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Study of the stability of polarization mode dispersion in fibre*

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Polarization mode dispersion (PMD) is the ultimate limitation to high bit-rate fibre communication system. The stability of PMD is very important to its measurement and compensation. This paper puts forward a method to measure the stability of PMD by measuring the stability of the state of polarization (SOP) and introduces the conception of time evolution vector (TEV) of SOP. We observe the fact that the regularity of the principal state of polarization changing with time is the same as other SOPs', if we neglect the dependence of TEV on wavelength. We also measure the SOP's stability of some fibres with different lengths, and obtain results of PMD changing with time.

Keywords: polarization mode dispersion, state of polarization, stability, time evolution vector
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1. Introduction

Polarization mode dispersion (PMD) can broaden the optical pulse and cause extra bit-errorrate (BER), so it is a severely limiting factor in the high-rate optical fibre communication systems over 10 Gb/s, especially in communication systems over 40 Gb/s. As a result, PMD compensation has become a major research subject worldwide. Based on the concept of the principal state of polarization (PSP), Poole has suggested the differential group delay (DGD) of the PSP as the measurement of the PMD.[1] PSP is a special pair of states of polarization (SOP), which does not change with optical frequency on the condition of the first order approximation. But SOP is unstable under random changes.

It is well known that the so-called "single-mode" fibres are, in fact, "two-mode" fibres because two different SOPs exist due to birefringence caused by internal and external stress or by imperfect circularity of fibre core. The field of normal fibre can be written as [2]

$$E(x,y,z) = c_x(z)e_x\exp(\mathrm{j}\beta_x z) + c_y(z)e_y\exp(\mathrm{j}\beta_y z), \quad (1)$$

here $e_x \neq e_y$, β_x and β_y are propagation constant of the two modes. The $c_x(z)$ and $c_y(z)$ can be determined by the equation of mode-coupling. From this equation, we see that the amplitude and phase of SOP $E(x,y,z)$ will change continuously along the fibre, appearing as linear SOP, circular SOP and elliptical SOP. This only shows the SOP distribution along the longitudinal direction, and we still do not know the change of SOP with time at a given position. If β_x is equal to $\beta_y, c_x(z)$, and $c_y(z)$ does not change with time, the SOP at the same position is stable. Due to the effect of time, temperature, environment and techniques of fibre fabrication, SOP is generally unstable. For example, as temperature rises, the difference of thermal expansion coefficients will bring about a thermal-stress at the transverse sections of the fibre, which induces stress birefringence by opto-elastic effect. The formula can be written as [3]

$$\Delta\beta = \frac{1}{2}kn^3(p_{11}-p_{12})N\frac{1+v_\mathrm{p}}{1-v_\mathrm{p}} \times (S_2-S_1)\Delta T\frac{a-b}{a+b}, \quad (2)$$

where n is the refractive index of the fibre, N is the normalized effective refractive index of propagating

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mode, p_{11}=0.12 and p_{12}=0.27 are opto-elastic coefficients of SiO_2, v_p is the Poisson ratio of SiO_2, S_2 and S_1 are the thermal expansion coefficients of core material and cladding material, and ΔT is the difference between fibre fabricating and environment temperatures. From this formula, we know that environment temperature alteration will change the birefringence of fibre, leading to the SOP instability.

Another factor is mode coupling, which means energy exchange between two propagating modes. Mode coupling results from longitudinal inhomogeneity, which is sensitive to environment, for instance microbend, twist, and lateral stress. In a word, we cannot get a stable SOP in reality because of these two factors (mode coupling and birefringence).

Obviously, investigating the stability of SOP is the basis of the stability of PMD research. Both the PMD compensation and the PMD measurement need the knowledge of the stability of PMD. As it is difficult to measure the stability of PMD, we solve this problem by studying the stability of SOP.

2. Relationship between the stability of SOP and the stability of PMD

As proven, PMD can be expressed by the PMD vector.[4] If $\boldsymbol{S}_{\rm out} = M\boldsymbol{S}_{\rm in}$, where $\boldsymbol{S}_{\rm out}$, $\boldsymbol{S}_{\rm in}$ and M are the Stocks vector of the input SOP, the Stocks vector of the output SOP and the Miller matrix of the fibre, respectively. The PMD vector Ω can then be written as

$$\frac{\partial \boldsymbol{S}}{\partial \omega} = \Omega \times \boldsymbol{S},$$

where

$$\Omega\times = \frac{\partial M}{\partial \omega} M^{-1}. \tag{3}$$

The direction of Ω is the direction of the Stocks vector corresponding to PSP; its magnitude is the DGD. If we keep input SOP stable, we get a slowly changing output SOP, which means that the Miller matrix $\boldsymbol{M}$ is a slowly changing function of time, namely $\boldsymbol{S}_{\rm out}(t) = M(t)\boldsymbol{S}_{\rm in}$. Then PMD vector $\Omega(t)$ is also a slowly changing function of time. Following the definition of the PMD vector, we introduce a new vector $\boldsymbol{T}$, which denotes a SOP changing with time:

$$\frac{\partial \boldsymbol{S}}{\partial t} = \boldsymbol{T} \times \boldsymbol{S}, \tag{4}$$

$\boldsymbol{T}\times = \dfrac{\partial M}{\partial t} M^{-1} = \boldsymbol{T}$ is the time evolution vector (TEV). Assuming $M(t)$ to be a slowly changing function, the second derivative to time is close to zero. We take $\boldsymbol{T}$ as a constant vector, whose magnitude shows the speed of change and its direction is the axis of SOP rotation. We do a time derivative to Eq.(3) and a frequency derivative to Eq.(4). Assuming these equations to be equal, we get

$$\frac{\partial \Omega}{\partial t} \times \boldsymbol{S} + \Omega \times \frac{\partial \boldsymbol{S}}{\partial t} = \frac{\partial \boldsymbol{T}}{\partial \omega} \times \boldsymbol{S} + \boldsymbol{T} \times \frac{\partial \boldsymbol{S}}{\partial \omega}. \tag{5}$$

Simplifying Eq.(5) using Eqs.(3) and (4), we have

$$\frac{\partial \Omega}{\partial t} \times \boldsymbol{S} = \frac{\partial \boldsymbol{T}}{\partial \omega} \times \boldsymbol{S} + \boldsymbol{S} \times (\Omega \times \boldsymbol{T}). \tag{6}$$

We assume any arbitrary Stocks vector $\boldsymbol{S}$ must satisfy this equation, so

$$\frac{\partial \Omega}{\partial t} = \frac{\partial \boldsymbol{T}}{\partial \omega} + (\boldsymbol{T} \times \Omega). \tag{7}$$

$\boldsymbol{T}$ is the axis of rotation of SOP changing with time. Because it has little correlation to optical frequency, its magnitude is approximately zero. So we get an important equation:

$$\frac{\partial \Omega}{\partial t} \approx \boldsymbol{T} \times \Omega. \tag{8}$$

Equation (8) shows that, even though requiring only input SOP, not the input PSP, to be kept constant, the latter being difficult to attain in practice, we can nevertheless conclude that on condition that the dependence of TEV on wavelength can be neglected, the PSP varies with time according to the same law as other SOPs do. It should be noted that in a short period of time, Ω only rotates around $\boldsymbol{T}$ but its magnitude does not change. In other words, in a short period, the value of PMD (i.e. the time delay between two PSPs) is stable, only the mode of PSP (circular, elliptical or linear) changes. This is very important. It follows that in designing a compensator consisting of two parts, one for transforming the PSP to the state of orthogonal linear polarization, the other for compensating the time delay, the latter part needs hardly any adjustment in short term, while the former can be adjusted under the guidance of Eq.(8). It can be further concluded that the magnitude of $\boldsymbol{T}$ represents the rate of change of PSP, which rotates around $\boldsymbol{T}$ as the axis.

Because $\boldsymbol{T}\times = \dfrac{\partial M}{\partial t} M^{-1} = T$, we have

$$\frac{\partial \Omega}{\partial t} \approx T\,\Omega. \tag{9}$$

Its solution is

$$\Omega(t) \approx \exp(Tt)\Omega_0, \tag{10}$$

where Ω_0 is a constant vector. So based on the $\boldsymbol{S}(t)$ and $\dfrac{\partial \boldsymbol{S}}{\partial t}$, we get $\boldsymbol{T}$, and sequentially find $\Omega(t)$.

3. Experiment

The experimental arrangement is shown in Fig.1, where a distributed feedback (DFB) laser at wavelength 1550 nm is used as the light source with output power 400 μW. Polarizer and analyser are made from the Green–Tailor prism. To get a stable input SOP, we make the laser beam pass through the polarizer to attain the linear SOP. By adjusting the polarization controller (PC) we get maximal optical power from the polarizer. The output SOP can be checked using the quarter-wave plate, analyser and power meter.

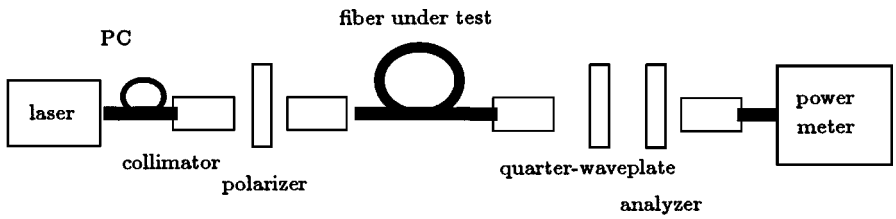

Fig.1. Experiment setup.

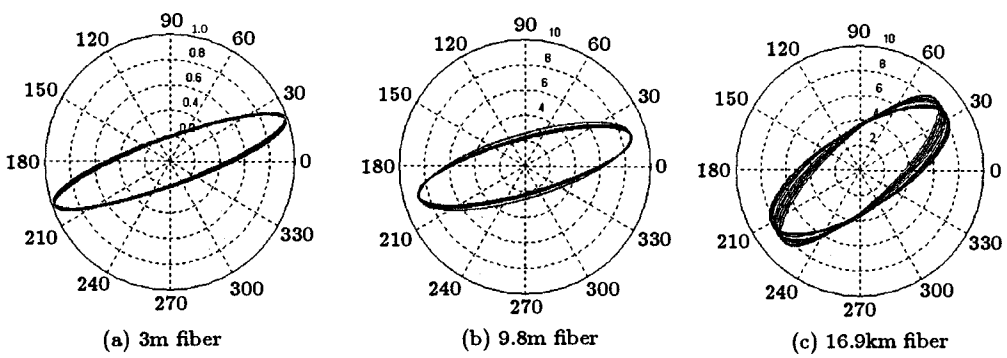

Fig.2. Different length fibre's output SOP evolution.

The procedure of checking the SOP is as follows: after removing the quarter-wave plate, we adjust the optical axis of the polarizer to the vertical position and record the value I_1 from the power meter. Then adjust the optical axis to the horizontal position, and to positions at $+45^\circ$ and -45° to the vertical position, respectively, record the values I_2, I_3 and I_4. Then fit in the quarter-wave plate and set its fast axis at positions $+45^\circ$ and -45° to the vertical position, and record the values I_5 and I_6 respectively. From these data we calculate the four components of the Stocks vector of the output SOP:

$$s_0 = I_1 + I_2, \quad s_1 = I_1 - I_2, \quad s_2 = I_3 - I_4,$$
$$s_3 = I_5 - I_6.$$

Before the experiment we need to make fine adjustment using the micro-adjustor carefully. The criteria are that after rotating the analyser full round and recording serial values, we get a plot of approximately a figure of eight. Finally, after setting the quarter-wave plate, we use the same criteria to monitor the correctness of the result. Theoretically it should satisfy the equations

$$\begin{aligned} S_0 =& I(0^\circ, 0^\circ) + I(90^\circ, 90^\circ) \\ =& I(0^\circ, 45^\circ) + I(0^\circ, 135^\circ) \\ =& I(45^\circ, 45^\circ) + I(135^\circ, 135^\circ). \end{aligned} \tag{11}$$

In reality all the experimental data cannot fit Eq.(11) fully. The smaller the difference between three groups is, the more precise experimental data are. We must point out that the delay from the quarter-wave plate is affected by temperature. Of course we can omit this effect by assuming constant temperature in the lab. We measure four pieces of G.652 fibre of different lengths, namely 3 m, 9.8 m, 12.7 km and 16.9 km. The 3 m and 9.8 m fibres are kept unaffected; the oth-

ers are tightly wound on a drum 160 mm in diameter. We do not apply any pressure to the fibre during experiment, only record the output SOP changing with time. The 3 m, 9.8 m and 16.9 km samples are measured every ten minutes for two hours. We plot the 12 output elliptical SOP as shown in Fig.2. (Radial coordinate denotes the optical power, its unit has been normalized.)

From Fig.2, SOP in long fibre changes more notably than in short fibre. We calculated values of the rate of change of three elliptical SOP's direction angle to be 0.0003, 0.00047 and 0.00191 rad/min for the 3 m, 9.8 m and 16.9 km fibres, respectively.

Figure 3 shows the change of the component of SOP. From the figure, we can see that all components change regularly with duration of time. SOP slowly rotates around the centre (see Fig.2), the change of phase difference results from the change of the two components of refractive index. Changes of x component and y component roots in the mode coupling. Theoretically, short fibre has little mode coupling, which can be omitted in our discussion. But we cannot omit it for the long fibre. Figure 3 shows that the x component is complementary with the y component: as it increases the other decreases. This proves that energy exchange happens due to mode coupling. As our lab's environment is slowly changing (only temperature alters), without outside interference applied, all components in Fig.3 slowly change.

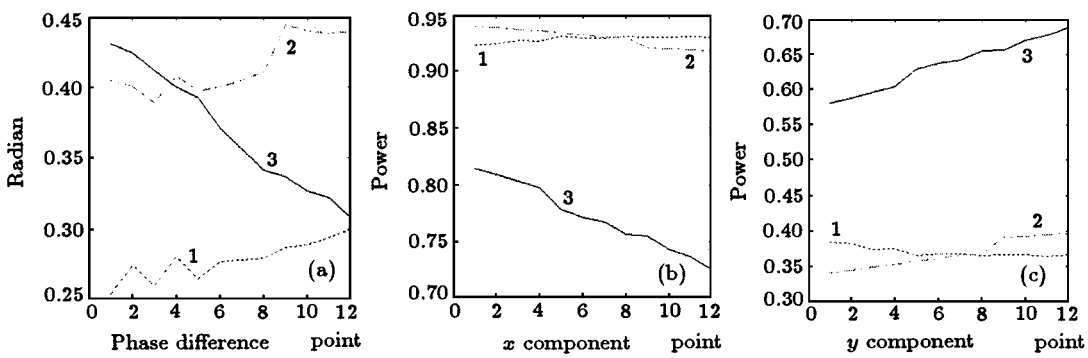

Fig.3. Component of different length fibre's SOP evolution (line 1:3 m fibre, line 2:9.8 m fibre, line 3:16.9 km fibre).

Change of Stocks vector of the three samples are plotted on Fig.4. The 3 m fibre's Stocks vector little changes. The 9.8 m fibre's Stocks vector has a slow change, only between point 8 and point 10 there is a break point, which may be caused by accidental disturbance. Breaks are also perceptible in the corresponding SOP curves in Fig.3. Obviously the 16.9 km fibre changes more seriously than other fibre. All the experiments were done in summer, so the long fibre has a drastic change as temperature varies. If we had controlled the temperature during measurement to reduce the temperature effect, we believe that change of output SOP would be slow.

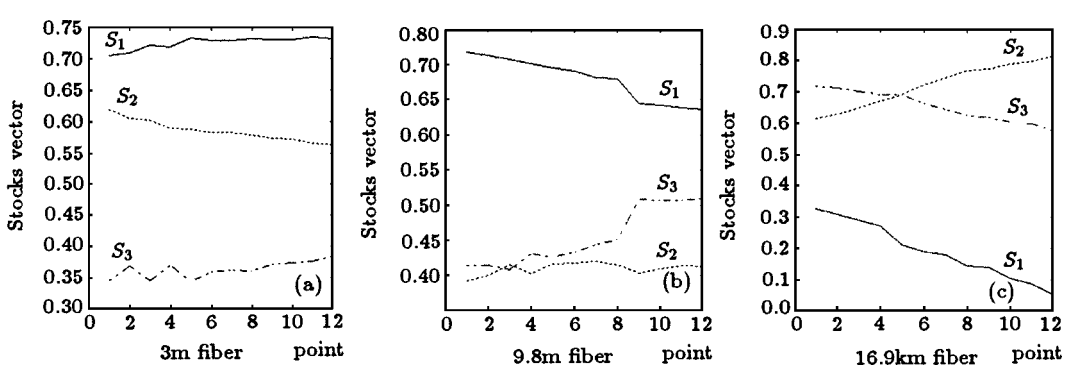

Fig.4. Stocks vector's component of different length fibre's SOP evolution.

On 30 August we measured a 12.7 km long fibre from 9:30 to 11:40. During the experiment the temperature rose from 21°C to 21.7°C. The result is shown in Fig.5(a), which is similar to that obtained before. Figure 5(b) is the result measured from 19:45 to 21:35 on the same day. The temperature was constant at 27°C, so the output SOP had little change. It confirmed that the temperature affects the output SOP, which was also confirmed by other measurements. Figure 5(c) is the result measured with the same fibre from 8:30 to 22:30 on 3 September.

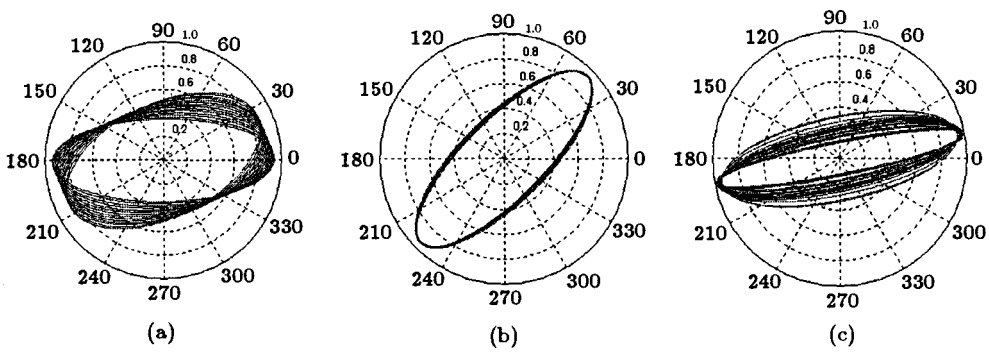

Fig.5. Different length fibre's SOP evolution measured at different time.

Figure 6(a)–(c) is the Stocks vector figure corresponding to Fig.5(a)–(c). Solid line denotes S_1, dashed line denotes S_2, dash-dotted line denotes S_3. Fig.6(c) has some fluctuations due to temperature change at point 8 and point 14. Points from 1 to 7 were measured in the morning, points from 8 to 13 were done in the afternoon, while the remaining points were measured at night. Temperature changed during measurement, as the temperature dropped from 24.9°C at point 13 to 24.6°C at point 14, we see a break point of output SOP. In additions, we can find that both point 14 and point 4 have the same SOP, because temperature of point 4 was also 24.6°C. This is another example for temperature affecting the output SOP.

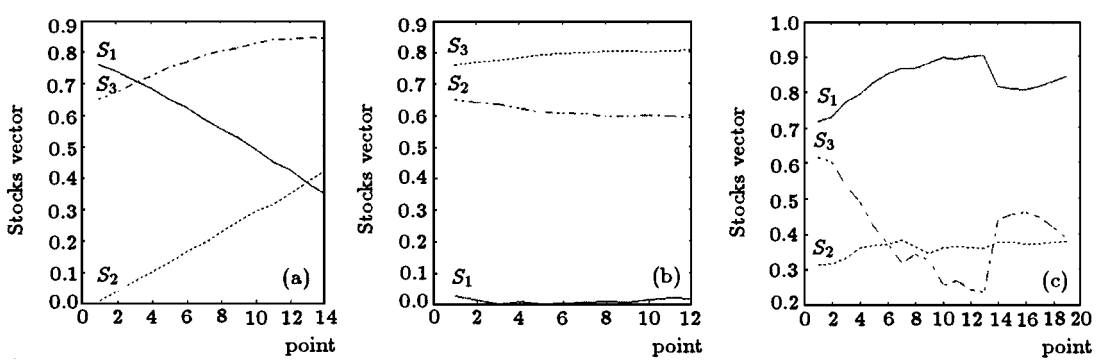

Fig.6. Same fibre's Stocks vector's component evolution at different time.

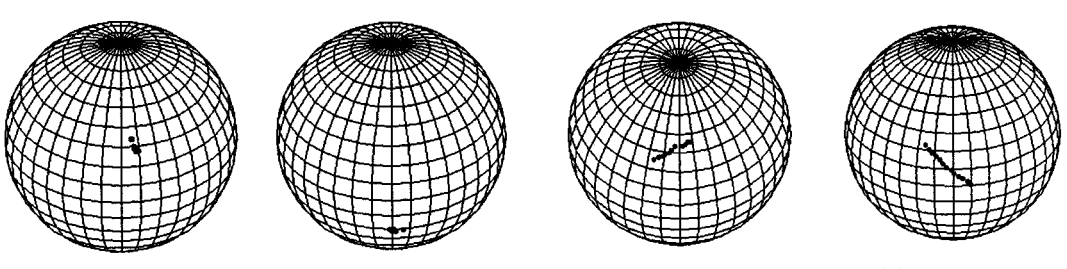

Fig.7. Stocks vector's component evolution on Poincaré Sphere.

The SOPs of all the measured fibres are plotted on a Poincaré sphere (Fig.7). It is clear that output SOP of short fibre has little change while great change takes place in the long fibre. We calculate TEV $\boldsymbol{T}$ of all fibre samples using Eq.(4). The results are given in Table 1.

Table 1. TEV value of fibre of different lengths.

length/m	3	9.8	16900	12700
$\|T\|/(1/\mathrm{h})$	0.1032	0.0702	0.2094	0.2892

We can judge the change speed of PMD vector by TEV value. In the lab's environment TEV is 10^{-1} per hour.

4. Conclusion

PMD is the ultimate limitation to high bit-rate fibre communication system; the stability of PMD is vital to PMD measurement and compensation. In this paper, we discussed the cause of PMD instability, analysed the relationship between SOP stability and PMD stability and introduced the concept of TEV of SOP. We have proved that both PSP and the other SOP (keeping the input SOP stable) have the same varying law with time on condition of neglecting the dependence of TEV on wavelength, and made SOP stability measurements using samples of fibres of different lengths. We obtained the result that the time rate of change of PMD vector is 10^{-1} per hour.

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