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Three-dimensional nonlinear theory of travelling wave tubes and simulation*

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A three-dimensional (3D) nonlinear theory of travelling wave tubes (TWTs) is developed, which includes a fundamental radio frequency (RF) and harmonics. When the instantaneous bandwidth exceeds an octave, the harmonic is generated and the mutual coupling between the harmonic and the fundamental RF can be observed in TWTs due to nonlinear interaction between the electron beam and the RF. At low frequencies the harmonic has an obvious effect. Based upon Tien's disc model, a plastic 3D super-particle model is proposed to improve the nonlinear analysis of TWTs. Numerical results employing a periodic magnetic focusing field are presented.

Keywords: TWTs, nonlinear theory, harmonic, simulation
PACC: 4100, 4180D

1. Introduction

Helix travelling wave tubes (TWTs) are broadband, low-to-medium power, high-gain microwave amplifiers that are widely used in communications, electronic warfare and radar systems.[1,2] When the instantaneous bandwidth exceeds an octave, the harmonic is generated and the mutual coupling between the harmonic and the fundamental radio frequency (RF) can be observed in TWTs due to the nonlinear interaction between electron beams and the RF. At low frequencies the harmonic has an obvious effect. In Ref. [3], the beam charge density $\rho(y, \Phi)$, which is rich in harmonics, is expanded into Fourier series in the phase variable $\Phi = -\omega t + \beta z$, and the harmonic voltages may be accounted for by retaining ρ_n. However, the phase constant β is a function of frequency, which is determined by the dispersion relation. When frequency changes linearly, β changes nonlinearly, then it is not reasonable that $\rho(y, \Phi)$ is expanded in $k\Phi$. In this paper the RF voltage and the beam charge density are expanded into Fourier series in the electron phase $\Phi_e = -\omega t + \beta_e z$.

In a large-signal analysis of TWTs, the space-charge force is both important and difficult to analyse. There are some useful approaches to it, in which those of Rowe[3], Tien and Walker[4] and Vainshtein[5] are representative.

For calculating the space-charge force, a disc model was put forward by Tien and Walker.[4] In essence, it is a one-dimensional super-particle model proposed by Dawson.[6] The motions of the disc and the space-charge forces are described only in one dimension. In the model, the electron beam is subdivided into several electron layers, which are charged discs just like rigid bodies. When the discs approach each other from infinity, the forces exerted between them remain constant, being independent of distance, and electron overtaking is possible. In this case, the discs can be treated as transparent meshes. For the three-dimensional (3D) case the entering beam charge can be divided into 3D super-particles, each possessing the same charge, which should be trailed through the interaction region.

In the following, 3D large signal working equations, including space harmonics and employing a periodic magnetic focusing field, are developed; and a detailed calculation for the 3D space-charge forces is given.

2. 3D large-signal working equations

For the 3D nonlinear analysis of the beam-wave interaction of TWTs, the RF circuit structure can be

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treated as an equivalent-circuit transmission-line. The continuous beam can be subdivided into definite discrete super-particles. So, the 3D large-signal working equations are obtained by means of a nonlinear Lagrangian hydrodynamic analysis treatment.[3,7,8]

2.1. Circuit equations

Including the contribution of the space harmonics, the circuit equation is given by

$$\frac{\partial^2 V}{\partial t^2} - v_{0k}^2 \frac{\partial^2 V}{\partial z^2} + 2\omega C_1 d_k \frac{\partial V}{\partial t} = v_{0k} z_{0k} \left[\frac{\partial^2 \sigma}{\partial t^2} + 2\omega C_1 d_k \frac{\partial \sigma}{\partial t} \right]. \tag{1}$$

The RF voltage is expanded into Fourier series in $\Phi_{\rm e}$

$$V(z,r,t) = {\rm Re}\left\{ \sum [A_{1k}(r,z) + {\rm j}A_{2k}(r,z)] {\rm e}^{-{\rm j}k\Phi_{\rm e}} \right\}. \tag{2}$$

The normalized RF voltage is

$$V(y,x,\Phi_{\rm e}) = \frac{Z_{01} I_0}{C_1} {\rm Re}\left\{ \sum [A_{1k}(y) + {\rm j}A_{2k}(y)] \psi_k(x) {\rm e}^{{\rm j}\theta_{kx}} {\rm e}^{-{\rm j}k\Phi_{\rm e}} \right\}. \tag{3}$$

RF voltage along the circuit ($x = x_a$) is

$$\begin{aligned} & V_k(y,x,\Phi_{\rm e}) \\ =& \frac{Z_{01} I_0}{C_1} {\rm Re}\{[A_{1k}(y) + {\rm j}A_{2k}(y)] {\rm e}^{-{\rm j}k\Phi_{\rm e}}\} \\ =& \frac{Z_{01} I_0}{C_1} [A_{1k}(y)\cos(k\Phi_{\rm e}) + A_{2k}(y)\sin(k\Phi_{\rm e}]. \end{aligned} \tag{4}$$

In Eqs. (1)–(4), r and z are the radial and axial directions, respectively; x and y are the normalized radial and axial directions, respectively.

The linear charge density is defined as

$$\sigma = -\int_0^{b'} \int_0^{2\pi} \rho\psi(r) r {\rm d}r {\rm d}\varphi. \tag{5}$$

Conservation of charge is expressed as

$$\rho r {\rm d}r {\rm d}\varphi {\rm d}z = \rho_0 r_0 {\rm d}r_0 {\rm d}\varphi_0 {\rm d}z_0. \tag{6}$$

In Eq. (6), ρ is the volume charge density, $\rho_0 = I_0/(\pi b'^2 u_0)$ is the DC charge density of the entrance. Substituting Eq. (6) into Eq. (5) and normalizing gives the following result:

$$\sigma = -\int_0^{b'} \int_0^{2\pi} \rho_0 r_0 \psi(r) {\rm d}r_0 {\rm d}\varphi_0 \left| \frac{\partial z_0}{\partial z} \right|. \tag{7}$$

Expanding Eq. (7) into Fourier series in $\Phi_{\rm e}$, we have

$$\begin{aligned} \sigma(y,\Phi_{\rm e}) =& \frac{-|I_0|}{\pi^2 x_{b'}^2 u_0} \\ & \times \sum_{k=1}^{\infty} \left[\int_0^{x_{b'}} \int_0^{2\pi} \int_0^{2\pi} \frac{\psi_k(x) x_0 \sin(k\Phi_{\rm e0})}{1 + 2C_1 u_y} \right. \\ & \times {\rm d}x_0 {\rm d}\varphi_0 {\rm d}\Phi_{\rm e0} \sin(k\Phi_{\rm e}) \\ & + \int_0^{x_{b'}} \int_0^{2\pi} \int_0^{2\pi} \frac{\psi_k(x) x_0 \cos(k\Phi_{\rm e0})}{1 + 2C_1 u_y} \\ & \left. \times {\rm d}x_0 {\rm d}\varphi_0 {\rm d}\Phi_{\rm e0} \cos(k\Phi_{\rm e}) \right]. \end{aligned} \tag{8}$$

Substitute the derivatives of Eqs. (4) and (8) into Eq. (1). Since the coefficients of $\sin(-k\Phi)$ and $\cos(-k\Phi)$ on each side of the equality sign are independent of $k\Phi$, and sine and cosine are orthogonal, we equate coefficients and evolve the following $2k$ circuit equations:

$$\begin{aligned} & \frac{{\rm d}^2 A_{1k}}{{\rm d}y^2} - k^2 A_{1k} \left[\frac{1}{C_1^2} - \left(\frac{1 + C_1 b_k}{C_1} \right)^2 \right] \\ & + \frac{2k}{C_1} \frac{{\rm d}A_{2k}}{{\rm d}y} + 2k d_k A_{2k} \frac{(1 + C_1 b_k)^2}{C_1} \\ =& - \frac{Z_{0k}}{Z_{01}} \frac{(1 + C_1 b_k)}{C_1 \pi^2 x_b^2} k^2 \\ & \times \left\{ \int_0^{2\pi} \int_0^{2\pi} \int_0^{x_b} \frac{\psi_k x_0 \cos(k\Phi_{\rm e0})}{1 + 2C_1 u_y} {\rm d}x_0 {\rm d}\Phi_{\rm e0} {\rm d}\varphi_0 \right. \\ & \left. + 2\frac{C_1}{k} d_k \int_0^{2\pi} \int_0^{2\pi} \int_0^{x_b} \frac{\psi_k x_0 \sin(k\Phi_{\rm e0})}{1 + 2C_1 u_y} {\rm d}x_0 {\rm d}\Phi_{\rm e0} {\rm d}\varphi_0 \right\}, \end{aligned} \tag{9}$$

$$\begin{aligned} & \frac{{\rm d}^2 A_{2k}}{{\rm d}y^2} - k^2 A_{2k} \left[\frac{1}{C_1^2} - \left(\frac{1 + C_1 b_k}{C_1} \right)^2 \right] \\ & - \frac{2k}{C_1} \frac{{\rm d}A_{1k}}{{\rm d}y} - 2k d_k A_{1k} \frac{(1 + C_1 b_k)^2}{C_1} \\ =& - \frac{Z_{0k}}{Z_{01}} \frac{(1 + C_1 b_k)}{C_1 \pi^2 x_b^2} k^2 \\ & \times \left\{ \int_0^{2\pi} \int_0^{2\pi} \int_0^{x_b} \frac{\psi_k x_0 \sin(k\Phi_{\rm e0})}{1 + 2C_1 u_y} {\rm d}x_0 {\rm d}\Phi_{\rm e0} {\rm d}\varphi_0 \right. \\ & \left. - 2\frac{C_1}{k} d_k \int_0^{2\pi} \int_0^{2\pi} \int_0^{x_b} \frac{\psi_k x_0 \cos(k\Phi_{\rm e0})}{1 + 2C_1 u_y} {\rm d}x_0 {\rm d}\Phi_{\rm e0} {\rm d}\varphi_0 \right\}, \end{aligned} \tag{10}$$

where $k = 1, 2, \cdots, n$ represents the kth harmonic, (k=1 represents the fundamental RF), C_1 is the gain parameter of fundamental RF. b_k, d_k and Z_{0k} are the relative injection velocity parameter, circuit loss parameter and impedance of the RF circuit, respectively. $\Phi_{\rm e0}$ is the entrance phase of an electron.

2.2.Force equations

The motion of charged particles in the presence of electric and magnetic fields is described by the Lorentz force equation. The beam velocity and wave phase velocity are assumed to be small compared with the velocity of light. As a result, non-relativistic mechanics are applicable, and the force due to RF magnetic fields and the magnetic field generated by the motion of electrons are negligibly small, thus equations of electrostatics can be used to calculate the space-charge field.

Under these conditions, in an axially symmetric system, by using periodic magnetic fields, the electrons force equations should be as follows:

Axial force equation:

$$\begin{aligned}&[1+2C_1u_y]\frac{\partial u_y}{\partial y}\\ =&C_1\sum_{k=1}^{\infty}\psi_k\left\{\left[\frac{\mathrm{d}A_{1k}}{\mathrm{d}y}+\frac{k}{C_1}A_{2k}\right]\cos(k\,\Phi_{\mathrm{e}}-\theta_{kx})\right.\\ &\left.+\left[\frac{\mathrm{d}A_{2k}}{\mathrm{d}y}-\frac{k}{C_1}A_{1k}\right]\sin(k\,\Phi_{\mathrm{e}}-\theta_{kx})\right\}\\ &+\frac{1}{2}\frac{\omega_{c1}}{C_1\omega}\frac{2\pi}{y_L}x\sin\left(\frac{2\pi}{y_L}y+\phi_m\right)u_\varphi\\ &-\frac{|\eta|}{2C_1\omega u_0}E_{sc-z}.\end{aligned}\tag{11}$$

Radial force equation:

$$\begin{aligned}&[1+2C_1u_y]\frac{\partial u_x}{\partial y}\\ =&C_1\sum_{k=1}^{\infty}\left\{\left[A_{1k}\frac{\mathrm{d}\psi_k}{\mathrm{d}x}-A_{2k}\psi_k\frac{\mathrm{d}\theta_{kx}}{\mathrm{d}x}\right]\cos(k\,\Phi_{\mathrm{e}}-\theta_{kx})\right.\\ &\left.+\left[A_{2k}\frac{\mathrm{d}\psi_k}{\mathrm{d}x}+A_{1k}\psi_k\frac{\mathrm{d}\theta_{kx}}{\mathrm{d}x}\right]\sin(k\,\Phi_{\mathrm{e}}-\theta_{kx})\right\}\\ &+\frac{1}{4C_1^3}\left(\frac{\omega_p}{\omega}\right)^2\frac{x_0^2}{x}+\frac{2C_1}{x}u_\varphi^2\\ &-\frac{\omega_{c1}}{C_1\omega}\cos\left(\frac{2\pi}{y_L}y+\phi_m\right)u_\varphi\\ &-\frac{|\eta|}{2C_1\omega u_0}E_{sc-r}.\end{aligned}\tag{12}$$

Angular force equation:

$$\begin{aligned}&[1+2C_1u_y]\frac{\partial u_\varphi}{\partial y}\\ =&\frac{\omega_{c1}}{C_1\omega}\cos\left(\frac{2\pi}{y_L}y+\phi_m\right)u_x\\ &-\frac{1}{4}\frac{\omega_{c1}}{C_1^2\omega}\frac{2\pi}{y_L}x\sin\left(\frac{2\pi}{y_L}y+\phi_m\right)(1+2C_1u_y)\\ &-\frac{4C_1}{x}u_xu_\varphi-\frac{|\eta|}{2C_1\omega u_0}E_{sc-\varphi}.\end{aligned}\tag{13}$$

Velocity-phase equation:

$$\frac{\partial\Phi_{\mathrm{e}}}{\partial y}=\frac{2u_y}{1+2C_1u_y}.\tag{14}$$

Super-particle position equation:

$$\frac{\partial x}{\partial y}=\frac{2C_1u_x}{1+2C_1u_y}.\tag{15}$$

Super-particle angular position equation:

$$\frac{\partial\varphi}{\partial y}=\frac{2C_1u_\varphi}{x(1+2C_1u_y)}.\tag{16}$$

Calculations of space-charge forces by using Eqs. (11)–(13) will be discussed in the next section.

3. Calculation of space-charge forces

A large-signal analysis for TWTs requires the following individual super-particles. We use the finite size particle model (Dawson 1983), in which the continuous beam is represented by the limited discrete 3D super-particles. Thus, the research on bunching is simplified to the analysis of motion status of these super-particles.

In a cylindrical coordinate system, the Green's function is written as

$$\begin{aligned}&G(r,\varphi,z,r',\varphi',z')\\ =&\sum_{l=1}^{\infty}\sum_{s=0}^{\infty}\frac{(2-\delta_s^0)}{2\pi\varepsilon_0 a}\frac{J_s\left(v_{ls}\frac{r'}{a}\right)J_s\left(v_{ls}\frac{r}{a}\right)}{v_{ls}J_{s+1}^2(v_{ls})}\\ &\times\cos[s(\varphi-\varphi')]\mathrm{e}^{-v_{ls}\frac{|z-z'|}{a}},\end{aligned}\tag{17}$$

where $\delta_s^0=\begin{cases}1, s=0\\ 0, s\neq 0\end{cases}$, and v_{ls} is determined from the successive zeros of $J_s(v_{ls})$, i.e. $J_s(v_{ls})=0$. $J_s(0)$ is an sth-order Bessel function. a is the RF circuit radius. ε_0 is the permittivity of free space.

Assume that the radial subdivision number for the beam is M, the angular subdivision number is N, the total charge is Q. For the initial input beam radius b', $\pi b'^2/(MN)$ is the transverse section area of a super-particle. Thus the potential produced by the super-particle at (r',φ',z') is

$$V_{sc}=\frac{Q}{\Delta S}\int_{r'-\Delta r'/2}^{r'+\Delta r'/2}\int_{\varphi'-\pi/N}^{\varphi'+\pi/N}Gr'\mathrm{d}r'\mathrm{d}\varphi',\tag{18}$$

where $\Delta r=b'^2/(2Mr')$, and $\Delta r=b'^2/(2Mr)$, which are used in the following. $\Delta r'$ and Δr represent the

radial dimensions of the exerting and accepting super-particles respectively. This expression indicates that a super-particle should be 'plastic', and its radial dimension can change dynamically to keep the volume constant; in other words, its charge quantity remains the same as the other super-particles.

Accordingly, the average potential of the super-particle at (r', φ', z') is

$$V_{Msc} = \frac{1}{\Delta S} \int_{r-\Delta r/2}^{r+\Delta r/2} \int_{\varphi-\pi/N}^{\varphi+\pi/N} V_{sc} r \mathrm{d}r \mathrm{d}\varphi. \tag{19}$$

Assuming the instant volume charge distribution for the bunched exerting super-particles be $\rho(r', \varphi', z')$, we obtain the total space-charge potential at (r, φ, z).

$$\begin{aligned} V_{Tsc} =& \frac{1}{\Delta S^2} \int_0^{+\infty} \int_0^{2\pi} \int_{-\infty}^{+\infty} \left[\int_{r-\Delta r/2}^{r+\Delta r/2} \int_{\varphi-\pi/N}^{\varphi+\pi/N} r \mathrm{d}r \mathrm{d}\varphi \right. \\ & \times \left. \left(\int_{r'-\Delta r'/2}^{r'+\Delta r'/2} \int_{\varphi'-\pi/N}^{\varphi'+\pi/N} G r' \mathrm{d}r' \mathrm{d}\varphi' \right) \right] \\ & \times \rho(r', \varphi', z') r' \mathrm{d}r' \mathrm{d}\varphi' \mathrm{d}z'. \end{aligned} \tag{20}$$

Making use of conservation of charge, and transforming space distribution of charges to time distribution of them, the normalized space-charge potential can be obtained. And the three components of the space-charge fields at (r, φ, z) can be derived from the partial difference.

Axial space-charge field:

$$\begin{aligned} E_{sc-z} =& -\left(\frac{u_0}{\omega}\right) \frac{\omega_p^2}{|\eta|} \\ & \times \int_0^{\chi_{b'}} \int_0^{2\pi} \int_0^{2\pi} \frac{\chi_0' \mathrm{d}\chi_0' \mathrm{d}\varphi_0' \mathrm{d}\Phi_{\mathrm{e}0}'}{2\pi\chi_a^2} F_{3-z} \\ & \times \mathrm{sgn}(\Phi_{\mathrm{e}} - \Phi_{\mathrm{e}}'), \end{aligned} \tag{21}$$

$$\begin{aligned} F_{3-z} =& \frac{1}{\Delta s'^2} \sum_{l=1}^{\infty} \sum_{s=0}^{\infty} (2 - \delta_s^0) \\ & \times \mathrm{e}^{-v_{ls} \frac{C_1}{x_a} |\Phi_{\mathrm{e}} - \Phi_{\mathrm{e}}'|} \\ & \times g_s^0 \int_{\chi'-\Delta\chi'/2}^{\chi'+\Delta\chi'/2} J_s\left(v_{ls} \frac{\chi'}{\chi_a}\right) \chi' \mathrm{d}\chi' \\ & \times \frac{\int_{\chi-\Delta\chi/2}^{\chi+\Delta\chi/2} J_s\left(v_{ls} \frac{\chi}{\chi_a}\right) \chi \mathrm{d}\chi}{J_{s+1}^2(v_{ls})}. \end{aligned} \tag{22}$$

Radial space-charge field:

$$\begin{aligned} E_{sc-r} =& -\left(\frac{u_0}{\omega}\right) \frac{\omega_p^2}{|\eta|} \int_0^{\chi_{b'}} \int_0^{2\pi} \int_0^{2\pi} \\ & \times \frac{\chi_0' \mathrm{d}\chi_0' \mathrm{d}\varphi_0' \mathrm{d}\Phi_{\mathrm{e}0}'}{2\pi\chi_a^2} \times F_{3-r}, \end{aligned} \tag{23}$$

$$\begin{aligned} F_{3-r} =& \frac{1}{\Delta s'^2} \sum_{l=1}^{\infty} \sum_{s=0}^{\infty} (2 - \delta_s^0) \mathrm{e}^{-v_{ls} \frac{C}{x_a} |\Phi_{\mathrm{e}} - \Phi_{\mathrm{e}}'|} g_s^0 \int_{\chi'-\Delta\chi'/2}^{\chi'+\Delta\chi'/2} J_s\left(v_{ls} \frac{\chi'}{\chi_a}\right) \chi' \mathrm{d}\chi' \\ & \times \frac{\int_{\chi-\Delta\chi/2}^{\chi+\Delta\chi/2} \left[J_{s+1}\left(v_{ls} \frac{\chi}{\chi_a}\right) - J_{s-1}\left(v_{ls} \frac{\chi}{\chi_a}\right) \right] \chi \mathrm{d}\chi}{2 J_{s+1}^2(v_{ls})}. \end{aligned} \tag{24}$$

Angular space-charge field:

$$E_{sc-\varphi} = -\left(\frac{u_0}{\omega}\right) \frac{\omega_p^2}{|\eta|} \int_0^{\chi_{b'}} \int_0^{2\pi} \int_0^{2\pi} \frac{\chi_0' \mathrm{d}\chi_0' \mathrm{d}\varphi_0' \mathrm{d}\Phi_{\mathrm{e}0}'}{2\pi\chi_a^2} \times F_{3-\varphi}, \tag{25}$$

$$\begin{aligned} F_{3-\varphi} =& \frac{1}{\Delta s'^2} \sum_{l=1}^{\infty} \sum_{s=1}^{\infty} (2 - \delta_s^0) \\ & \times \frac{\sin\left[s\left(\varphi - \varphi' + \frac{2\pi}{N}\right)\right] + \sin\left[s\left(\varphi - \varphi' - \frac{2\pi}{N}\right)\right] - 2\sin[s(\varphi - \varphi')]}{s} \\ & \times \int_{\chi'-\Delta\chi'/2}^{\chi'+\Delta\chi'/2} J_s\left(v_{ls} \frac{\chi'}{\chi_a}\right) \chi' \mathrm{d}\chi' \frac{\int_{\chi-\Delta\chi/2}^{\chi+\Delta\chi/2} J_s\left(v_{ls} \frac{\chi}{\chi_a}\right) \chi \mathrm{d}\chi}{\left(v_{ls} \frac{\chi}{\chi_a}\right) J_{s+1}^2(v_{ls})} \mathrm{e}^{-v_{ls} \frac{C}{\chi_a} |\Phi_e - \Phi_{\mathrm{e}}'|}. \end{aligned} \tag{26}$$

In Eqs. (21)–(26), we have

$$\mathrm{sgn}(\Phi_{\mathrm{e}} - \Phi'_{\mathrm{e}}) = \begin{cases} 1, \Phi_{\mathrm{e}} > \Phi'_{\mathrm{e}} \\ 0, \Phi_{\mathrm{e}} = \Phi'_{\mathrm{e}} \\ -1, \Phi_{\mathrm{e}} < \Phi'_{\mathrm{e}}, \end{cases}$$

$$g_s^0 = \begin{cases} \left(\dfrac{2\pi}{N}\right)^2, s = 0, \\ \dfrac{2\cos[s(\varphi - \varphi')] - \cos\left[s\left(\varphi - \varphi' + \dfrac{2\pi}{N}\right)\right] - \cos\left[s\left(\varphi - \varphi' - \dfrac{2\pi}{N}\right)\right]}{s^2}, s \neq 0. \end{cases} \tag{28}$$

Here, Φ is the instantaneous phase of the RF wave along the circuit. ω is the radian frequency of the RF wave. u_0 is the average initial axial velocity component of electrons in the beam, η is the charge-to-mass ratio, $\chi_{b'}$ is the normalized initial beam radius, χ_a is the normalized mean circuit radius. The plasma frequency $\omega_p^2 = |\eta| I_0/(\pi\varepsilon_0 b^2 u_0)$, I_0 is the dc beam current, χ'_0 is the normalized radial position of an electron on the input plane, φ'_0 is the angular position of an electron on the input plane and $\Phi'_{\mathrm{e}0}$ is the entrance phase of an electron. $\Delta s' = \pi\chi_{b'}^2/(MN)$, $2\pi/N$ is the angular width of the super-particle, $\chi_{b'}/M$ is the normalized radial thickness, χ and χ' are the normalized radial positions of the exerting and accepting superparticles.

4. Numerical results

It is a boundary-value problem to solve the 3D large-signal working equations of the TWTs. An iterative solution, using the Runge–Kutta algorithm, from the input would be required since the RF conditions are not known at the output. Four-dimensional linear interpolation is used to calculate space-charge forces. Calculation precision is guaranteed during estimating conversation of energy at every calculating step.

4.1.Effects of harmonic

When the instantaneous bandwidth exceeds octave, the harmonic is generated and the mutual coupling between harmonic and fundamental RF can be observed in TWTs due to the nonlinear interaction between electron beam and RF. The second harmonic would probably have a higher output power, if special harmonic suppression technique is not used.[9,10] Fundamental RF has shorter saturation length and lower output power, because the harmonics absorb electron energy, if special harmonic suppression technique is not used. Figures 1 and 2 represent this result. Dispersion shaping technique is used to suppress harmonics, Figs. 3 and 4 represent this result. Moreover, the second harmonic's saturation length is shorter than the fundamental RFs at any frequency. This is valuable for choosing available length of TWTs.

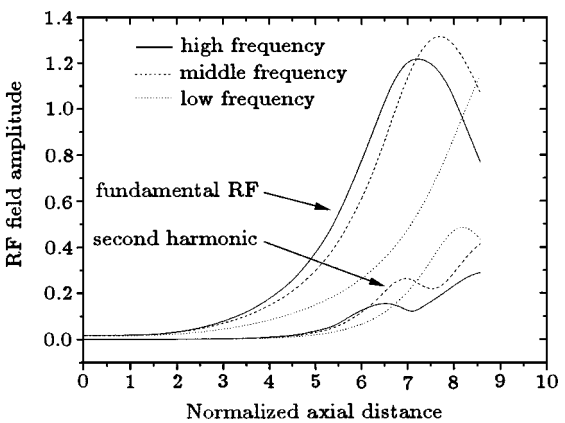


Fig.1. RF field amplitude versus normalized axial distance unused harmonic suppression technique.

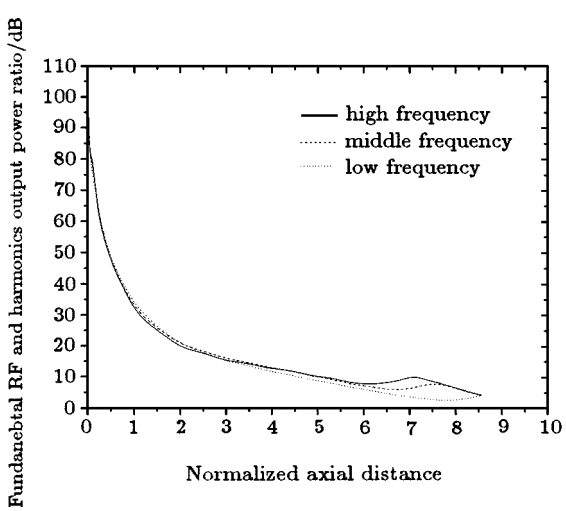


Fig.2. Fundamental RF and harmonics output power ratio versus normalized axial distance unused harmonics suppression technique.

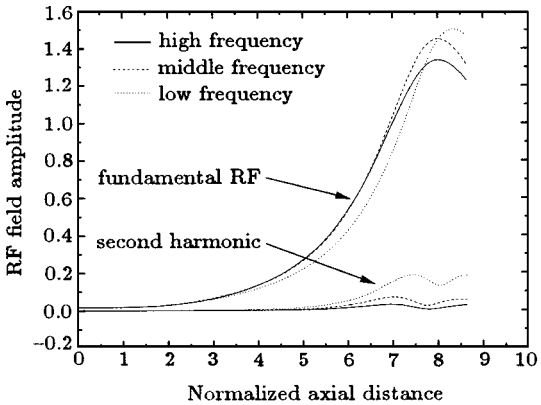


Fig.3. RF field amplitude versus normalized axial distance used harmonics suppression technique.

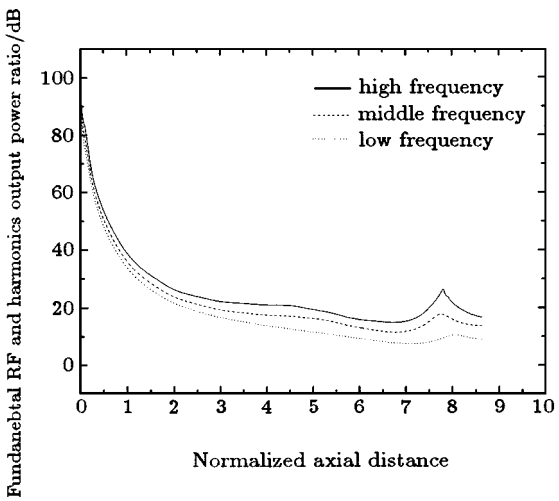


Fig.4. Fundamental RF and harmonics output power ratio versus normalized axial distance used harmonics suppression technique.

4.2.Effects of the space-charge force

In the numerical solution for the 3D large-signal working equations of TWTs, most of the time should be used to calculate space-charge forces. The solutions of TWTs are obtained, using the 3D super-particle model and considering the effects of the space-charge forces. It is shown that (1) the calculated results obtained by employing only the axial and radial space-charge forces have a slight difference in saturation length and saturation gain, compared with the full 3D computation results including the axial, radial and angular space-charge forces, but the calculating time is saved beyond 90%. So it is safely bartered for calculating time with the ignored angular space-charge force. (2) Numerical results, calculated by employing only the axial space-charge force, are equivalent to those without considering space-charge force. We will study this phenomenon in our future work.

5. Conclusions

A 3D nonlinear theory of TWTs, including space harmonics, has been developed. When the instantaneous bandwidth exceeds an octave, the harmonic has an obvious effect on the performance of tubes at low frequencies. And the harmonic has a shorter saturation length than the fundamental RF.

A 3D super-particle model is used to calculate the space-charge force. The numerical results show that (1) the space-charge forces cannot be ignored because of their significant effects on the electron phase bunching; (2) because the angular electron bunching results from the axial spatial bunching in the helical 3D electron motions, there exists an angular space-charge force, despite the RF potentials and magnetic focusing fields being symmetrical around the axis; (3) the angular space-charge force can be safely ignored, so as to reduce computation time largely, whereas the radial and the axial space-charge forces must be included in the 3D calculation. More accurate results can be gained using 3D code based on the improved 3D nonlinear theory of TWTs from this paper.

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