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Controlling hyperchaos in erbium-doped fibre laser

Zhang Sheng-Hai(张胜海)† and Shen Ke(沈 柯)

Department of Physics, Changchun University of Science and Technology, Changchun 130022, *China*

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The dual-ring erbium-doped fibre laser shows a hyperchaotic behaviour under some conditions. The hyperchaotic behaviour can be well controlled to enter into periodicity by modulating the pumping in one of the two rings. The period is different for different modulation index at the same modulation frequency, or for different modulation frequency at the same modulation index.

Keywords: hyperchaos, controlling hyperchaos, dual-ring erbium-doped fibre laser, Lyapunov exponent

PACC: 0545, 4255

1. Introduction

Erbium-doped fibre lasers appear very attractive for long haul communications over optical fibres as the lasing wavelength about 1.55 μm is near the minimum attenuation and dispersion point of a standard single-mode optical fibre. During the previous 30 years, many researchers have studied the erbium-doped fibre lasers.[1,2] We know that a chaotic signal is a very good carrier of information for its enormous bandwidth; meanwhile, chaotic signals used to mask the information also have an important application in secure communication. Many scientists have carried out theoretical and experimental investigations of the dynamics in erbium-doped fibre lasers.[3−11] Van Wiggeren and Roy[12,13] first realized secure communication experimentally by using chaos of erbium-doped fibre lasers. Our group has investigated the inverse synchronization of chaos in a dual-ring erbium-doped fibre laser via mutual coupling.[14] A hyperchaos system is a system which has two positive Lyapunov exponents at least; in other words, it is unstable in two directions at least. A secure communication which uses a hyperchaotic signal as a carrier can strengthen its security. It is very important to study the hyperchaos in erbium-doped fibre lasers. However, only a few researchers have investigated the hyperchaos in erbium-doped fibre laser by studying the delayed differential equation mode.[7] Especially, few researchers study the control of the hyperchaos in erbium-doped fibre laser as a general low-dimensional system. Since Ott *et al*[15] presented the method of controlling chaos in 1990, controlling chaos or hyperchaos has been an important aspect in the chaos field,[16−18] and many experts have successfully controlled the chaos in CO_2 laser,[19,20] semiconductor laser[21,22] and neodymium-doped yttrium aluminium garnet laser[23] using different methods. We have also successfully controlled the chaos in the dual-ring erbium-doped fibre laser using a delayed feedback method.[24] In this paper, we investigate the hyperchaos in a dual-ring erbium-doped fibre laser under some conditions, and this hyperchaotic behaviour can be controlled to enter into a stable periodic state by modulating the pumping in one of the two rings.

2. Hyperchaos in a dual-ring erbium-doped fibre laser

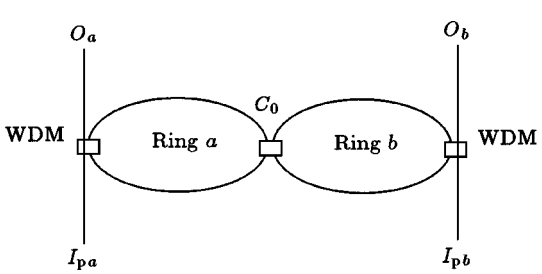


Fig.1. Dual-ring doped-erbium fibre laser system. C_0 is coupler; WDM denotes wavelength division multiplexer; I_{pa} and I_{pb} are the pump intensities.

The dual-ring fibre laser system is shown in Fig.1. In both rings a and b, each contains a wavelength-division-multiplexing (WDM) coupler and an erbium-

†E-mail address: zhangshenghai999@sohu.com

doped fibre, and the two rings are coupled with each other by a coupler C_0. I_{pa} and I_{pb} are the pump intensities of ring a and ring b respectively. When the lasing field in the two rings are frequency-locked through the coupler C_0 with a phase change $\pi/2$ from one ring to the other, the equations for the fundamental system are the following[11]

$$\frac{\mathrm{d}E_a}{\mathrm{d}\tau} = -k_a(E_a + C_0E_b) + g_aE_aD_a, \qquad (1)$$

$$\frac{\mathrm{d}E_b}{\mathrm{d}\tau} = -k_b(E_b - C_0E_a) + g_bE_bD_b, \qquad (2)$$

$$\frac{\mathrm{d}D_a}{\mathrm{d}\tau} = -(1 + I_{pa} + E_a^2)D_a + I_{pa} - 1, \qquad (3)$$

$$\frac{\mathrm{d}D_b}{\mathrm{d}\tau} = -(1 + I_{pb} + E_b^2)D_b + I_{pb} - 1, \qquad (4)$$

where E_a and E_b are the lasing fields, and D_a and D_b are the population inversions in rings a and b respectively; k_a and k_b are the decay rates and g_a and g_b are the gain coefficients of the lasing fields of rings a and b respectively. Here, we have $\tau = \gamma t$, and in an erbium-doped fibre laser, $\gamma = 1/\tau_2$, with τ_2 being the decay time of the lasing upper level, which is ~ 10 ms. So we take τ_2=10 ms.

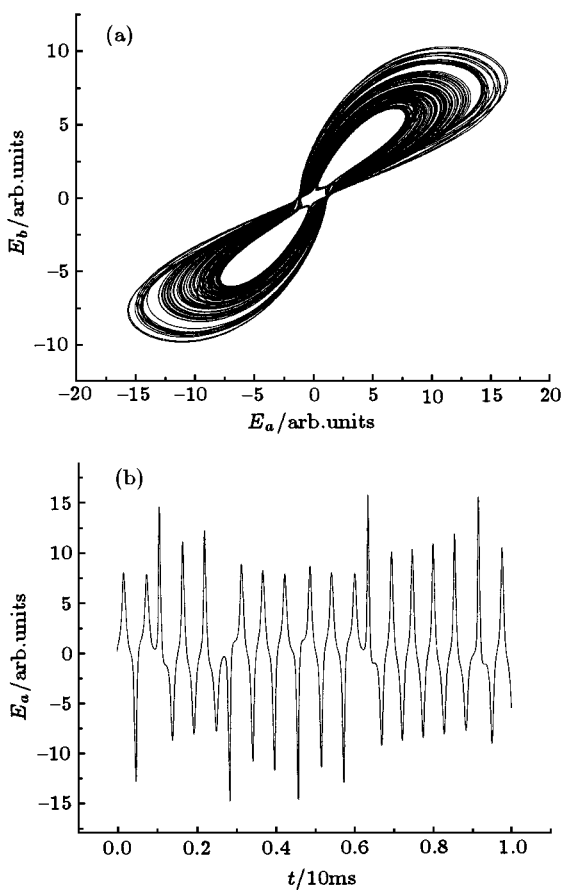


Fig.2. The strange attractor projection on plane E_a–E_b and the time series of E_a. The parameters are: k_a, k_b = 1000; g_a = 10500; g_b = 4700; C_0 = 0.2; I_{pa}, I_{pb}=4.

To study the dynamics of the system, we solve Eqs.(1)–(4) numerically using the fourth Runge–Kutta algorithms. The parameters in Eqs.(1)–(4) are listed as follows:[11] k_a=k_b=1000; g_a=10500, g_b=4700; C_0=0.2; I_{pa}=I_{pb}=4. Figure 2(a) shows the strange attractor of chaos or hyperchaos projected on to the E_a–E_b plane; however, we cannot tell whether this attractor is chaotic or hyperchaotic. We calculate the Lyapunov exponent spectrum of the dual-ring erbium-doped fibre laser system using the algorithms presented by Wolf *et al.*[25] The Lyapunov exponent spectrum is $(11.0443, 0.0264, -49.4674, -69.2164)$ correspond to the directions of E_a, E_b, D_a and D_b (the Lyapunov spectra in the next sections also correspond to the directions of E_a, E_b, D_a and D_b). The system lies in hyperchaotic state as there exist two positive Lypunov exponents. Figure 2(b) shows the time series of lasing field E_a, and one can find that the value seems random, but it is different from those noise signals without rules and seems to change following some regularity.

3. Controlling the hyperchaos in an erbium-doped fibre laser

In order to control the hyperchaos in an erbium-doped fibre laser, we modulate the pump intensity I_{pb} of ring b as follows

$$I_{pb} = I_{pb0}(1 + m\cos\omega t), \qquad (5)$$

where I_{pb0} is the original pump intensity of ring b, m is the modulation index, $\omega = 2\pi f$, with f being the modulation frequency. m, f, and I_{pb0} can be taken as the modulation parameters. Here we only modulate m and f as an example.

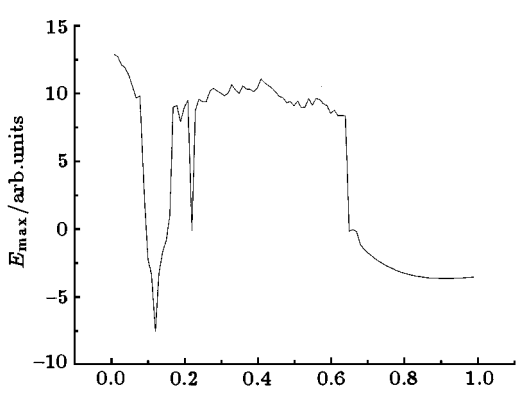


Fig.3. The leading Lyapunov exponent λ_{max} as a function of the modulation index m. The parameters are the same as in Fig.2, except I_{pb}, I_{pb0} = 4; f=4 kHz. g_a = 10500; g_b = 4700; C_0 = 0.2; I_{pa}, I_{pb}=4.

We calculate numerically the leading Lyapunov exponent of the system when f=4 kHz, other parameters being the same as in Fig.2 except I_{pb}, and we take $I_{pb0} = 4$. Figure 3 shows the leading Lyapunov exponent as a function of the modulation index m. We find that there appear two "valleys" in the range of $0.09 < m < 0.16$ and near $m = 0.22$ respectively, and the leading Lyapunov exponent is negative when m is in these two ranges. Especially, when $m > 0.655$, the leading Lyapunov exponent is negative and is almost constant. If the modulation index takes the value in these three ranges, the dual-ring erbium-doped fibre laser would be in a periodic state.

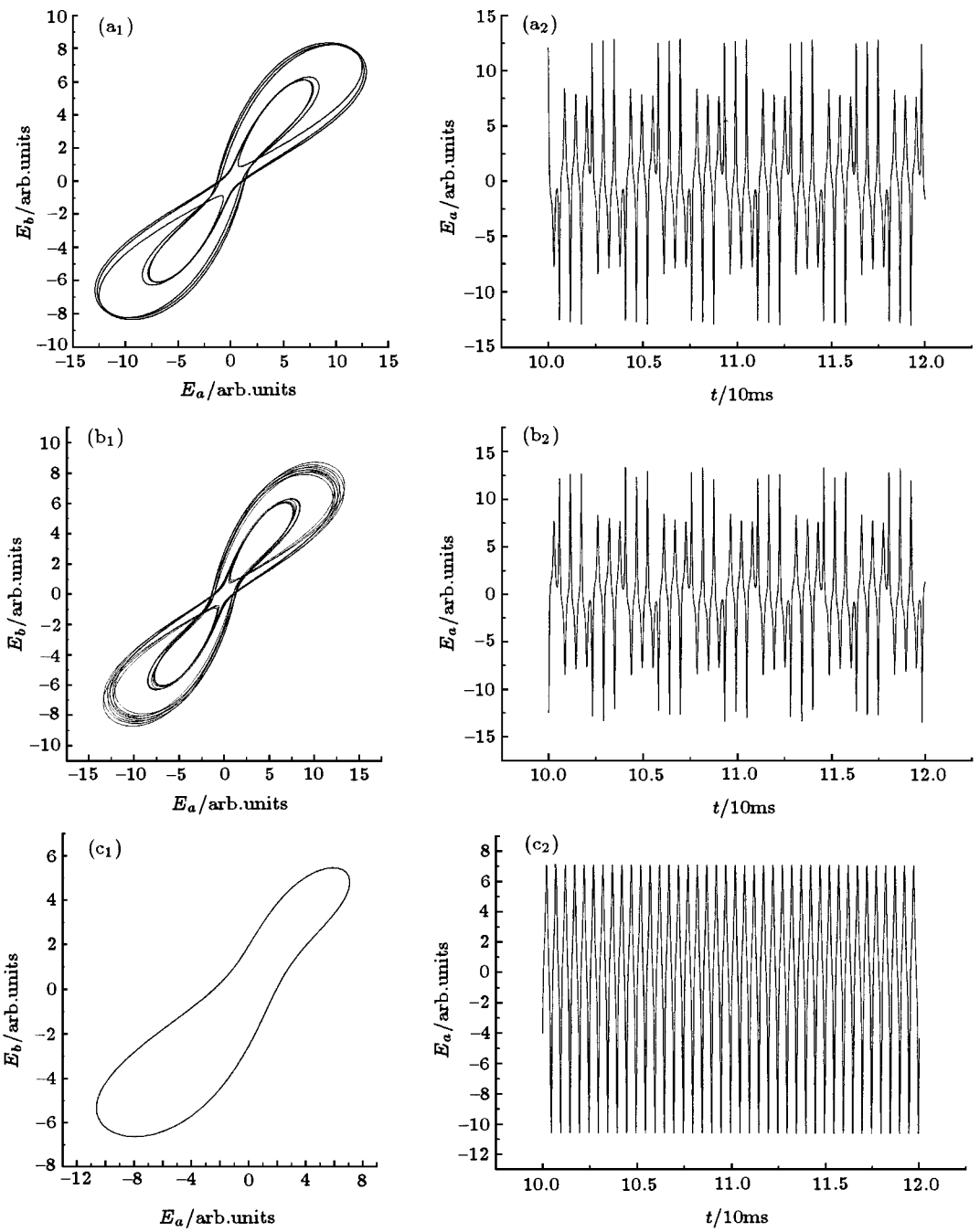


Fig.4. The attractor projection on plane $E_a - E_b$ and the time series of E_a at different modulation indices. The parameters are the same as in Fig.2 except I_{pb}, f=4 kHz and the modulation indices are (a) 0.13, (b) 0.15, (c) 0.70.

The Lyapunov exponent spectra are respectively $(-3.2605,\ -4.0446,\ -41.3214,\ -59.8482)$, $(-8.3760,\ -8.3045, -42.3252, -56.8475)$ and $(-1.7720, -1.7585,\ -36.5688,\ -77.2479)$ when the modulation indices m take values as 0.13, 0.15, 0.70, indicating that the dual-ring erbium-doped fibre laser is in the periodic state. Figure 4 shows the attractor projection on the $E_a - E_b$ plane and the time series of the lasing field E_a. One can find that the system is in periods $7, 21, 1$ (P7, P21, P1) respectively when $m = 0.13$, 0.15, 0.70. By detailed investigation of the behaviour of the attractor when m is in the range of $0.09 < m < 0.16$ and $m > 0.6548$, we find that the period number changes with m when $0.09 < m < 0.16$, which is indicated in Figs.4(a) and 4(b); however, when $m > 0.655$, the system is always in period 1. In one word, one can force the system to enter into a periodic state from the hyperchaotic state by modulating the pumping intensity. The system may be in different periodic states under different modulation indices with the same modulation frequency. In our method, we use a strong modulation to change the system from the hyperchaotic state into a new periodic state which is not the original unstable periodic state embedded in chaotic orbits. This is different from the approach presented by Ott *et al.*[15] In their method, one of the unstable periodic orbit embedded in the hyperchaotic orbit is stabilized using weak periodic perturbations, and the chaotic state is controlled to enter into an unstable periodic state. The dual-ring erbium-doped fibre laser can be controlled into a periodic state if the modulation strength is in the range $0.09 < m < 0.16$, which should be easily realized in experiment in the future. Of course, the realization in experiment may be difficult for $m > 0.65$.

It should be pointed out that when $m = 0.15$, the second Lyapunov exponent (-8.3045) is larger than the first one (-8.3760) (in general, the first one is larger than the second one). The reason may be that the system is more stable in direction E_a than in direction E_b at this modulation index.

Secondly, we investigate the behaviour of the dual-ring erbium-doped fibre laser with a modulation index $m = 0.1$ and a variational modulation frequency. The parameters are the same as in Fig.3 except m and f. Figure 5 shows the leading Lyapunov exponent as a function of modulation frequency. One can find that the leading Lyapunov exponent is always positive except for the five "valleys", where the modulation frequencies are close to 1.8, 3.5, 5.1, 6.7 and 8.4 kHz respectively. There exists no negative leading Lyapunov exponent in the range of $f > 8.4$ kHz. In other words, the hyperchaotic erbium-doped fibre dual-ring laser can be controlled into periodic state under a given modulation frequency (near 1.8, 3.5, 5.1, 6.7 and 8.4 kHz) at the modulation index $m = 0.1$; however, if the modulation frequency is larger than 8.4 kHz, the laser cannot be controlled into a periodic state and it is always in a chaotic state.

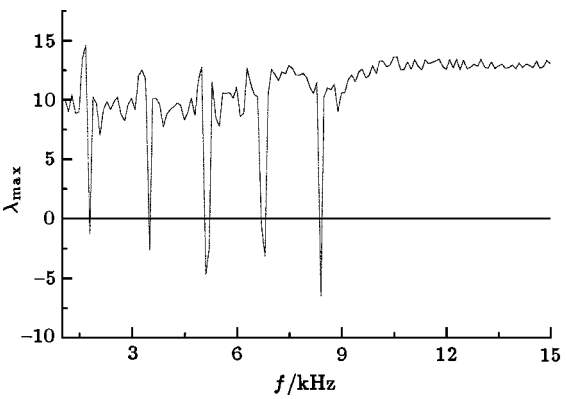


Fig.5. The leading Lyapunov exponent $\lambda_{\max}$ as a function of the modulation frequency f. The parameters are the same as in Fig.3 except f and m, m=0.1.

Figure 6 shows that the laser is controlled into P1 and P5 at the modulation frequencies $f = 6.7$, 8.4 kHz respectively. The laser can also be controlled into P1 if $f = 1.8$, 3.5, 5.1 kHz, which are similar to Figs.6 (a) and 6(b).

4. Conclusion

The dual-ring erbium-doped fibre laser shows a hyperchaotic behaviour with the parameters we have chosen. We can control the hyperchaos to enter into a stable period state by modulating the pumping in one of the two rings in a dual-ring erbium-doped fibre laser. The pumping modulation can be achieved by modulating index m, frequency f or pumping intensity I_{pb0}. This is easily realized by modulating the electric current intensity in the diode laser used as a pump. For a given modulation index (for example, $m = 0.1$), the dual-ring erbium-doped fibre laser can be controlled into a periodic state only at a given modulation frequency (for example, 1.8, 3.5, 5.1, 6.7 or 8.4 kHz), and for a given modulation frequency (for example, 4 kHz). The laser can also be controlled into different periodic states in several ranges of modulation indices.

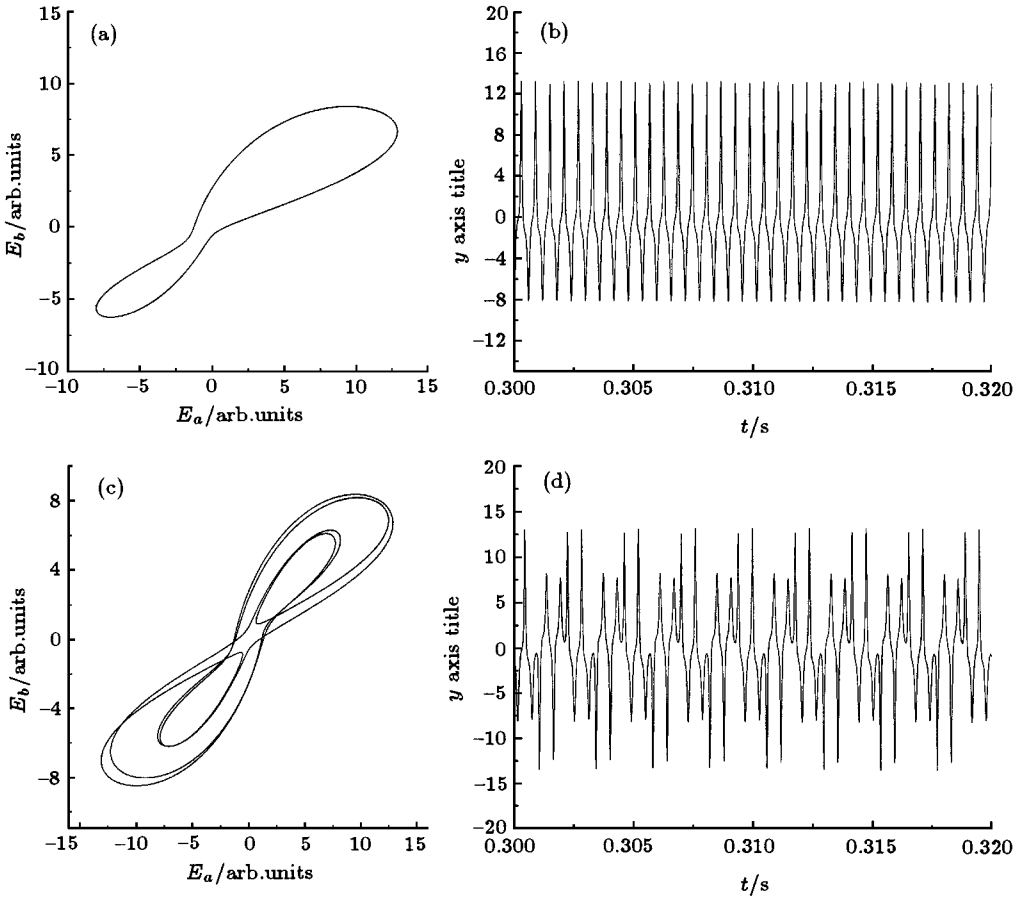


Fig.6. The attractor projection on plane E_a–E_b and the time series of E_a at different modulation frequencies. The parameters are the same as in Fig.5. (a) 6.7 kHz, (b) 8.4 kHz.

References

[1] Leboudec P *et al* 1993 *Opt. Quantum Electron.* **25** 501

[2] Leboudec P *et al* 1993 *Opt. Quantum Electron.* **25** 359

[3] Sanchez F *et al* 1995 *IEEE. J. Quantum Electron.* **31** 481

[4] Lacot E, Stoeckel F and Chenvier M 1994 *Phys. Rev.* A **49** 3997

[5] Sanchez F and Stephan G 1996 *Phys. Rev.* E **53** 2110

[6] Williams Q L, Garca-Ojalvon J and Roy R 1997 *Phys. Rev.* A **55** 2386

[7] Thernburg K S, Möller M Jr and Roy R 1997 *Phys. Rev.* E **55** 3865

[8] Pessina E M *et al* 1997 *Phys. Rev.* A **56** 4086

[9] Luo L G, Tee T J and Chu P L 1998 *Opt. Commun.* **146** 151

[10] Abarbanel H D I *et al* 1999 *Phys. Rev.* A **60** 2360

[11] Luo L G, Tee T J and Chu P L 1998 *J. Opt. Soc. Am.* B **15** 972

[12] Van Wiggeren G H and Roy R 1998 *Phys. Rev. Lett.* **81** 3547

[13] Van Wiggeren G H and Roy R 1998 *Science* **279** 1198

[14] Wang R and Shen K 2001 *Chin. Phys.* **10** 711

[15] Ott E, Grebogi C and Yorke J A 1990 *Phys. Rev. Lett.* **64** 1196

[16] Ciofini M, Meucci R and Arecchi F T 1995 *Phys. Rev.* E **52** 94

[17] Luo X S 2001 *Chin. Phys.* **10** 17

[18] Li Z and Han C Z 2001 *Chin. Phys.* **10** 494

[19] Labate A, Ciofini M and Meucci R 1998 *Phys. Rev.* E **57** 5320

[20] Gu C M and Shen K 1999 *Acta Phys. Sin.* (Overseas Edition) **8** 733

[21] Li H J and Chern J L 1996 *Phys. Rev.* E **54** 2118

[22] Li G H, Zhou S P and Xu D M 2002 *Acta Phys. Sin.* **51** 736 (in Chinese)

[23] Colet P and Braiman Y 1996 *Phys. Rev.* E **53** 200

[24] Wang R and Shen K 2001 *Acta Phys. Sin.* **50** 1024 (in Chinese)

[25] Wolf A *et al* 1985 *Physica* D **16** 285