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Double-wave description of mesoscopic resistance-inductance-capacitance coupled circuit with power source*

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Quantum fluctuations in the mesoscopic capacitance-inductance-resistance coupled circuit with a power source are investigated using canonical transformation and a double wavefunction. We confirm that the fluctuations are not influenced by the power source. As a new method, the double wavefunction describes a single system of the coupled circuit, whereas the single wavefunction describes a quantum ensemble.

Keywords: mesoscopic circuit, double wavefunction, canonical transformation
PACC: 0365, 0250

1. Introduction

In recent years, the rapid development of nanotechnology has made it possible to minimize the integrated circuit and components towards atomic dimensions. It is well known that when the scale of fabricated electric materials reach the dimension of the Fermi wavelength, the quantum effect of the circuit or component must be considered. So research on the fluctuations of the mesoscopic circuit can provide theoretical guidance for the designing of micro-circuits and minimization of the noise in them.

Since 1973, when Louisell[1] discussed the quantum effect of the LC (inductance-capacitance) circuit and calculated the quantum fluctuations of the vacuum state, many papers[2−10] have been published concerning the quantum effect of the mesoscopic circuit. In recent years, the coupled circuit has aroused increasing concern. Liang[11] discussed the fluctuations in a damped inductance-coupled circuit according to the mechanism of the generation of resistance. We also[12,13] studied the quantum effect of the mesoscopic RL and RLC coupled circuits, by the method of canonical transformation and unitary transformation respectively.

In this paper, we investigate the fluctuations of the mesoscopic RLC coupled circuit with a power source using the quantum double wavefunction. The double wavefunction is a new theory developed in recent years and has been used successfully in treating problems of harmonic oscillators,[14] and Long *et al*[15] have applied this theory to LC mesoscopic circuits. In Section 2, we build the canonical transformation matrices which transform the Hamiltonian of the RLC (resistance-inductance-capacitance) coupled circuit with power source to a system composed of two linear independent harmonic oscillators. In Section 3, we introduce the quantum double wavefunction and calculate the values of charges and currents which are related to initial phases of the circuit. In Section 4, we obtain the quantum fluctuations of the circuit by calculating the statistic average values of the ensemble. These statistic average values of the ensemble are just the same as those calculated with the single wavefunction.

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2. Canonical transformation

We consider two loops of an RLC circuit that are coupled via inductance, capacitance and resistance as shown in Fig.1. There is a power source in the left side loop of the circuit.

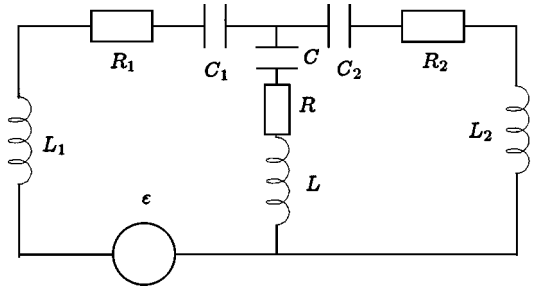

Fig.1. Diagram of mesoscopic resistance-capacitance-inductance coupled circuit.

The classical equation of motion for charges can be obtained from Kirchoff's law as

$$L_1\frac{\mathrm{d}^2q_1}{\mathrm{d}t^2}+R_1\frac{\mathrm{d}q_1}{\mathrm{d}t}+\frac{q_1}{C_1}+L\left(\frac{\mathrm{d}^2q_1}{\mathrm{d}t^2}-\frac{\mathrm{d}^2q_2}{\mathrm{d}t^2}\right)+R\left(\frac{\mathrm{d}q_1}{\mathrm{d}t}-\frac{\mathrm{d}q_2}{\mathrm{d}t}\right)+\frac{(q_1-q_2)}{C}=\varepsilon(t), \tag{1}$$

$$L_2\frac{\mathrm{d}^2q_2}{\mathrm{d}t^2}+R_2\frac{\mathrm{d}q_2}{\mathrm{d}t}+\frac{q_2}{C_2}-L\left(\frac{\mathrm{d}^2q_1}{\mathrm{d}t^2}-\frac{\mathrm{d}^2q_2}{\mathrm{d}t^2}\right)-R\left(\frac{\mathrm{d}q_1}{\mathrm{d}t}-\frac{\mathrm{d}q_2}{\mathrm{d}t}\right)-\frac{(q_1-q_2)}{C}=0, \tag{2}$$

where q_i $(i=1,2)$ represents electric charges, L_i inductances and C_i capacitances of the loops, and L, C and R the coupling inductance, capacitance and resistance of the mesoscopic circuit, respectively.

To simplify the problem, we suppose that

$$\frac{R_1}{L_1}=\frac{R_2}{L_2}=\frac{R}{L}\equiv\gamma. \tag{3}$$

The equations of motion, Eqs.(1) and (2), can be formulated from the following Hamiltonian:

$$\begin{aligned}H_q=&\mathrm{e}^{-\gamma t}\left[\frac{p_1^2}{2L_1}+\frac{p_2^2}{2L_2}-\frac{1}{2}\frac{LL_1L_2}{LL_1+LL_2+L_1L_2}\left(\frac{p_1}{L_1}-\frac{p_2}{L_2}\right)^2\right]\\&+\mathrm{e}^{\gamma t}\left(\frac{q_1^2}{2C_1}+\frac{q_2^2}{2C_2}+\frac{(q_1-q_2)^2}{2C}\right)-\mathrm{e}^{\gamma t}\varepsilon(t)q_1+\delta(t),\end{aligned} \tag{4}$$

where $\delta(t)$ is a parameter related to time. An arbitrary $\delta(t)$ satisfies Eqs.(1) and (2).Meanwhile we note that q_i is substituted for coordinates and its conjugate variables p_i for momenta.

To simplify Hamiltonian (4), we transform the variables (q_i,p_i) to the variables (Q_i,P_i) by the following relations:

$$\begin{pmatrix}Q_1\\Q_2\end{pmatrix}=\mathrm{e}^{\frac{\gamma}{2}t}\begin{pmatrix}\mu\cos\frac{\varphi}{2}&-\nu\sin\frac{\varphi}{2}\\\mu\sin\frac{\varphi}{2}&\nu\cos\frac{\varphi}{2}\end{pmatrix}\begin{pmatrix}q_1\\q_2\end{pmatrix}+\mathrm{e}^{\frac{\gamma}{2}t}\begin{pmatrix}f_1(t)\\f_2(t)\end{pmatrix}, \tag{5}$$

$$\begin{pmatrix}P_1\\P_2\end{pmatrix}=\mathrm{e}^{-\frac{\gamma}{2}t}\begin{pmatrix}\frac{1}{\mu}\cos\frac{\varphi}{2}&-\frac{1}{\nu}\sin\frac{\varphi}{2}\\\frac{1}{\mu}\sin\frac{\varphi}{2}&\frac{1}{\nu}\cos\frac{\varphi}{2}\end{pmatrix}\begin{pmatrix}p_1\\p_2\end{pmatrix}+\mathrm{e}^{\frac{\gamma}{2}t}\begin{pmatrix}\frac{\gamma}{2}\alpha_1&0\\0&\frac{\gamma}{2}\alpha_2\end{pmatrix}\times\left[\begin{pmatrix}\mu\cos\frac{\varphi}{2}&-\nu\sin\frac{\varphi}{2}\\\mu\sin\frac{\varphi}{2}&\nu\cos\frac{\varphi}{2}\end{pmatrix}\begin{pmatrix}q_1\\q_2\end{pmatrix}+\begin{pmatrix}f_1(t)\\f_2(t)\end{pmatrix}\right]+\begin{pmatrix}g_1(t)\\g_2(t)\end{pmatrix}, \tag{6}$$

where $\mu,\nu,\alpha_1,\alpha_2$ and φ are given by

$$\mu=\sqrt[4]{\frac{L_1/C-L/C_1}{L_2/C-L/C_2}}, \tag{7}$$

$$\nu=\sqrt[4]{\frac{L_2/C-L/C_2}{L_1/C-L/C_1}}, \tag{8}$$

$$\alpha_1=\frac{LL_1+LL_2+L_1L_2}{(L+L_1)\nu^2\sin^2\frac{\varphi}{2}+(L+L_2)\mu^2\cos^2\frac{\varphi}{2}-\mu\nu L\sin\varphi}, \tag{9}$$

$$\alpha_2=\frac{LL_1+LL_2+L_1L_2}{(L+L_1)\nu^2\cos^2\frac{\varphi}{2}+(L+L_2)\mu^2\sin^2\frac{\varphi}{2}+\mu\nu L\sin\varphi}, \tag{10}$$

$$\tan\varphi = \frac{2\mu\nu/C}{(1/C+1/C_1)\nu^2-(1/C+1/C_2)\mu^2} = \frac{2L\mu\nu}{(L+L_1)\nu^2-(L+L_2)\mu^2}, \tag{11}$$

and $f_1(t), f_2(t), g_1(t)$, and $g_2(t)$ are induced parameters which depend on the initial conditions of the circuit and are determined by the following expressions:

$$\mathrm{e}^{\frac{\gamma t}{2}} f_i'(t) - \frac{1}{\alpha_i} g_i(t) = 0, \quad (i=1,2) \tag{12}$$

$$\left(\beta_1 + \frac{\alpha_1\gamma^2}{4}\right) f_1(t) + \frac{1}{2}\alpha_1\gamma f_1'(t) + g_1'(t)\mathrm{e}^{-\frac{\gamma t}{2}} + \frac{\varepsilon(t)}{\mu}\cos\frac{\varphi}{2} = 0, \tag{13}$$

$$\left(\beta_2 + \frac{\alpha_2\gamma^2}{4}\right) f_2(t) + \frac{1}{2}\alpha_2\gamma f_2'(t) + g_2'(t)\mathrm{e}^{-\frac{\gamma t}{2}} + \frac{\varepsilon(t)}{\mu}\sin\frac{\varphi}{2} = 0, \tag{14}$$

$$\sum_{i=1}^{2}\left[\frac{\mathrm{e}^{\gamma t}}{2}\left(\beta_i + \frac{\alpha_i\gamma^2}{4}\right) f_i^2(t) + \frac{g_i^2(t)}{2\alpha_i} - \mathrm{e}^{\frac{\gamma t}{2}} f_i'(t) g_i(t)\right] + \frac{\varepsilon(t)}{\mu}\mathrm{e}^{\gamma t}\left[f_1(t)\cos\frac{\varphi}{2} + f_2(t)\sin\frac{\varphi}{2}\right] + \delta(t) = 0. \tag{15}$$

To make sure that the transformation Eqs.(5) and (6) are canonical transformation (that is, (Q_i, P_i) are canonical variables), these variables (q_i, p_i) and (Q_i, P_i) in the two representations must satisfy the following relation:

$$\sum_{i=1}^{2} P_i\dot{Q}_i - H_Q = \sum_{i=1}^{2} p_i\dot{q}_i - H_q + \frac{\mathrm{d}F}{\mathrm{d}t}, \tag{16}$$

where the generating function F is a time-dependent function in phase space. Because the variables (Q_i, P_i) are mutually independent, by comparing the coefficients of $(\dot{Q}_i, \dot{P}_i)$ on two sides of Eq.(16), we can obtain the following relations:

$$P_j - \sum_{i=1}^{2} p_i\frac{\partial q_i}{\partial Q_j} = \frac{\partial F}{\partial Q_j}, \tag{17}$$

$$-\sum_{i=1}^{2} p_i\frac{\partial q_i}{\partial P_j} = \frac{\partial F}{\partial P_j}, \tag{18}$$

and

$$H_Q = H_q - \sum_{i=1}^{2} p_i\frac{\partial q_i}{\partial t} - \frac{\partial F}{\partial t}. \tag{19}$$

From Eqs.(5), (6), (17) and (18), the generating function F is calculated as

$$F = \sum_{i=1}^{2}\left\{\frac{\gamma}{4}\alpha_i Q_i^2 + g_i(t)Q_i\right\}. \tag{20}$$

Substituting Eqs.(4)–(6) and (20) into Eq.(19), we obtain the Hamiltonian represented by (Q_i, P_i) as

$$H_Q = \sum_{i=1}^{2}\left\{\frac{P_i^2}{2\alpha_i} + \frac{\beta_i}{2}Q_i^2\right\}, \tag{21}$$

where

$$\beta_1 = \frac{\cos^2\frac{\varphi}{2}}{\mu^2}\left(\frac{1}{C}+\frac{1}{C_1}\right) + \frac{\sin^2\frac{\varphi}{2}}{\nu^2}\left(\frac{1}{C}+\frac{1}{C_2}\right) + \frac{\sin\varphi}{C\mu\nu} - \frac{\alpha_1}{4}\gamma^2, \tag{22}$$

$$\beta_2 = \frac{\sin^2\frac{\varphi}{2}}{\mu^2}\left(\frac{1}{C}+\frac{1}{C_1}\right) + \frac{\cos^2\frac{\varphi}{2}}{\nu^2}\left(\frac{1}{C}+\frac{1}{C_2}\right) - \frac{\sin\varphi}{C\mu\nu} - \frac{\alpha_2}{4}\gamma^2. \tag{23}$$

From Eq.(21), we can see that the mesoscopic RLC coupled circuit with source is equivalent to a system of two linear independent harmonic oscillators. According to the standard quantization principle, a pair of the observable quantities Q_i and P_i can be written as a pair of linear Hermitian operators, namely $\hat{Q}_i$ and $\hat{P}_i$. For the quantum version, we can reexpress Eq.(21) as

$$\hat{H}_Q = \sum_{i=1}^{2}\left[\frac{\hat{P}_i^2}{2\alpha_i} + \frac{1}{2}\alpha_i\omega_i^2\hat{Q}_i^2\right], \tag{24}$$

where

$$\omega_i = \sqrt{\frac{\beta_i}{\alpha_i}}. \tag{25}$$

3. Double-wave description of mesoscopic RLC coupled circuit with a power source

Because the Hamiltonian of the coupled circuit has been written as of two independent harmonic oscillators Eq.(24), by solving the Schrödinger equation, we can obtain the quantum energy spectrum as

$$E_{n_1,n_2} = \sum_{i=1}^{2}\left(n_i + \frac{1}{2}\right)\hbar\omega_i, \tag{26}$$

and the wavefunction of the coupled circuit in Q representation as

$$\Psi_{n_1,n_2}(Q_1,Q_2)=\psi_{n_1}(Q_1)\psi_{n_2}(Q_2), \tag{27}$$

$$\psi_{n_i}(Q_i)=N_{n_i}\mathrm{e}^{-\frac{1}{2}\lambda_i^2Q_i^2}H_{n_i}(\lambda_iQ_i), \tag{28}$$

where H_{n_i} $(n_i=1,2)$ are Hermite polynomials and N_{n_i} and λ_i are given, respectively by

$$N_{n_i}=\sqrt{\frac{\lambda_i}{\sqrt{\pi}2^{n_i}n_i!}}\ ,\quad \lambda_i=\sqrt{\frac{\alpha_i\omega_i}{\hbar}}. \tag{29}$$

Equation (27), which is the traditional wavefunction in quantum dynamics, is called the single wavefunction. According to the quantum double-wave theory, the state of coupled circuit is described by a pair of wavefunctions (called double wavefunction)

$$\begin{aligned}\Psi_{n_1,n_2}(Q_1,Q_2,t)=&\Psi_{n_1,n_2}(Q_1,Q_2)\\&\times\exp\Big\{-\mathrm{i}\Big[\Big(n_1+\frac{1}{2}\Big)\omega_1(t-t_1)\\&+\Big(n_2+\frac{1}{2}\Big)\omega_2(t-t_2)\Big]\Big\},\end{aligned} \tag{30}$$

$$\begin{aligned}\Phi(Q_1,Q_2,t)=&\sum_{n_1',n_2'}\Psi_{n_1',n_2'}(Q_1,Q_2,t)\\=&\sum_{n_1',n_2'}\Psi_{n_1',n_2'}(Q_1,Q_2)\\&\times\exp\Big\{-\mathrm{i}\Big[\Big(n_1'+\frac{1}{2}\Big)\omega_1(t-t_1)\\&+\Big(n_2'+\frac{1}{2}\Big)\omega_2(t-t_2)\Big]\Big\},\end{aligned} \tag{31}$$

where t_1 and t_2 are real parameters which are dependent on initial phase. The value of a dynamic quantity of the system at time t is given by the expression

$$\begin{aligned}\langle f(t)\rangle=&\mathrm{Re}\int_{-\infty}^{\infty}\mathrm{d}Q_1\int_{-\infty}^{\infty}\mathrm{d}Q_2\,\Phi^*(Q_1,Q_2,t)\\&\times\hat{f}\,\Psi_{n_1,n_2}(Q_1,Q_2,t).\end{aligned} \tag{32}$$

Let $f=1$ in Eq.(32), the normalization condition of the double wavefunction is obtained as

$$\begin{aligned}&\mathrm{Re}\int_{-\infty}^{\infty}\mathrm{d}Q_1\int_{-\infty}^{\infty}\mathrm{d}Q_2\,\Phi^*(Q_1,Q_2,t)\\&\times\Psi_{n_1,n_2}(Q_1,Q_2,t)=1.\end{aligned} \tag{33}$$

Consider the following relations:

$$\begin{aligned}&Q_i\psi_{n_i}(Q_i)\\=&\frac{1}{\lambda_i}\left[\sqrt{\frac{n_i}{2}}\psi_{n_i-1}(Q_i)+\sqrt{\frac{n_i+1}{2}}\psi_{n_i+1}(Q_i)\right],\end{aligned} \tag{34}$$

and

$$\begin{aligned}&P_i\psi_{n_i}(Q_i)\\=&-\mathrm{i}\hbar\lambda_i\left[\sqrt{\frac{n_i}{2}}\psi_{n_i-1}(Q_i)-\sqrt{\frac{n_i+1}{2}}\psi_{n_i+1}(Q_i)\right].\end{aligned} \tag{35}$$

The values of the canonical variables (Q_i,P_i) can be calculated as

$$\langle Q_i\rangle=\frac{1}{\lambda_i}\left[\sqrt{\frac{n_i}{2}}+\sqrt{\frac{n_i+1}{2}}\right]\cos\omega_i(t-t_i), \tag{36}$$

$$\langle P_i\rangle=\lambda_i\hbar\left[\sqrt{\frac{n_i}{2}}+\sqrt{\frac{n_i+1}{2}}\right]\sin\omega_i(t-t_i). \tag{37}$$

From Eqs.(5), (6), (36) and (37), the values of electric charges and currents of the coupled circuit are obtained as

$$\begin{aligned}\langle q_1(t)\rangle=&\frac{\mathrm{e}^{-\frac{\gamma t}{2}}}{\mu}\left[\frac{1}{\lambda_1}\left(\sqrt{\frac{n_1}{2}}+\sqrt{\frac{n_1+1}{2}}\right)\right.\\&\times\cos\frac{\varphi}{2}\cos\omega_1(t-t_1)\\&+\frac{1}{\lambda_2}\left(\sqrt{\frac{n_2}{2}}+\sqrt{\frac{n_2+1}{2}}\right)\\&\left.\times\sin\frac{\varphi}{2}\cos\omega_2(t-t_2)\right]\\&-\frac{1}{\mu}\left[f_1(t)\cos\frac{\varphi}{2}+f_2(t)\sin\frac{\varphi}{2}\right],\end{aligned} \tag{38}$$

$$\begin{aligned}\langle q_2(t)\rangle=&-\frac{\mathrm{e}^{-\frac{\gamma t}{2}}}{\nu}\left[\frac{1}{\lambda_1}\left(\sqrt{\frac{n_1}{2}}+\sqrt{\frac{n_1+1}{2}}\right)\right.\\&\times\sin\frac{\varphi}{2}\cos\omega_1(t-t_1)\\&-\frac{1}{\lambda_2}\left(\sqrt{\frac{n_2}{2}}+\sqrt{\frac{n_2+1}{2}}\right)\\&\left.\times\cos\frac{\varphi}{2}\cos\omega_2(t-t_2)\right]\\&+\frac{1}{\nu}\left[f_1(t)\sin\frac{\varphi}{2}-f_2(t)\cos\frac{\varphi}{2}\right],\end{aligned} \tag{39}$$

$$\begin{aligned}\langle p_1(t)\rangle=&\mathrm{e}^{\frac{\gamma t}{2}}\mu\cos\frac{\varphi}{2}\left(\sqrt{\frac{n_1}{2}}+\sqrt{\frac{n_1+1}{2}}\right)\\&\times\left[\frac{\alpha_1\gamma}{2\lambda_1}\cos\omega_1(t-t_1)-\lambda_1\hbar\sin\omega_1(t-t_1)\right]\\&+\mathrm{e}^{\frac{\gamma t}{2}}\mu\sin\frac{\varphi}{2}\left(\sqrt{\frac{n_2}{2}}+\sqrt{\frac{n_2+1}{2}}\right)\\&\times\left[\frac{\alpha_2\gamma}{2\lambda_2}\cos\omega_2(t-t_2)-\lambda_2\hbar\sin\omega_2(t-t_2)\right]\\&+\mathrm{e}^{\frac{\gamma t}{2}}\mu\left(g_1(t)\cos\frac{\varphi}{2}+g_2(t)\sin\frac{\varphi}{2}\right),\end{aligned} \tag{40}$$

$$\begin{aligned}\langle p_2(t)\rangle =&\mathrm{e}^{\frac{\gamma t}{2}}\nu\sin\frac{\varphi}{2}\left(\sqrt{\frac{n_1}{2}}+\sqrt{\frac{n_1+1}{2}}\right)\\&\times\left[\frac{\alpha_1\gamma}{2\lambda_1}\cos\omega_1(t-t_1)-\lambda_1\hbar\sin\omega_1(t-t_1)\right]\\&-\mathrm{e}^{\frac{\gamma t}{2}}\nu\cos\frac{\varphi}{2}\left(\sqrt{\frac{n_2}{2}}+\sqrt{\frac{n_2+1}{2}}\right)\\&\times\left[\frac{\alpha_2\gamma}{2\lambda_2}\cos\omega_2(t-t_2)-\lambda_2\hbar\sin\omega_2(t-t_2)\right]\\&+\mathrm{e}^{\frac{\gamma t}{2}}\nu\left(g_1(t)\sin\frac{\varphi}{2}-g_2(t)\cos\frac{\varphi}{2}\right).\end{aligned}\tag{41}$$

From Eqs.(38)–(41), we can see that the values of the variables (q_i, p_i) are related to the state of the coupled circuit as well as the initial phases. We can calculate the average values of the ensemble by calculating the integral to initial phases $\omega_1 t_1$ and $\omega_2 t_2$ within a cycle period. These average values of the ensemble are just the expected values solved by the single wavefunction. Therefore we confirm that the double wavefunction can describe a single system of the coupled circuit which has determinate initial phases, but the single wavefunction only describes an ensemble composed of many circuits whose initial phases distribute irregularly.

4. Quantum fluctuation

According to the double-wave theory, the statistic average value of a dynamic quantity is given by the formula

$$\overline{\langle f\rangle}=\frac{1}{(2\pi)^2}\int_0^{2\pi}\mathrm{d}\theta_1\int_0^{2\pi}\mathrm{d}\theta_2\langle f(t)\rangle,\tag{42}$$

where $\theta_1=\omega_1 t_1$ and $\theta_2=\omega_2 t_2$ are initial phases of two loops, respectively.

Substituting Eqs.(38)–(41) to Eq.(42), we obtain the following values:

$$\langle q_1\rangle=-\frac{1}{\mu}\left[f_1(t)\cos\frac{\varphi}{2}+f_2(t)\sin\frac{\varphi}{2}\right],\tag{43}$$

$$\langle q_2\rangle=\frac{1}{\nu}\left[f_1(t)\sin\frac{\varphi}{2}-f_2(t)\cos\frac{\varphi}{2}\right],\tag{44}$$

$$\langle p_1\rangle=\mathrm{e}^{\frac{\gamma t}{2}}\mu\left(g_1(t)\cos\frac{\varphi}{2}+g_2(t)\sin\frac{\varphi}{2}\right),\tag{45}$$

$$\langle p_1\rangle=\mathrm{e}^{\frac{\gamma t}{2}}\nu\left(g_1(t)\sin\frac{\varphi}{2}-g_2(t)\cos\frac{\varphi}{2}\right).\tag{46}$$

Following the same steps of calculation as above, we obtain the average values of q_1^2, q_2^2, p_1^2 and p_2^2. Noting Eq.(29), we obtain the quantum fluctuations of the coupled circuit as

$$\left(\Delta\hat{q}_1\right)^2=\mathrm{e}^{-\gamma t}\nu^2\left[(2n_1+1)\frac{\cos^2\frac{\varphi}{2}}{\omega_1\alpha_1}+(2n_2+1)\frac{\sin^2\frac{\varphi}{2}}{\omega_2\alpha_2}\right]\frac{\hbar}{2},\tag{47}$$

$$\left(\Delta\hat{q}_2\right)^2=\mathrm{e}^{-\gamma t}\mu^2\left[(2n_1+1)\frac{\sin^2\frac{\varphi}{2}}{\omega_1\alpha_1}+(2n_2+1)\frac{\cos^2\frac{\varphi}{2}}{\omega_2\alpha_2}\right]\frac{\hbar}{2},\tag{48}$$

$$\left(\Delta\hat{p}_1\right)^2=\mathrm{e}^{\gamma t}\mu^2\left[\omega_1\alpha_1(2n_1+1)\left(1+\frac{\gamma^2}{4\omega_1^2}\right)\cos^2\frac{\varphi}{2}+\omega_2\alpha_2(2n_2+1)\left(1+\frac{\gamma^2}{4\omega_2^2}\right)\sin^2\frac{\varphi}{2}\right]\frac{\hbar}{2},\tag{49}$$

$$\left(\Delta\hat{p}_2\right)^2=\mathrm{e}^{\gamma t}\nu^2\left[\omega_1\alpha_1(2n_1+1)\left(1+\frac{\gamma^2}{4\omega_1^2}\right)\sin^2\frac{\varphi}{2}+\omega_2\alpha_2(2n_2+1)\left(1+\frac{\gamma^2}{4\omega_2^2}\right)\cos^2\frac{\varphi}{2}\right]\frac{\hbar}{2}.\tag{50}$$

It can be seen from Eqs.(47)–(50) that the quantum fluctuations of electric charges and currents are related to the state, as well as the components of the circuit, but not influenced by the power source.

5. Discussion and conclusion

The above research on mesoscopic RLC coupled circuit with power source can be applied to the cases of mesoscopic RL or LC coupled circuit with a power source.

In the case $1/C=0$, the circuit in Fig. 1 becomes an inductance-resistance coupled circuit. The fluctuations of the RL coupled circuit with a power source can be obtained from Eqs.(7), (8) and (47)–(50). In the case $R=L=0$, the circuit in Fig.1 becomes a capacitance coupled circuit. We can obtain the fluctuations of the C coupled circuit with a power source in the same way.

We should mention the idea proposed by Li and Chen [10] that charge is discrete in the mesoscopic circuit. In this paper, we quantized the circuit on the basis of continuous charge. This method simplifies the quantization process. For a more realistic study, the charge discreteness should be considered.

In this paper, we investigated the quantum fluctuations of electric charges and those conjugate currents

in the mesoscopic capacitance-inductance-resistance coupled circuit on the basis of classical equation of motion and the results can be applied to RL or C coupled circuits. We built the canonical transformation matrices which transform the Hamiltonian of the coupled circuit to that of two independent linear harmonic oscillators, and thus made it possible to introduce the double wavefunction of a two-dimensional harmonic oscillator. Using this double wavefunction, we obtained the values of charges and currents in connection with the initial phases. So we confirm that the double wavefunction describes a single system of mesoscopic circuit which has determinate initial phases, while the traditional single wavefunction describes a quantum ensemble. Finally, the fluctuations are not influenced by the power source of the circuit.

References

[1] Louisell W H 1973 *Quantum Statistical Properties of Radiation* (New York: Wiley)

[2] Chen B, Li Y Q, Fang H, Jiao Z K and Zhang Q R 1995 *Phys. Lett.* A **205** 121

[3] Gu Y J 2000 *Acta Phys. Sin.* **49** 965 (in Chinese)

[4] Gu Y J 1999 *Acta Photo. Sin.* **28** 1138 (in Chinese)

[5] Wang J S and Sun C Y 1997 *Acta Phys. Sin.* **46** 2007 (in Chinese)

[6] Wang J S, Liu T K and Zhan M S 2000 *Acta Phys. Sin.* **49** 2271 (in Chinese)

[7] Ji Y H, Lei M S and Ouyang C Y 2002 *Chin. Phys.* **11** 0163

[8] Chen B, Li Y Q, Sha J and Zhang Q R 1996 *Chin. Sci. Bull.* **41** 1084

[9] Chen B and Li Y Q 1997 *Acta Phys. Sin.* **46** 129 (in Chinese)

[10] Li Y Q and Chen B 1996 *Phys. Rev.* B **53** 4027

[11] Liang M L and Yuan B 2002 *J. Beijing Inst. Technol.* **22** 727

[12] Zhang S, Choi J R, Um C I and Yeon K H 2002 *J. Korean Phys. Soc.* **40** 325

[13] Zhang S, Choi J R, Um C I and Yeon K H 2002 *Phys. Lett.* A **294** 294

[14] Huang X Y 1987 *Chin. Phys. Lett.* **4** 154

[15] Long C Y and Liu B 2001 *Acta Phys. Sin.* **50** 1011 (in Chinese)