Turing instability. Since the first experimental observation of optical transverse instability,[8] the potential significance both in theoretical and application aspects has stimulated great investigation activities in this field.[9−15]

Real pattern effects should include boundaries, but they are rarely discussed in many cases. For simplicity, periodic boundary conditions are usually assumed in theoretical studies. Under such conditions spatial translation invariance homogenizes out any time average (unless some unexpected ergodicity breaking takes place[16]). In fact, periodic boundary conditions are not exact for infinite transverse solutions of extended systems. Here we suggest a simple universal boundary condition. The numerical results show that the system boundary conditions have important effects on pattern characteristics. The appearance of boundary may be explained as a necessary condition of pattern formation, for the boundary effects may be interpreted as sources or sinks of defects, reflection sites for waves or as a resonant force which balances the losses and affects the solutions of the system.[17−19] It has also been reported that modulation effects induced by boundaries may lead to spatial chaos.[20,21] Different pattern characteristics influenced by system boundary conditions in a passive optical system are discussed in this paper.

2.Model

We consider one of the prototype equations which describes the mean-field model in a unidirectional ring cavity or Fabry–Perot cavity, with plane mirrors enclosing a Kerr medium and driven by a coherent plane-wave field. Assuming that only one longitudinal mode

of the cavity is excited so it is possible to neglect the z dependence of the field, the evolution of the cavity field is governed by a nonlinear partial differential equation[22,23]

$$\partial E/\partial\tau = -E + E_i + \mathrm{i}\eta(|E|^2 - \theta)E + \mathrm{i}D\nabla_\perp^2 E, \quad (1)$$

where E is the normalized slowly varying envelope of the electric field, E_i is the normalized input field, assumed real and positive for definiteness, θ is the detuning parameter, and η equals 1 for self-focusing Kerr medium or -1 for self-defocusing Kerr medium. We only consider self-focusing condition in this paper. The time τ is normalized to the mean lifetime t_{ph} of the field in the cavity, and parameter D describes the constant of diffraction which is usually chosen to be unity. $\nabla_\perp^2$ is Laplacian operator. All variables in Eq.(1) are dimensionless. Equation (1) also holds for the cavity filled with a two-level system in the pure dispersive limit.[24]

The system's transverse homogeneous stationary solutions E_{s} satisfy the equation

$$E_i^2 = |E_{\mathrm{s}}|^2 \left[1 + (\theta - |E_{\mathrm{s}}|^2)^2\right], \quad (2)$$

which is known to give rise to optical bistability when the detuning parameter θ is greater than $3^{1/2}$, and both nonlinear stability analysis and numerical simulation predict the bifurcation of subcritical hexagons close to modulation instability threshold for $\theta > 41/30$. For simplicity, the stability of a dynamic system is usually analysed according to its linear stability. Some linear stability analysis of Eq. (1) has been done by Lugiato, Firth and co-workers.[23–25] Transverse homogeneous stationary solutions of Eq.(1) are shown in Fig.1. The curve with unstable part marked by crosses corresponds to focusing case $\theta = 1$, the curve with unstable part marked by dots corresponds to focusing case $\theta = 1.5$. We will choose the latter case in this paper, i.e. the system is located below the bistability threshold. Figure 1 shows that at threshold $I_{\mathrm{s}} = |E_{\mathrm{s}}|^2 = 1$, which is always true, the input intensity needed in the case of $\theta = 1.5$ is higher than that in the case of $\theta = 1$.

The selection of the critical wavenumber k_{c} is determined by a balance of the diffraction phase modulation Dk_{c}^2, the nonlinear phase modulation $I_{\mathrm{s}} = |E_{\mathrm{s}}|^2$ and the cavity detuning θ. Above the threshold, the most unstable wavevectors are $k_m = \sqrt{(2I_{\mathrm{s}} - \theta)/D}$. Their relations are shown in Fig.2. The dotted curve corresponds to $\theta = 1.5$, the crossed curve corresponds to $\theta = 1$; and the solid line corresponds to the most unstable modes above threshold in the case of $\theta = 1.5$. With detuning becoming larger, the linear instability boundary curve will move to the left, the instability threshold does not change, but the critical wavenumber will become smaller.

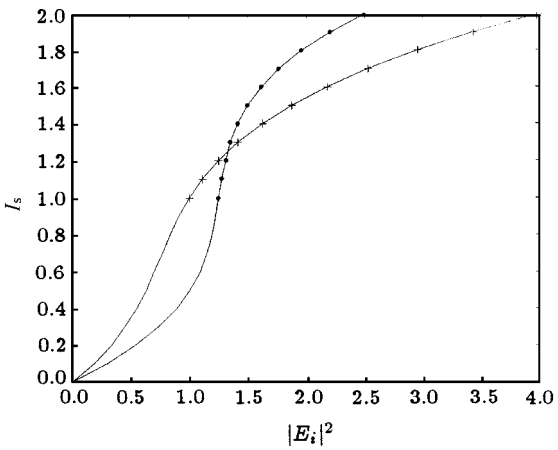


Fig.1. Transverse homogeneous stationary solutions of Eq.(1). The curve with unstable part marked by crosses corresponds to focusing case θ=1, the curve with unstable part marked by dots corresponds to focusing case θ=1.5.

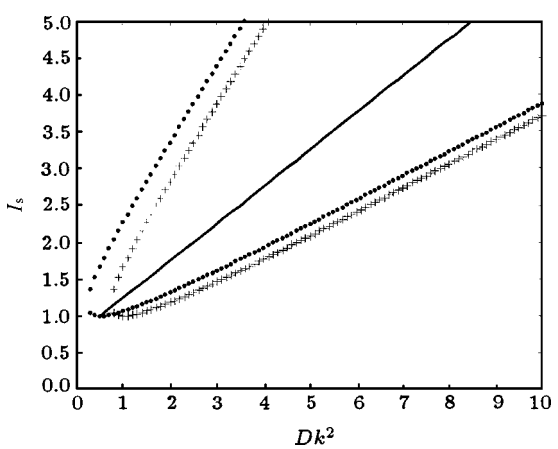


Fig.2. Linear instability boundaries for Eq. (1). The dotted curve corresponds to θ=1.5, the crossed curve corresponding to θ=1; and the solid line corresponds to the most unstable modes above threshold when θ=1.5.

Boundaries should be considered in real pattern effects. Here we discuss null Dirichlet boundary conditions that obey

$$E(-L/2, \tau) = E(L/2, \tau) = 0. \quad (3)$$

Physically, one can assume $E = 0$ on the boundary in the case of a dielectric waveguide in appropriate limits,

or in the case of a typical large-aperture laser having a kind of absorbing boundary arising from fast decay of the pumping amplitude away from the beam axis in an ideal situation (very sharp decay). It is important to know how effective the boundary conditions are in the process of pattern formation. System pattern should reveal the symmetries expressed by both the equation and the boundary conditions. In the case of plane-wave pumping, the system is invariant under translation, reflection and rotation in the transverse phase. The presence of boundary condition (3) breaks the translational symmetry of equation (1) and will influence the solutions of the system greatly. And such boundaries can act as sources of plane waves bringing perturbations or a forcing term, with which the growing of the unstable modes leads to the formation of patterns. We prove that the boundary effects will be stronger in relatively smaller systems and the most unstable modes of the system are on boundaries[3] under condition (3).

3. Numerical results

It is interesting to compare the simulation results of different computation domains. We will change the size and geometry of the computation domains and discuss the influences of the system boundaries on the characteristics of system patterns. In two dimensions with Dirichlet boundary condition (3), equation (1) was solved with random initial conditions and system parameters are chosen to be $\theta = 1.5$, $E_i = 1.13$ throughout. E_i is slightly above the threshold according to the linear stability analysis discussed above. First, grey level map in Fig.3 shows the numerical results in system square domain $L(x, y) \in [-4\ 4]$.

Under the above conditions, a soliton-like solution arises in the centre of pattern (a). From its time evolution trace (b) and correlative characteristic curve (c), we confirm it is weak chaos. Boundary effects enhance the instability, leading to a value high above the critical instability that will bring about a Hopf bifurcation and the appearance of oscillations breaks the time-translation invariance. In general, system pattern are strongly influenced by boundary condition in narrow containers although it is difficult to discuss in detail because the linear stability analysis is not true in conditions high above the threshold.

Figure 4 shows a typical pattern formation course when the domain of the system is appropriately chosen to be $L(x, y) \in [-40\ 40]$.

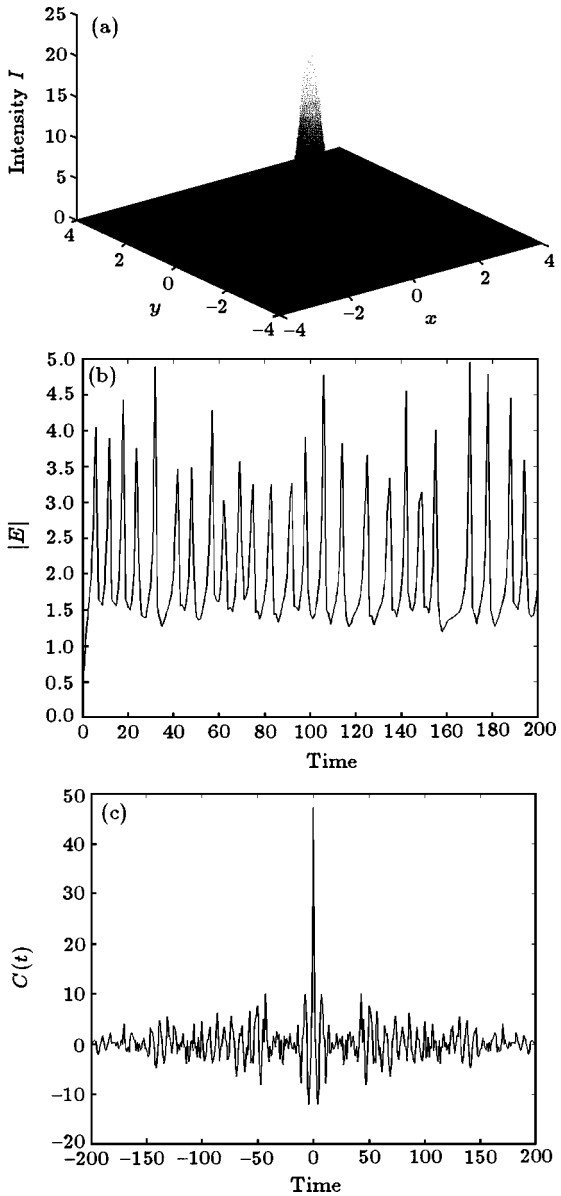


Fig.3. (a) Three-dimensional intensity distribution; (b) time evolution trace of $|E|$; (c) the correlative characteristic curve.

From Fig.4(a) and (b) we can see that pattern peaks arise at four corners first, then evolve around the boundaries, i.e. the patterns always form initially along the boundaries. Thus waves emitting from the corners and boundaries eventually dominate the pattern evolution. The amplitudes of the oscillations are strongest near sidewalls and attenuate with increasing distance form them. This shows that the most unstable modes of the systems are on the boundaries. If we take the average distance between two consecutive maxima as system scale wavelength, then it will be always greater than that of the fastest growing mode

obtained by linear stability analysis. These results are similar to those of Ref.[16]. After the transient, stationary spatial dissipative structures or structure stabilized patterns whose symmetries are mainly determined by the boundaries in the transverse profile are built as shown in Fig.4(c), and the positions of the peaks will not change. This is the general case that the first symmetry of system pattern is governed by system boundary conditions.[3] We also observed the same phenomena when we changed the system geometries to circular or triangular. The stability of pattern structure means that the distribution of pattern peaks is spatially more uniform and their intensities become constant with respect to time, as shown in Fig.4(d).

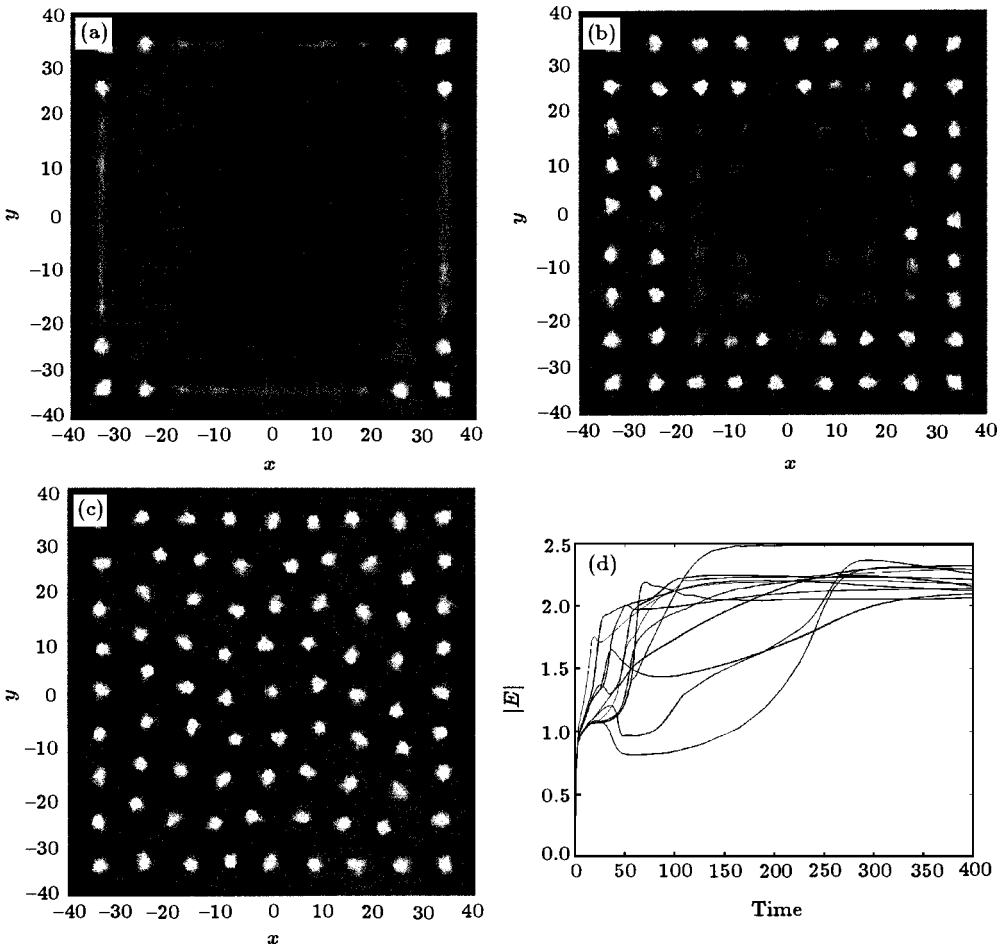


Fig.4. (a), (b) and (c) are contours of peak intensities corresponding to the sample time 25, 50, and 400, respectively; (d) time traces of the peaks relative to modulus $|E|$.

With the size of the system increasing, the number of peaks will increase, but synchronously their structure strengths, represented by the averaged peak intensities after systems being stable, will decrease. Quantitative results corresponding to different system sizes are given in Fig.5.

We obtain the figure by sampling the average values of the stationary ordered hexagonal patterns observed in ten times of runs after transient. The curve with circles shows that pattern structure strength will decrease with system size increasing, however, the curve with diamonds shows that pattern peak numbers (reduced ten times) will increase with system size increasing. And we always find the distribution of the peaks has the same symmetries as that of the calculation windows and the cavity field receives the largest contribution from system boundaries.

When the size of the system becomes large enough, the influence of the system boundaries becomes weak, and system solutions exhibit localized structures with a small number of peaks as shown in Fig.6 when the system domain is chosen as $L(x, y) \in [-70\ 70]$. But if we further increase the system size, patterns will evolve only around system sidewalls, i.e. there is no evolution at the centre of the pattern until the system is already stable. For large enough sys-

tem, a surface instability develops, leading to a solution localized near the boundary and decaying in the bulk for control parameter less than the value of that in the ideal bulk instability. These are the balanced results induced by the interactions between the system instability and its boundary conditions, and the larger the system domain, the more the instability system modes and the weaker the boundary influences on pattern formation.

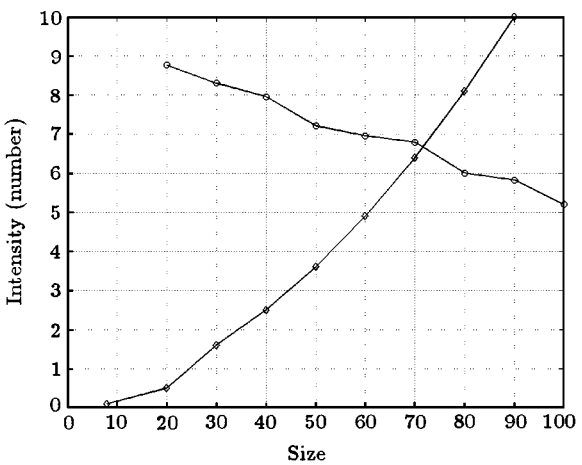


Fig.5. Intensity and peak numbers of the patterns corresponding to different system sizes.

Although the systematic study on how the magnitude of patterns decreases with aspect ratio has not been done thoroughly both numerical simulation and experiment are consistent with the expected behaviour that the magnitude of the time-averaged patterns should vanish in the relative infinite-aspect-ratio limit.[16,17]

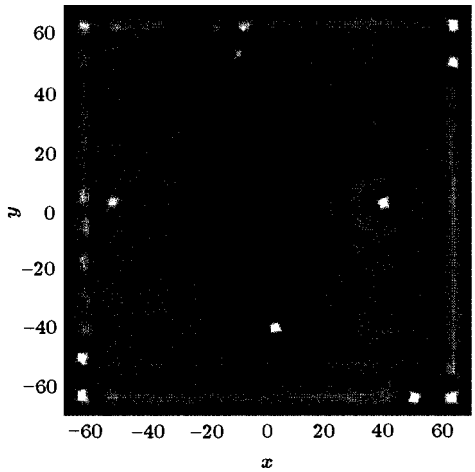


Fig.6. Localized structure corresponding to system domain within $L(x, y) \in [-70\ 70]$.

4. Conclusions

Our numerical simulations demonstrate the important roles played by system boundaries in pattern formation. Different boundary conditions may lead to substantially different dynamic behaviours. Our results show that the cavity field receives the largest contribution from system boundaries, especially in small systems. With system increasing, the temporal oscillatory soliton-like localized structure or instability patterns will evolve to stable and uniformly distributed hexagonal patterns after transient near the threshold. And pattern characteristics such as peak number, structure strength, distribution symmetry, evolution course and stability are all closely related to the system boundary conditions, or precisely, they all depend on both the system boundary conditions and the system control parameters.

References

[1] Abraham N B and Firth W J 1990 *J. Opt. Soc. Am.* B **7** 951
[2] Arecchi F T, Boccaletti S and Ramazza P 1999 *Phys. Reports* **318** 1
[3] Cross M C and Henberg P C 1993 *Rev. Mod. Phys.* **65** 851
[4] Alessandro G D and Firth W J 1991 *Phys. Rev. Lett.* **66** 2597
[5] Luo X S and Fang J Q 2000 *Chin. Phys.* **9** 333
[6] Zhang X and Shen K 2001 *Chin. Phys.* **10** 624
[7] Wang Z B and Hu G 2001 *Chin. Phys.* **10** 1666
[8] Giusfredi G, Valley J F, Pon R, Khitrova G *et al* 1988 *J. Opt. Soc. Am.* B **5** 1181
[9] Wang P Y, Xie P, Dai J H and Zhang H J 1998 *Phys. Rev. Lett.* **80** 4669
[10] Svetlana L L and Lu W P 2001 *Phys. Rev.* E **64** 026207
[11] Hegarty S P *et al* 1999 *Phys. Rev. Lett.* **82** 1434
[12] Tornkvist O *et al* 1997 *Phys. Rev. Lett.* **78** 1908
[13] Yu D J, Lu W P and Harrison R G 1999 *J. Opt. B: Quantum Semiclass.* **1** 25
[14] Weaire D, Kermode J P and Dwyer V M 1985 *Opt. Commun.* **55** 223
[15] Wang R and Shen K 2001 *Chin. Phys.* **10** 711
[16] Eguiluz V M, Alstrom P, Hernandez-Garcia E and Piro O 1999 *Phys. Rev.* E **59** 2822
[17] Egolf D A and Greenside H S 1994 *Phys. Lett.* A **185** 395
[18] Feng Q, Moloney J V and Newell A C 1994 *Phys. Rev.* A **50** R3601
[19] Harkness G K, Firth W J, Geddes J B *et al* 1994 *Phys. Rev.* E **50** 4310
[20] Eguiluz V M, Hernandez-Garcia E, Piro O *et al* 1999 *Phys. Rev.* E **60** 6571
[21] Rudroff S and Rehberg I 1997 *Phys. Rev.* E **55** 2742
[22] Firth W J and Lord A 1996 *J. Mod. Opt.* **43** 1071
[23] Firth W J, Scroggie A J, McDonald G S and Lugiato L A 1992 *Phys. Rev.* A **46** R3609
[24] Lugiato L A and Lefever R 1987 *Phys. Rev. Lett.* **58** 2209
[25] Lugiato L A and Oldano C 1988 *Phys. Rev.* A **37** 3896