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Fokker-planck study of tokamak electron cyclotron resonance heating*

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A Fokker–Planck study is carried out for tokamak electron cyclotron resonance heating by writing the quasi-linear diffusion operator into a form adaptive to the collision operator and considering the wave absorption characteristics of both the O-mode and the X-mode in different magnetic surfaces. Though the Fokker–Planck code is non-relativistic in nature, however, if the relativistic resonance condition for the nearly perpendicularly propagating waves is treated suitably, we can obtain correct results. The energy loss mechanism through anomalous transport is also modelled using a suitable loss term. In the heating phase, the electron distribution deviates from the Maxwellian distribution substantially, which leads to non-linear absorption characteristics. The wave damping rate is non-linear and changes with time. The electron pressure is usually anisotropic under different conditions: $p_{e\perp}/p_{e\parallel} > 2$ for different D_0 and τ_e.

Keywords: Fokker–Planck equation, electron cyclotron resonance heating, anomalous heat loss
PACC: 5250, 5255, 5265

1. Introduction

Electron cyclotron resonance heating (ECRH) is important for plasma heating and for plasma profile control in magnetic confinement devices. The usual way to do this is to employ the fundamental ordinary mode (O-mode) injection from the weak magnetic field side and the second harmonic extraordinary mode (X-mode) injection from the strong field side that is realized by some reflecting mirror.[1,2] To determine the absorption profile of the wave power, one needs to combine the ray tracing calculation and the linear wave damping calculation, and for the latter, it is enough to consider the local dispersion relation of plasma with Maxwellian distribution. This kind of analysis is simple and can be applied in interpretation of experimental results. However, the real wave absorption process is much complicated because the wave damping is mainly due to the electron-wave resonance and related to the wave field polarization on the one hand, and the electron distribution evolves nonlinearly during heating on the other. Many important aspects of the problem, such as the nonlinear wave damping, the anisotropy of the electron pressure and the form of energetic electrons are missing in the linear calculation. In this study, the tokamak ECRH is studied by the Fokker–Planck code supplemented by some subroutines describing the ECRH. The wave-induced quasi-linear diffusion coefficients are determined from analysing the wave-electron resonance condition and the fast motion of electrons along the magnetic surface. In the present form of the code, we also add an energy loss term that keeps the total particle number constant to describe some anomalous electron energy transfer found in experiments.

2. Quasi-linear diffusion operator and determination of quasi-linear diffusion coefficients

The general form of the Fokker–Planck equation for wave heating is[3]

$$\frac{\partial f_e}{\partial t} = \left(\frac{\partial f_e}{\partial t}\right)_c + Q - L, \tag{1}$$

where the three terms on the right-hand side describe the collision, wave-induced diffusion and energy loss, respectively. In Ref. [3], the nonlinear term for electron–electron and electron–ion collision is written

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in terms of the velocity variables v, θ so that it can be expanded by the Legendre polynomial, the standard form being

$$\begin{aligned}\frac{\partial f}{\partial t} =&\frac{1}{v^2}\frac{\partial}{\partial v}\left(Af + B\frac{\partial f}{\partial v}\right) + C\frac{\partial f}{\partial \theta} \\ &+ \frac{1}{v^2\sin\theta}\frac{\partial}{\partial\theta}\left(Df + E\frac{\partial f}{\partial v} + F\frac{\partial f}{\partial\theta}\right) \\ &+ Kf + J. \end{aligned} \tag{2}$$

For oblique propagating electron cyclotron waves, the general non-relativistic quasi-linear operator is written in variables $v_\parallel, v_\perp$ as

$$\begin{aligned}Q =&\frac{1}{v_\perp}\frac{\partial}{\partial v_\perp}\left[v_\perp\left(D_{22}\frac{\partial f}{\partial v_\perp} + D_{21}\frac{\partial f}{\partial v_\parallel}\right)\right] \\ &+ \frac{\partial}{\partial v_\parallel}\left[D_{12}\frac{\partial f}{\partial v_\perp} + D_{11}\frac{\partial f}{\partial v_\parallel}\right], \end{aligned} \tag{3}$$

where $D_{12} = D_{21}$. To adapt it to the Fokker–Planck code written in variables v, θ, we must make the transformation:

$$v = \sqrt{u_\parallel^2 + u_\perp^2}, \qquad \cos\theta = u_\parallel/v. \tag{4}$$

Through a lengthy manipulation, expression (3) can be rewritten as:

$$\begin{aligned}Q =&\frac{1}{v^2}\frac{\partial}{\partial v}\Big[v^2(\sin^2\theta D_{22} + \cos^2\theta D_{11} \\ &+ 2\cos\theta\sin\theta D_{12})\frac{\partial f}{\partial v} \\ &+ v(\sin\theta\cos\theta(D_{22} - D_{11}) \\ &+ (\cos^2\theta - \sin^2\theta)D_{12})\frac{\partial f}{\partial\theta}\Big] \\ &+ \frac{1}{v^2\sin\theta}\frac{\partial}{\partial\theta}\Big[v\sin\theta\{(\cos\theta\sin\theta(D_{22} - D_{11}) \\ &+ (\cos^2\theta - \sin^2\theta)D_{12})\}\frac{\partial f}{\partial v} \\ &+ \sin\theta(\cos^2\theta D_{22} + \sin^2\theta D_{11} \\ &- 2\cos\theta\sin\theta D_{12})\frac{\partial f}{\partial\theta}\Big], \end{aligned} \tag{5}$$

or

$$\begin{aligned}Q =&\frac{1}{v^2}\frac{\partial}{\partial v}\Big[v^2(\sin^2\theta D_{22} + \cos^2\theta D_{11} \\ &+ \sin 2\theta D_{12})\frac{\partial f}{\partial v} \\ &+ v\left(\frac{\sin 2\theta}{2}(D_{22} - D_{11}) + \cos 2\theta D_{12}\right)\frac{\partial f}{\partial\theta}\Big] \\ &+ \frac{1}{v^2\sin\theta}\frac{\partial}{\partial\theta}\Big[v\sin\theta\Big\{\frac{\sin 2\theta}{2}(D_{22} - D_{11}) \\ &+ \cos 2\theta D_{12}\Big\}\frac{\partial f}{\partial v} \\ &+ \sin\theta(\cos^2\theta D_{22} + \sin^2\theta D_{11} \\ &- \sin 2\theta D_{12})\frac{\partial f}{\partial\theta}\Big]. \end{aligned} \tag{6}$$

The diffusion coefficients for an oblique propagating wave are as follows[4]:

$$D_{22} = \frac{\pi}{2\omega}\left(\frac{Ze}{m}\right)^2\sum_{n,k}|d_2^n|^2\delta\left(1 - \frac{n\Omega}{\omega} - \frac{k_\parallel v_\parallel}{\omega}\right), \tag{7}$$

$$D_{12} = \frac{\pi}{2\omega}\left(\frac{Ze}{m}\right)^2\sum_{n,k}RE\{d_1^{n^*}d_2^n\}\delta\left(1 - \frac{n\Omega}{\omega} - \frac{k_\parallel v_\parallel}{\omega}\right), \tag{8}$$

$$D_{11} = \frac{\pi}{2\omega}\left(\frac{Ze}{m}\right)^2\sum_{n,k}|d_1^n|^2\delta\left(1 - \frac{n\Omega}{\omega} - \frac{k_\parallel v_\parallel}{\omega}\right), \tag{9}$$

$$\begin{aligned}d_2^n =&\frac{1}{\sqrt{2}}\left(1 - \frac{k_\parallel v_\parallel}{\omega}\right)\left[J_{n-1}\left(\frac{k_\perp v_\perp}{\Omega}\right)E_{+,k}\mathrm{e}^{-\mathrm{i}\psi_k}\right. \\ &\left. + J_{n+1}\left(\frac{k_\perp v_\perp}{\Omega}\right)E_{-,k}\mathrm{e}^{\mathrm{i}\psi_k}\right] \\ &+ \left(\frac{n\Omega}{\omega}\right)\frac{v_\parallel}{v_\perp}J_n\left(\frac{k_\perp v_\perp}{\Omega}\right)E_{\parallel,k}, \end{aligned} \tag{10}$$

$$\begin{aligned}d_1^n =&\frac{1}{\sqrt{2}}\frac{k_\parallel v_\perp}{\omega}\left[J_{n-1}\left(\frac{k_\perp v_\perp}{\Omega}\right)E_{+,k}\mathrm{e}^{-\mathrm{i}\psi_k}\right. \\ &\left. + J_{n+1}\left(\frac{k_\perp v_\perp}{\Omega}\right)E_{-,k}\mathrm{e}^{\mathrm{i}\psi_k}\right] \\ &+ \left(1 - \frac{n\Omega}{\omega}\right)J_n\left(\frac{k_\perp v_\perp}{\Omega}\right)E_{\parallel,k}. \end{aligned} \tag{11}$$

The polarization components involved should be determined by the dispersion relation. The resonant condition can be expressed as

$$v_\parallel = \left(\frac{\omega}{k_\parallel}\right)\left(\frac{R - R_\Omega}{R}\right) = \frac{c}{n_\parallel}\frac{x}{R}. \tag{12}$$

It can be seen from Eq. (12) that the absorption region is symmetric about the resonant point for the non-relativistic case and the effective width of this region is about a few $(n_\parallel v_{\mathrm{eT}}/c)R$. For the nearly perpendicular propagating case, we have $n_\parallel/n_\perp \ll 1$. However, the non-relativistic condition is $n_\parallel/n_\perp > v_{\mathrm{eT}}/c$, so that the incident angle should be around 80°. In this case, the energetic electrons can be produced in relatively distant places from the resonant region. In the above expressions, $\Omega_{\mathrm{e}} < 0$, the fundamental mode corresponds to $n = -1$, however, in the arguments of the Bessel functions we should take the absolute value of Ω.

It is noted that though the above Fokker–Planck equation is non-relativistic in nature, it is still applicable for treating relativistic absorption approximately

because the resonant absorption can be treated as process independent of the collision process. More rigorous relativistic Fokker–Planck code is being developed with the cooperation of foreign co-workers.

Polarization vector of the ECRH O-mode The polarization vector component (i.e., the E-field) for O-mode is mainly along the magnetic line of force. This means that the $E_\parallel$ component is dominant. However, for oblique propagation, there is also right circularly polarized component. In the nearly resonant region we have

$$\frac{|E_-|^2}{|E_\parallel|^2} = \frac{n_\perp^2}{4}\frac{v_{\mathrm{eT}}^2}{c^2}\left|\frac{Z'(\zeta_1)}{Z(\zeta_1)}\right|^2, \tag{13}$$

where Z is the plasma dispersion function with its argument

$$\zeta_1 = \frac{\omega-\Omega}{k_\parallel v_{\mathrm{eT}}} = \frac{c}{n_\parallel v_{\mathrm{eT}}}\left(\frac{R-R_\Omega}{R}\right). \tag{14}$$

Because the perpendicular refractive index $n_\perp$ of the O-mode is nearly equal to unity, the arguments of the Bessel functions J_n are always less than one, we can expand them and neglect the left-hand component. Approximately we have

$$d_2^1 = \frac{1}{\sqrt{2}}E_{-,k} + \frac{1}{2}\frac{n_\perp v_\parallel}{c}E_{\parallel,k}, \tag{15}$$

$$d_1^1 = \frac{1}{\sqrt{2}}\frac{n_\parallel v_\perp}{c}E_{-,k} + \left(\frac{R-R_\Omega}{R}\right)\frac{n_\perp v_\perp}{c}E_{\parallel,k}. \tag{16}$$

The above coefficients can be separated as the product of two factors, one depends only on the space variables and the other on velocity variables. The space-dependent factor stems from: (1) the microwave source is space-dependent; (2) the wave power damping is space-dependent. In general, the above-mentioned three diffusion coefficients can be written as $D \propto f(x)g(v,\theta)\delta\left(\frac{x}{R} - \frac{n_\parallel v_\parallel}{c}\right)$. Integrating them along the magnetic surface, we obtain

$$\int_{x_1}^{x_2} D\mathrm{d}x = f\left(\frac{n_\parallel v\cos\theta}{c}R\right)g(v,\theta)$$

$$\text{for}\quad v_{\parallel 1} \le v\cos\theta \le v_{\parallel 2}, \tag{17}$$

where $v_{\parallel 1}, v_{\parallel 2}$ are related to two extreme positions x_1 and x_2, where the wave rays intersect with the magnetic surface considered through Eq. (12), representing the contribution of electrons effectively interacting with the wave. Function f also consists of two factors: $f = f_1 f_2$, representing respectively the power distribution within the ray and the power damping during propagation. If we are interested in the power deposition, we should specify these factors according to the experimental situation about the wave rays, the position of the resonance and the structure of the magnetic surfaces.

Relationship between the initial heating power and the quasi-linear diffusion coefficients We consider the simplest model: the constant diffusion coefficient D_0 model for the whole velocity space. In this case,

$$\begin{aligned} p &= \int \mathrm{d}\boldsymbol{v}\frac{1}{2}mv^2\left(\frac{\partial f}{\partial t}\right)_Q \\ &= \frac{1}{2}m\int_{-\infty}^{\infty}\mathrm{d}v_\parallel\int_0^{\infty}\mathrm{d}v_\perp v_\perp^2\frac{\partial}{\partial v_\perp}\left[v_\perp D\frac{\partial f}{\partial v_\perp}\right] \\ &= -\,m\int_{-\infty}^{\infty}\mathrm{d}v_\parallel\int_0^{\infty}\mathrm{d}v_\perp v_\perp^2 D\frac{\partial f}{\partial v_\perp}. \end{aligned}$$

At the very initial time, the electron distribution f can be substituted by the Maxwellian distribution, then we obtain

$$p = \frac{4D_0}{v_{\mathrm{eT}}^2}\iint \mathrm{d}\boldsymbol{v}\frac{1}{2}mv_\perp^2 f_m = 2n_{\mathrm{e}}mD_0. \tag{18}$$

This provides a test for the accuracy of the code and determines the time step. Results from different time steps are compared with that given by Eq. (18) in Fig. 1.

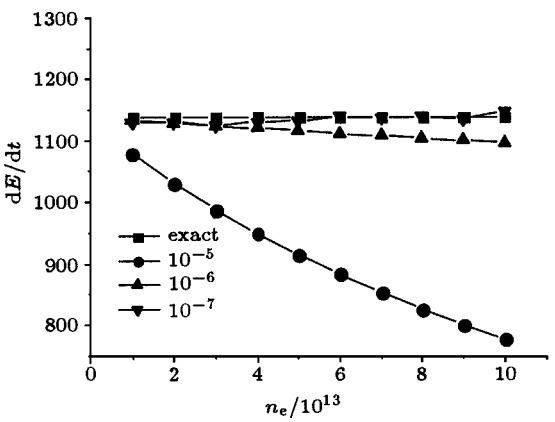


Fig.1. Initial wave power deposition rates for different values of the time step compared with that given by Eq. (18). A time step of 10^{-6}s is suitable.

3. Dependence of wave deposition rate on the loss time and the diffusion coefficients

The following case is for the perpendicularly propagating O-mode and X-mode at fixed magnetic surface. According to the polarization,[3] we can retain only one independent coefficient:[3,4]

$$D_{22}v = \begin{cases} D_0\left[1 + g\dfrac{v^2\cos^2\theta}{v_{\mathrm{eT}}^2}\right]h(y) & \text{for } v_2 > v > v_1, \\ 0 & \text{otherwise.} \end{cases} \tag{19}$$

For O-mode we select g=1, while for X-mode g=0.

For the perpendicularly propagating fundamental mode, the resonant region is located at the strong field side and asymmetric about the resonant position

$$v_1 = c\sqrt{2x_1/R}, \quad v_2 = c\sqrt{2x_2/R}, \tag{20}$$

where x_1 and x_2 represent the nearest and the most distant points from the resonant location, respectively. As indicated above, if the power deposition profile is of interest, we should determine the function $h(y)$ by the wave power profile within the ray and the power damping during propagation. This in turn relates to the experimental details (the width of the wave ray and the power profile, the resonant position and the structure of the magnetic surfaces, etc.) Now we are interested in the evolution of the electron distribution function, we can select some model function as

$$h(y) = 1 - y^2/b^2. \tag{21}$$

As for ECRH experiment on the HL-1A tokamak,[5,6] the width of wave ray b is smaller than the radius of the q=1 magnetic surface (b=2cm, r_1=4cm), the resonant position is selected at the inner side of the q=1 surface, this makes us to adopt the simplified model

$$h = \left(\frac{v - v_1}{v_2 - v_1}\right)^2. \tag{22}$$

Other relevant parameters are indicated in the figures.

It is found that the wave power deposition rate is not proportional to the wave power (the diffusion coefficient D_0), this can be seen from Fig. 2 which shows the ratio $\left(\dfrac{\mathrm{d}E}{\mathrm{d}t}/D_0\right)$ versus time for different D_0 without the energy loss. It can be seen from this figure that this ratio decreases with increasing D_0, implying that the wave deposition rate is slower compared with a linear increase. In addition to this, the wave power deposition rate changes nonlinearly with time. It usually arrives at a maximum then decreases with time. For higher power this happens earlier than for lower power. Figure 3 is the same characteristic with energy loss.

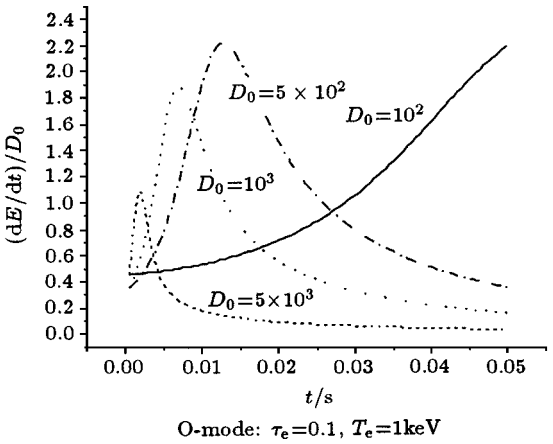


Fig.2. The ratio $\left(\dfrac{\mathrm{d}E}{\mathrm{d}t}/D_0\right)$ for O-mode for different D_0.

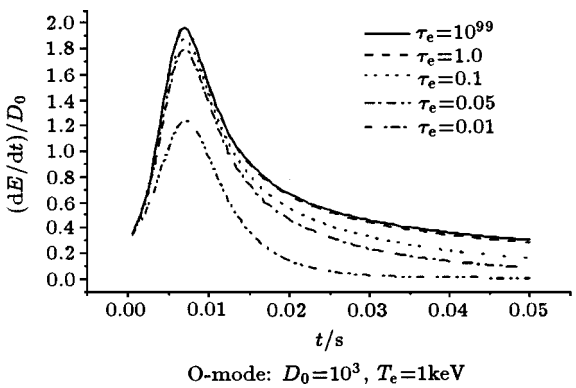


Fig.3. Same as in Fig. 2 for different energy loss time τ_{e}.

The above-mentioned non-linearity obtained from the Fokker–Planck calculation is very important. It manifests the effect of the nonlinear evolution of the electron distribution function on the resonance process. It can be seen from Fig. 3 that the larger loss could balance the wave power deposition (heating) and a stationary electron distribution can be approached.

4. Anisotropic electron pressure and characteristics

For the perpendicularly propagating O-mode or X-mode, the main acceleration of electrons by the wave is in the vertical direction. This makes the electron pressure anisotropic during heating. The degree of anisotropy depends mainly on the heating power (D_0) and the effective collision frequency which is a function of the electron density and temperature. We find that under conditions of the HL-1A experiments, the anisotropy is rather strong. Results for the O-mode and the X-mode are shown in Figs. 4 and 5, respectively. It can be seen that with moderate heating

power, the perpendicular component of the electron pressure is more than 2 times as large as its parallel component during heating, and for larger loss, the anisotropy can be maintained for a longer time. All the above results are beyond the scope of the ordinary linear calculations.

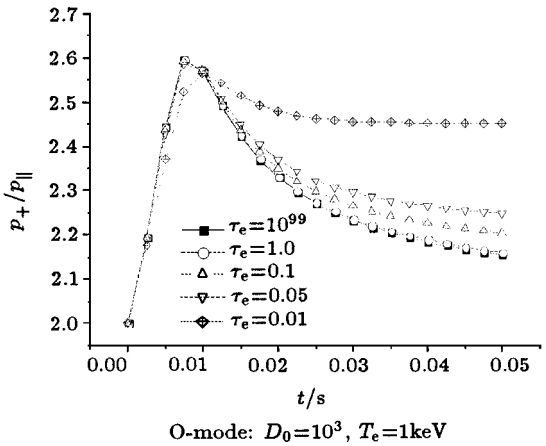


Fig.4. The electron pressure anisotropy versus time for O-mode for different τ_e.

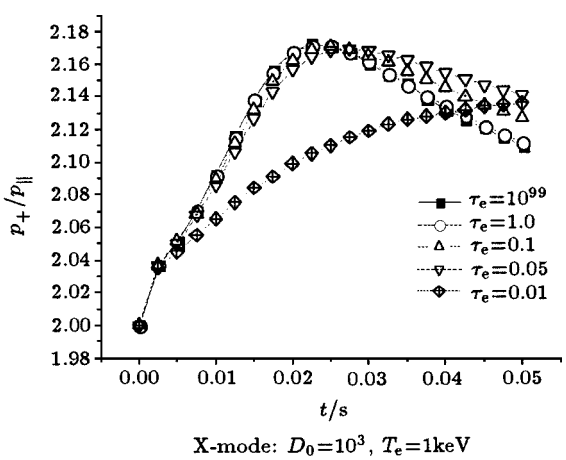


Fig.5. Same as in Fig.4, for X-mode.

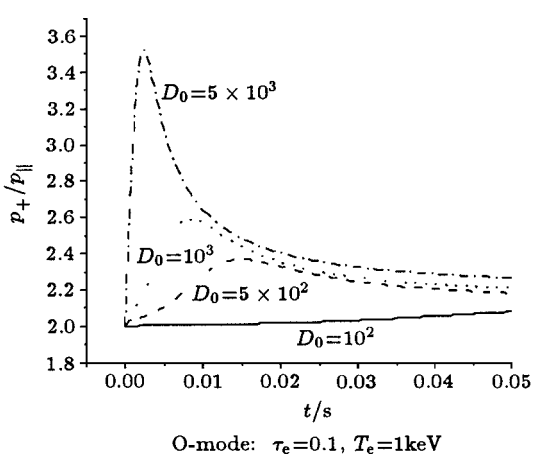


Fig.6. Electron anisotropy versus time for different D_0 for O-mode and τ=0.1s.

5. Evolution of electron distribution function

An important merit of the Fokker–Planck code is its ability to give information about the electron distribution function at any time. This is beneficial for studying problems such as the interaction of energetic electrons with MHD modes.[7]

Typical evolution of the electron distribution function during the O-mode heating are shown in Figs. 7(a) and 7(b). The main characteristic is the production of energetic tail that deviates from the Maxwellian distribution clearly.

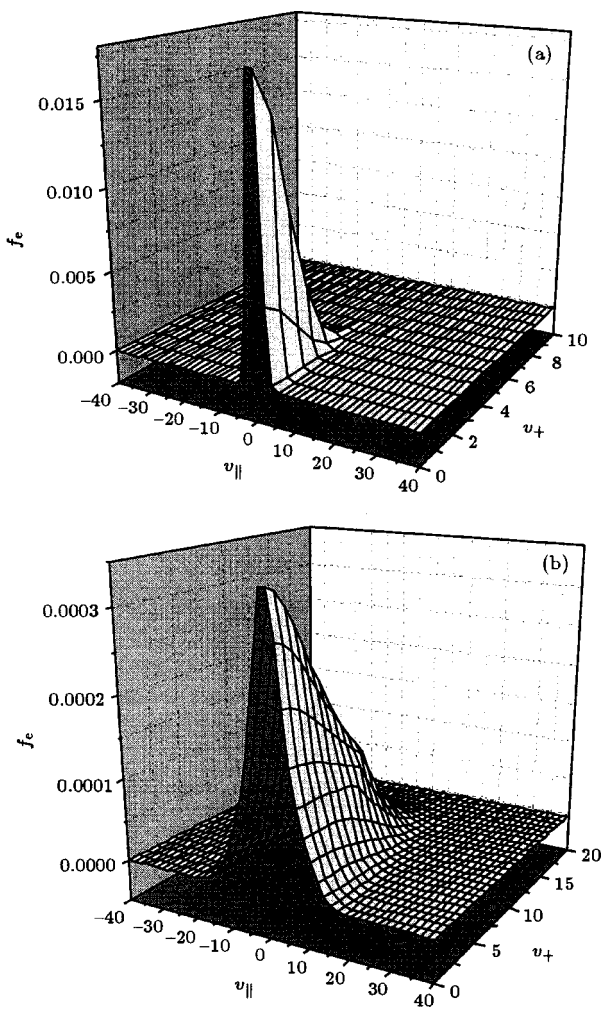


Fig.7. (a) Electron distribution function for the O-mode heating at time t=0.001. (b) The electron distribution function for the O-mode heating at tim t=0.02.

6. Discussion

Recently, the ion heating by the fast wave was investigated using the Fokker–Planck code.[8] Now

we accomplish the incorporation of ECRH with the Fokker–Planck code. Problems such as the determination of the diffusion coefficients, application to the experimental condition, etc., are discussed. It is also indicated that during the O-mode and the X-mode heating, there is strong anisotropy in electron pressure, which can be the main reason for the magnetohydrodynamic modes observed experimentally.

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