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Effect of four-wave mixing on electromagnetically induced transparency in Λ-type system with a two-photon probe field*

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Using density matrix equation, we calculate analytically as well as numerically the effect of the four-wave mixing (FWM) field on electromagnetically induced transparency (EIT) in the rubidium $5S_{1/2}$-$5P_{3/2}$-$5D_{5/2}$ Λ-type system with a two-photon probe field. The calculated results are in good agreement with Fulton's experimental results in 1995 *Phys. Rev.* A **52** 2302.

Keywords: asymmetry of EIT profile, two-photon probe field, four-wave mixing
PACC: 4250, 3270

1. Introduction

Electromagnetically induced transparency (EIT) is an induced transparency experienced by a weak-probe field transition due to the presence of strong-coupling field on a linked transition.[1,2] It plays a key role in many applications such as gain and lasing without inversion,[3] storage of quantum information[4] and optical atomic clock.[5] These applications have been investigated in V-type, Λ-type and Ξ-type configurations. To avoid optical pumping between the ground state hyperfine levels in Λ-type system, Fulton *et al* studied experimentally continuous EIT by setting a two-photon scanning probe field and observed the asymmetry of EIT profile as shown in Fig.1 (solid line),[6] but they did not discuss the reason.

In the above Λ-type system with a two-photon probe field and a strong coupling field, the four-wave mixing (FWM) can be generated simultaneously. The FWM has been observed in such Λ-type system recently.[7−9] But the effect of the FWM on EIT with the two-photon probe field has not been investigated up to now.

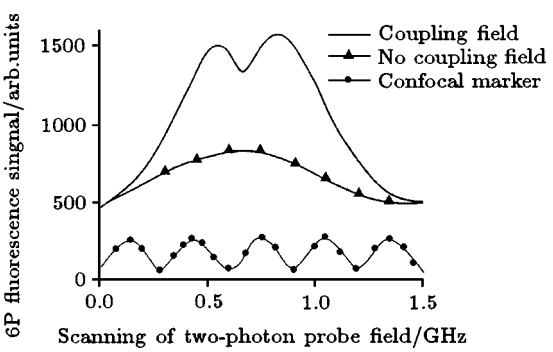


Fig.1. 6P (422 nm) fluorescence signal versus two-photon probe field in the Λ-type ^{85}Rb system with coupling field $\Omega_{32} \approx$190MHz (solid line) and without coupling field (marked by triangles). A 300MHz confocal signal is also present as a frequency reference.

In this paper, we calculate the effect of FWM on EIT in the rubidium $5S_{1/2}$-$5P_{3/2}$-$5D_{5/2}$ Λ-type system with a two-photon probe field as sketched in Fig.2. Obviously, the generated FWM field, coupling field and two-photon probe field should be considered together in the study. It is found that the FWM field can cause the asymmetry of EIT profile. The calculated result is consistent with the experimental result in Ref.[6].

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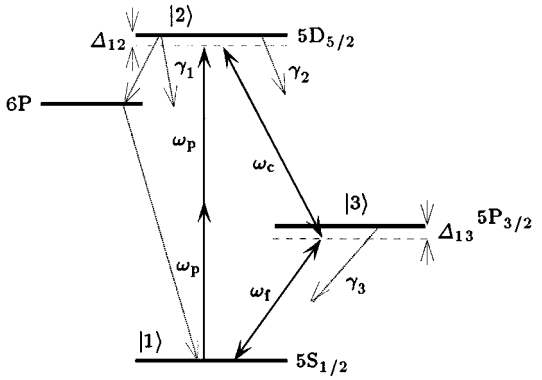


Fig.2. Rubidium $5S_{1/2}$-$5P_{3/2}$-$5D_{5/2}$ three-level Λ-type scheme.

2. The model and basic equations

The calculations are aimed at Fulton's experimental conditions so that the calculated results can be compared with their experimental results. The rubidium $5S_{1/2}$-$5P_{3/2}$-$5D_{5/2}$ Λ-type system as sketched in Fig.2 is considered, in which the two-photon probe field $\hbar\omega_p$ with Rabi frequency Ω_{21} couples with the $5S_{1/2}$-$5D_{5/2}$ transition, the coupling field $\hbar\omega_c$ with Rabi frequency Ω_{32} is resonant with the $5P_{3/2}$-$5D_{5/2}$ transition and the generated FWM field $\hbar\omega_f$ with Rabi frequency Ω_{31} couples with the $5S_{1/2}$-$5P_{3/2}$ transition. Δ_{ij} represents the detunings of laser frequency and FWM field frequency from the corresponding atomic resonances. The interaction Hamiltonian can be expressed as

$$\begin{aligned} H =&\hbar\Omega_{21}e^{i\Delta_{12}t}|2\rangle\langle 1| + \hbar\Omega_{32}e^{i\Delta_{23}t}|2\rangle\langle 3| \\ &+ \hbar\Omega_{31}e^{i\Delta_{13}t}|3\rangle\langle 1| + \text{c.c.}, \end{aligned} \tag{1}$$

where $\Delta_{12} = \omega_{12} - 2\omega_p$, $\Delta_{23} = \omega_{32} - \omega_c$, $\Delta_{13} = \omega_{31} - \omega_f$. The Liouville equation can be expressed as

$$\begin{aligned} \frac{\partial\rho_{nm}(t)}{\partial t} =& -\frac{i}{\hbar}\sum_k H_{nk}(t)\rho_{km}(t) \\ &+ \frac{i}{\hbar}\sum_k \rho_{nk}(t)H_{km}(t) + \Lambda\rho_{nm}, \end{aligned} \tag{2}$$

where ρ_{nm} is the density matrix element in the interaction picture, and Λ represents the phenomenologically added decay term corresponding to all the incoherent processes. The system is assumed to be closed, i.e. $\rho_{11}+\rho_{22}+\rho_{33}=1$, and thus reduce the number of equations by one. With the rotating wave approximation, we can obtain the following motion equations

$$\begin{aligned} \frac{\partial\rho_{22}}{\partial t} =& i\Omega_{32}\rho_{23} - i\Omega_{32}\rho_{32} + i\Omega_{21}\rho_{21} \\ &- i\Omega_{21}\rho_{12} - (\gamma_1+\gamma_2)\rho_{22}, \end{aligned} \tag{3a}$$

$$\begin{aligned} \frac{\partial\rho_{33}}{\partial t} =& i\Omega_{32}\rho_{32} - i\Omega_{32}\rho_{23} + i\Omega_{31}\rho_{31} \\ &- i\Omega_{31}\rho_{13} + \gamma_2\rho_{22} - \gamma_3\rho_{33}, \end{aligned} \tag{3b}$$

$$\begin{aligned} \frac{\partial\rho_{23}}{\partial t} =& i\Omega_{32}\rho_{22} - i\Omega_{32}\rho_{33} - i\Omega_{23}\rho_{23} \\ &+ \Lambda_{32}\rho_{23} + i\Omega_{31}\rho_{21} - i\Omega_{21}\rho_{13}, \end{aligned} \tag{3c}$$

$$\begin{aligned} \frac{\partial\rho_{32}}{\partial t} =& -i\Omega_{32}\rho_{22} + i\Omega_{32}\rho_{33} + i\Omega_{23}\rho_{32} \\ &+ \Lambda_{32}\rho_{32} - i\Omega_{31}\rho_{12} + i\Omega_{21}\rho_{31}, \end{aligned} \tag{3d}$$

$$\begin{aligned} \frac{\partial\rho_{12}}{\partial t} =& -2i\Omega_{21}\rho_{22} - i\Omega_{21}\rho_{33} - i\Omega_{31}\rho_{32} \\ &+ i\Omega_{32}\rho_{13} + i\Omega_{21} + i\Delta_{12}\rho_{12} + \Lambda_{21}\rho_{12}, \end{aligned} \tag{3e}$$

$$\begin{aligned} \frac{\partial\rho_{21}}{\partial t} =& 2i\Omega_{21}\rho_{22} + i\Omega_{21}\rho_{33} + i\Omega_{31}\rho_{23} \\ &- i\Omega_{32}\rho_{31} - i\Omega_{21} - i\Delta_{12}\rho_{21} + \Lambda_{21}\rho_{21}, \end{aligned} \tag{3f}$$

$$\begin{aligned} \frac{\partial\rho_{31}}{\partial t} =& i\Omega_{31}\rho_{22} + 2i\Omega_{31}\rho_{33} + i\Omega_{21}\rho_{32} \\ &- i\Omega_{32}\rho_{21} - i\Delta_{13}\rho_{31} - i\Delta_{31} + \Lambda_{31}\rho_{31}, \end{aligned} \tag{3g}$$

$$\begin{aligned} \frac{\partial\rho_{13}}{\partial t} =& -i\Omega_{31}\rho_{22} - 2i\Omega_{31}\rho_{33} - i\Omega_{21}\rho_{23} \\ &+ i\Omega_{32}\rho_{12} + i\Delta_{13}\rho_{13} + i\Delta_{31} + \Lambda_{31}\rho_{13}, \end{aligned} \tag{3h}$$

where $\gamma_i (i = 1, 2, 3)$ is the spontaneous emission rate of each level. The coherence-damping coefficient γ_{ij} due to spontaneous emission is given by

$$\Lambda_{32} = -\frac{1}{2}(\gamma_1 + \gamma_2 + \gamma_3),$$

$$\Lambda_{21} = -\frac{1}{2}(\gamma_3 + \gamma_2),$$

$$\Lambda_{31} = -\frac{1}{2}\gamma_3.$$

The stationary solution satisfying $\frac{\partial\rho}{\partial t}$=0 can be analytically found in the weak-field limit and numerically calculated.

3. Asymmetry of EIT profile

In Fulton's experiment, the incoherent fluorescence, which is proportional to the population in the $5D_{5/2}$ level, was observed. The presence of EIT was inferred by the observation of the dip in the fluorescence signal, so we calculate the profile of the population in $5D_{5/2}$ level.

Considering the probe field of Rabi frequency Ω_{21} and FWM field of Rabi frequency Ω_{31} are much weaker

than the coupling field of Rabi frequency Ω_{32}, we apply perturbation theory to simply the Eq.(3f) and Eq.(3g) for the off-diagonal elements and obtain

$$\frac{\partial \rho_{21}}{\partial t} = (\Lambda_{21} - \mathrm{i}\Delta_{12})\rho_{21} - \mathrm{i}\Omega_{21} - \mathrm{i}\Omega_{32}\rho_{31}, \quad (4)$$

$$\frac{\partial \rho_{31}}{\partial t} = (\Lambda_{31} - \mathrm{i}\Delta_{13})\rho_{31} - \mathrm{i}\Omega_{31} - \mathrm{i}\Omega_{23}\rho_{21}. \quad (5)$$

In deriving Eq.(4), we have neglected the terms $2\mathrm{i}\Omega_{21}\rho_{22}$, $\mathrm{i}\Omega_{21}\rho_{33}$ and $\mathrm{i}\Omega_{31}\rho_{23}$. For the same reason the terms $2\mathrm{i}\Omega_{31}\rho_{33}$, $\mathrm{i}\Omega_{31}\rho_{22}$ and $\mathrm{i}\Omega_{21}\rho_{32}$ have also been neglected in deriving Eq.(5). The perturbative stationary state solutions for ρ_{21} and ρ_{31} to the first order can be deduced as

$$\rho_{21} = \frac{\mathrm{i}\Omega_{21}(\Lambda_{31} - \mathrm{i}\Delta_{13}) - \Omega_{31}\Omega_{32}}{(\Lambda_{21} - \mathrm{i}\Delta_{12})(\Lambda_{31} - \mathrm{i}\Delta_{13}) + \Omega_{32}^2},$$

$$\rho_{31} = \frac{\mathrm{i}\Omega_{31}(\Lambda_{21} - \mathrm{i}\Delta_{12}) - \Omega_{21}\Omega_{32}}{(\Lambda_{21} - \mathrm{i}\Delta_{12})(\Lambda_{31} - \mathrm{i}\Delta_{13}) + \Omega_{32}^2}. \quad (6)$$

Inserting Eq.(6) and its complex components into Eqs.(3a)–(3d), we obtain the solution for the stationary state population ρ_{22} to the second order

$$\rho_{22} = k\mathrm{Im}\left(\frac{\mathrm{i}\Omega_{21}^2(\Lambda_{31} - \mathrm{i}\Delta_{13}) + \mathrm{i}\Omega_{31}^2(\Lambda_{21} - \mathrm{i}\Delta_{12}) - 2\Omega_{21}\Omega_{31}\Omega_{32}}{T}\right), \quad (7)$$

$$k = \frac{2\Lambda_{32}\Omega_{32}^2}{(\gamma_1 + \gamma_2)(\Lambda_{32}^2 + \Delta_{23}^2) + 2\Lambda_{21}\Omega_{32}^2},$$

$$T = \Omega_{32}^2 + (\Lambda_{31} - \mathrm{i}\Delta_{13})(\Lambda_{21} - \mathrm{i}\Delta_{12}).$$

The first term $\mathrm{i}\Omega_{21}^2(\Lambda_{31} - \mathrm{i}\Delta_{13})/T$ in Eq.(7) represents two-photon absorption for the probe field $\hbar\omega_{\mathrm{p}}$, the second term $\mathrm{i}\Omega_{31}^2(\Lambda_{21} - \mathrm{i}\Delta_{12})/T$ describes one-photon absorption for the FWM field $\hbar\omega_{\mathrm{f}}$, and the final term $-2\Omega_{21}\Omega_{31}\Omega_{32}/T$ is the three-photon interaction term. From the following calculation, we will see the final term causes the asymmetry of EIT profile.

In Fulton's experiment, the two-photon probe field, the coupling field and the FWM field are co-propagating. Therefore, we consider the Doppler broadening effect in the calculation as the following

$$\Delta_{12} \to \Delta_{12} + \frac{\omega_{12}v}{c},$$

$$\Delta_{23} \to \Delta_{23} + \frac{\omega_{32}v}{c},$$

$$\Delta_{13} \to \Delta_{13} + \frac{\omega_{31}v}{c},$$

where v is the atomic velocity. We then integrate Eq.(7) and obtain velocity-averaged ρ_{22} as a function of probe detuning. The parameters used in calculation are Ω_{21}=0.1MHz, γ_1=0MHz, γ_2=1MHz and γ_3=5MHz, ω_{12}=25693.7cm^{-1}, ω_{32}=12890cm^{-1} and ω_{31}=12820cm^{-1}.

Figure 3 (a) shows the calculated symmetry of EIT profile when the FWM field is ignored (the second term $\mathrm{i}\Omega_{31}^2(\Lambda_{21} - \mathrm{i}\Delta_{12})/T$ and the third term $-2\Omega_{21}\Omega_{31}\Omega_{32}/T$ are ignored). Figure 3 (b) shows the asymmetry of EIT profile in the presence of FWM field. If only the three-photon interaction term is ignored in Eq.(7), the symmetry profile of EIT will reappear as shown in Fig.3 (c). It is obvious that the asymmetry of EIT profile is caused by the three-photon interaction term $-2\Omega_{21}\Omega_{32}\Omega_{31}/T$ in Eq.(7).

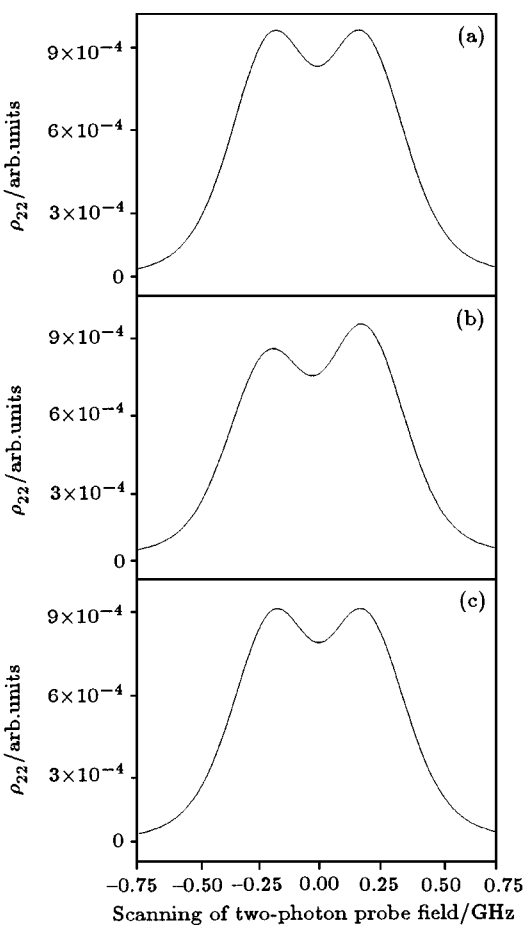


Fig.3. Analytically calculated population of $|2\rangle$ versus the detuning of two-photon probe field. (a) Ω_{31}=0MHz; (b) Ω_{31}=0.01MHz; (c) Ω_{31}=0.01MHz, but the three photon interaction term is ignored.

Figure 1 shows the Fulton's experimental result. The EIT profile (solid line) shows the asymmetry. It is found that the calculated result as shown in Fig.3 (b) is in good agreement with the experimental result.

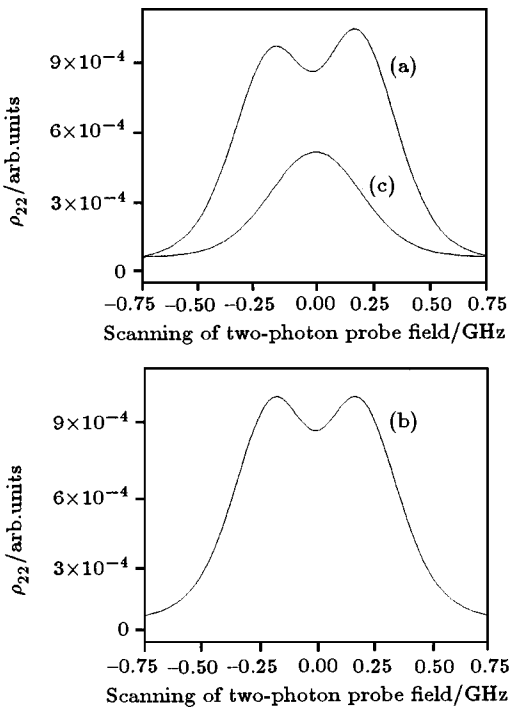


Fig.4. Numerically calculated population of $|2\rangle$ versus the detuning of two-photon probe field. (a) Ω_{32}=190MHz, Ω_{31}=0.01MHz; (b) Ω_{32}=190MHz, Ω_{31}=0MHz; (c) Ω_{32}=0MHz, Ω_{31}=0MHz.

To prove analytically that the calculated asymmetrical profile is not due to the assumption in deriving Eq.(7), the numerical calculation for Eq.(3) is performed. Figure 4 shows the numerically calculated results. When the generated field Ω_{31} is considered and ignored, the EIT profiles exhibit the asymmetry and symmetry as shown in Fig.4 (a) and Fig.4 (b), respectively. We can see clearly that the numerically calculated results for EIT profiles in Fig.4 are consistent with the analytically calculated results in Fig.3 (a) and (b) as well as the experimental result in Fig.1 (solid line).

The calculated maximum of the population intensity ρ_{22} for the case of no coupling field is about one half that in the presence of the coupling field, as shown in Fig.4 (a) and (c), which is well consistent with Fulton's experimental result in Fig.1. The reason for this is that the population of level $|2\rangle$ decaying into the level $|3\rangle$ can be re-excited back to the level $|2\rangle$ as the coupling field is on.

4. Conclusion

We have theoretically studied the effect of the FWM field on EIT in the rubidium $5S_{1/2}$-$5P_{3/2}$-$5D_{5/2}$ Λ-level scheme experimented by Fulton *et al.*[6] It is found that the FWM field can cause the asymmetry of the EIT profile, which is in good agreement with the experimental result of Ref.[6].

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