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On the interaction between dislocations and cracks in one-dimensional hexagonal quasi-crystals*

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The solution of elasticity problems involving the interaction between dislocations and cracks plays a fundamental role in many practical and theoretical applications. Although elasticity problems involving dislocation or cracks in quasi-crystals have been investigated in many papers, the analysis is limited to a single defect. This paper investigates the interaction of defects in one-dimensional hexagonal quasi-crystals using the complex variable function method. The interaction force between two parallel dislocations is presented and the analytic solutions of elastic fields of interaction between a dislocation and a crack are obtained. A version of the well-known Peach–Koehler formula in one-dimensional hexagonal quasi-crystals is given.

Keywords: one-dimensional hexagonal quasi-crystals, dislocation, crack, interaction
PACC: 6140M, 6220M, 4630N

1. Introduction

The discovery of quasi-crystals (QCs) around 1984[1−3] is a significant breakthrough for condensed matter physics. QCs are characterized by an unusual structure: they are orientationally ordered, but with noncrystallographic, e.g., five-, eight-, ten-, or 12-fold symmetry axes. Their translational order is not periodic, as in conventional crystals, but quasi-periodic. The quasi-periodic structure can be obtained by the cut-and-projection method from higher-dimensional space to physical space, so the elastic field of QCs is divided into two parts, namely, phonon field in physical space and phason field in the complementary orthogonal space, one interacting with the other. Due to the existence of phason field, the elastic theory of QCs is more complicated than the classical one.

Defectiveness of quasi-crystalline materials has been observed.[4] It is well known that defects greatly influence physical and mechanical properties of solid material including QCs. Experiments showed that QCs are quite brittle,[5] and brittle materials are generally sensitive to the defects. As far as the single defect problem of QCs is concerned, there are some research papers. Griffith crack problem in two-dimensional decagonal QCs was investigated by using the Fourier transform;[6−10] circular crack problem in three-dimensional icosahedral QCs was investigated by using perturbation method;[11] dislocation problems in decagonal and dodecagonal QCs were investigated by using Green function method;[12,13] crack problem and moving dislocation problem in one-dimensional(1D) hexagonal QCs was also investigated,[14,15], etc. But on interaction between defects in QCs, no research paper has been seen. The present paper is devoted to investigating the interaction between some defects in 1D hexagonal QCs, including interaction between dislocations and between a dislocation and a crack. The results show that the interaction force between defects in 1D hexagonal QCs is related to phason field, which is another special property of QCs.

2. Basic theory

Let $[\varepsilon_{ij}, w_{ij}]=[\varepsilon_{11}, \varepsilon_{22}, \varepsilon_{33}, 2\varepsilon_{23}, 2\varepsilon_{31}, 2\varepsilon_{12}, w_3, w_1, w_2]$ denote the phonon strains ε_{ij} and phason strains w_j and $[\sigma_{ij}, H_j]=[\sigma_{11}, \sigma_{22}, \sigma_{33}, \sigma_{23}, \sigma_{31}, \sigma_{12}, H_3, H_1, H_2]$ denote the phonon stress σ_{ij} and phason stress H_j, respectively, after Ref.[8], the generalized

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Hooke's laws of elasticity problem in 1D hexagonal QCs are given by

$$\begin{aligned}
\sigma_{11} &= C_{11}\varepsilon_{11} + C_{12}\varepsilon_{22} + C_{13}\varepsilon_{33} + R_1 w_3,\\
\sigma_{22} &= C_{12}\varepsilon_{11} + C_{11}\varepsilon_{22} + C_{13}\varepsilon_{33} + R_1 w_3,\\
\sigma_{33} &= C_{13}\varepsilon_{11} + C_{13}\varepsilon_{22} + C_{33}\varepsilon_{33} + R_2 w_3,\\
\sigma_{23} &= 2C_{44}\varepsilon_{23} + R_3 w_2,\\
\sigma_{31} &= 2C_{44}\varepsilon_{31} + R_3 w_1,\\
\sigma_{12} &= 2C_{66}\varepsilon_{12},\\
H_1 &= 2R_3\varepsilon_{31} + K_2 w_1,\\
H_2 &= 2R_3\varepsilon_{23} + K_2 w_2,\\
H_3 &= R_1(\varepsilon_{11} + \varepsilon_{22}) + R_2\varepsilon_{33} + K_1 w_3, \qquad (1)
\end{aligned}$$

where C_{ij} denotes the independent elastic constants of phonon field, K_i denotes the independent elastic constants of phason field and R_i denotes the coupling elastic constants of phonon field and phason field.

Besides, the geometry equations are given by

$$\begin{aligned}
\varepsilon_{ij} &= \frac{1}{2}(\partial_j u_i + \partial_i u_j),\\
w_j &= \partial_j v, \quad i,j = 1,2,3, \qquad (2)
\end{aligned}$$

where u_i and v denote the displacement of phonon field and phason field, respectively. Here we have used the tensor notation and $\partial_j u_i = \partial u_i/\partial x_j$, the same hereafter.

Equilibrium equations in the absence of body forces are given by

$$\begin{aligned}
&\partial_1\sigma_{11} + \partial_2\sigma_{12} + \partial_3\sigma_{13} = 0,\\
&\partial_1\sigma_{21} + \partial_2\sigma_{22} + \partial_3\sigma_{23} = 0,\\
&\partial_1\sigma_{31} + \partial_2\sigma_{32} + \partial_3\sigma_{33} = 0,\\
&\partial_1 H_1 + \partial_2 H_2 + \partial_3 H_3 = 0. \qquad (3)
\end{aligned}$$

Eqs.(1)–(3) are the basic equations of elasticity theory of 1D hexagonal QCs.

When defects such as infinitely long straight dislocations and cracks, etc parallel to the quasi-periodic axis of 1D hexagonal QCs exist, the geometric properties of the materials will not change in the quasi-periodic direction. If we take quasi-periodic axis of 1D hexagonal QCs for x_3 axis, then

$$\begin{aligned}
&\partial_3 u_i = 0, \quad \partial_3 v = 0, \quad \partial_3\sigma_{ij} = 0,\\
&\partial_3 H_j = 0, \quad i,j = 1,2,3. \qquad (4)
\end{aligned}$$

Substituting Eqs.(4) into Eqs.(1)–(3), the above problem turns into two independent problems as follows:

Problem I

$$\sigma_{23} = \sigma_{32} = 2C_{44}\varepsilon_{23} + R_3 w_2, \qquad (5)$$

$$\sigma_{31} = \sigma_{13} = 2C_{44}\varepsilon_{31} + R_3 w_1, \qquad (6)$$

$$H_1 = 2R_3\varepsilon_{31} + K_2 w_1, \quad H_2 = 2R_3\varepsilon_{23} + K_2 w_2, \qquad (7)$$

$$\varepsilon_{3j} = \varepsilon_{j3} = \frac{1}{2}\partial_j u_3, \qquad w_j = \partial_j v, \quad j = 1,2, \qquad (8)$$

$$\partial_1\sigma_{31} + \partial_2\sigma_{32} = 0, \quad \partial_1 H_1 + \partial_2 H_2 = 0. \qquad (9)$$

This is an antiplane elasticity problem of coupling phonon and phason fields.

Problem II

$$\sigma_{11} = C_{11}\varepsilon_{11} + C_{12}\varepsilon_{22}, \quad \sigma_{22} = C_{12}\varepsilon_{11} + C_{11}\varepsilon_{22}, \qquad (10)$$

$$\sigma_{12} = \sigma_{21} = (C_{11} - C_{22})\varepsilon_{12}, \qquad (11)$$

$$\sigma_{33} = C_{13}(\varepsilon_{11} + \varepsilon_{22}), \quad H_3 = R_1(\varepsilon_{11} + \varepsilon_{22}), \qquad (12)$$

$$\varepsilon_{ij} = \frac{1}{2}(\partial_j u_i + \partial_i u_j), \quad i,j = 1,2, \qquad (13)$$

$$\partial_1\sigma_{11} + \partial_2\sigma_{12} = 0, \quad \partial_1\sigma_{21} + \partial_2\sigma_{22} = 0. \qquad (14)$$

This is a plane elasticity problem of general crystal.

For Problem I, substituting Eq.(8) into Eqs.(5)–(7), then substituting the obtained results into Eq.(9), the equilibrium equations in terms of displacements are obtained as follows:

$$C_{44}\nabla^2 u_3 + R_3\nabla^2 v = 0, \quad R_3\nabla^2 u_3 + K_2\nabla^2 v = 0, \qquad (15)$$

where the harmonic operator $\nabla^2 = \partial^2/\partial x_1^2 + \partial^2/\partial x_2^2$. Further more, under the condition of $C_{44}K_2 - R_3^2 \neq 0$, one has

$$\nabla^2 u_3 = 0, \quad \nabla^2 v = 0. \qquad (16)$$

Eq.(16) indicate that the terminal governing equations of Problem I finally turn into two harmonic equations.

3. Solution of elastic field of a dislocation

The dislocation of QCs is described by Burgers vector in higher-dimensional space.[16] The dislocation of 1D QCs can be expressed by Burgers vector in four-dimensional space. Supposing there is an infinitely long straight dislocation parallel to the quasi-periodic direction in the body of an infinite 1D hexagonal QCs, in virtue of symmetry of the body, it is enough to investigate any plane perpendicular to the quasi-periodic direction of 1D hexagonal QCs. Let a dislocation be located at ξ point in x_1x_2-plane and its Burgers vector is $(b_1, b_2, b_3, b_\perp)$. In virtue of superposition principle, $(b_1, b_2, b_3, b_\perp)$=$(b_1, b_2, 0, 0)$+$(0, 0, b_3, b_\perp)$. The $(0, 0, b_3, b_\perp)$ together with Problem I constitutes a

boundary problem of antiplane elasticity of coupling phonon and phason field, whereas $(b_1, b_2, 0, 0)$ together with Problem II constitutes a boundary problem of plane elasticity.

In the following, we will solve Problem I at first. Its dislocation conditions are given by

$$\oint_\lambda \mathrm{d}u_3 = b_3, \qquad \oint_\lambda \mathrm{d}v = b_\perp, \tag{17}$$

where λ denotes Burgers contour surrounding the dislocation ξ. Note that both integrals are made in physical space.

First, complex representation of all field variables will be given. By means of properties of analytic function, from Eq.(16), we have

$$u_3 = \eta(z) + \overline{\eta(z)}, \qquad v = \zeta(z) + \overline{\zeta(z)}, \tag{18}$$

where $\eta(z)$ and $\zeta(z)$ are two arbitrary analytic functions; $\overline{\zeta(z)}$ denotes the conjugate function of $\zeta(z)$.

Substituting Eq.(18) into Eqs.(5)–(7), through some derivation, the complex representation of stresses are obtained as follows:

$$\sigma_{32} = \mathrm{i}C_{44}[\eta'(z) - \overline{\eta'(z)}] + \mathrm{i}R_3[\zeta'(z) - \overline{\zeta'(z)}], \tag{19}$$

$$\sigma_{31} = C_{44}[\eta'(z) + \overline{\eta'(z)}] + R_3[\zeta'(z) + \overline{\zeta'(z)}], \tag{20}$$

$$H_2 = \mathrm{i}R_3[\eta'(z) - \overline{\eta'(z)}] + \mathrm{i}K_2[\zeta'(z) - \overline{\zeta'(z)}], \tag{21}$$

$$H_1 = R_3[\eta'(z) + \overline{\eta'(z)}] + K_2[\zeta'(z) + \overline{\zeta'(z)}], \tag{22}$$

or

$$\sigma \equiv \sigma_{32} + \mathrm{i}\sigma_{31} = 2\mathrm{i}C_{44}\eta'(z) + 2\mathrm{i}R_3\zeta'(z), \tag{23}$$

$$H \equiv H_2 + \mathrm{i}H_1 = 2\mathrm{i}R_3\eta'(z) + 2\mathrm{i}K_2\zeta'(z), \tag{24}$$

where $\zeta'(z)$ denotes first-order derivative of $\zeta(z)$. Eqs.(17) and (18) yields

$$\eta'(z) = -\frac{\mathrm{i}b_3}{4\pi(z-\xi)}, \qquad \zeta'(z) = -\frac{\mathrm{i}b_\perp}{4\pi(z-\xi)}. \tag{25}$$

From Eqs.(18) and (25), we have

$$u_3 = \frac{b_3}{4\pi\mathrm{i}}[\ln(z-\xi) - \ln(\overline{z}-\overline{\xi})],$$

$$v = \frac{b_\perp}{4\pi\mathrm{i}}[\ln(z-\xi) - \ln(\overline{z}-\overline{\xi})]. \tag{26}$$

Here integral constants which denote rigid body motion are omitted.

From Eqs.(23)–(25), we have

$$\sigma = \frac{C_{44}b_\perp + R_3 b_3}{2\pi(z-\xi)}, \qquad H = \frac{R_3 b_3 + K_2 b_\perp}{2\pi(z-\xi)}. \tag{27}$$

Now we solve Problem II. Its dislocation condition is given by

$$\oint_\lambda \mathrm{d}(u_1 + \mathrm{i}u_2) = b_1 + \mathrm{i}b_2, \tag{28}$$

where λ also denotes Burgers contour surrounding the dislocation ξ.

By the route of linear elastic theory,[17,18] all field variables in Problem II can be represented by two analytic functions $\phi(z)$ and $\psi(z)$ as follows:

$$\begin{aligned} u \equiv & u_1 + \mathrm{i}u_2 \\ = & \frac{3C_{11} - C_{12}}{C_{11}^2 - C_{12}^2}\phi(z) \\ & - \frac{1}{C_{11} - C_{12}}[\overline{\psi(z)} + (z - \overline{z})\overline{\phi'(z)}], \end{aligned} \tag{29}$$

$$\sigma_{11} + \sigma_{22} = 2(\phi'(z) + \overline{\phi'(z)}), \tag{30}$$

$$\sigma_{22} - \mathrm{i}\sigma_{12} = \phi'(z) + \overline{\psi'(z)} + (z - \overline{z})\overline{\phi''(z)}, \tag{31}$$

$$\sigma_{33} = \frac{2C_{13}}{C_{11} + C_{12}}(\phi'(z) + \overline{\phi'(z)}), \tag{32}$$

$$H_3 = \frac{2R_1}{C_{11} + C_{12}}(\phi'(z) + \overline{\phi'(z)}). \tag{33}$$

From Eqs.(28) and (29), we have

$$\phi'(z) = \frac{2A}{z-\xi}, \qquad \psi'(z) = \frac{2\overline{A}}{z-\xi} - \frac{2(\xi - \overline{\xi})A}{(z-\xi)^2}, \tag{34}$$

where

$$A = \frac{C_{11}^2 - C_{12}^2}{8\pi\mathrm{i}}\left(\frac{b_1}{2C_{11}} + \frac{b_2}{C_{11} - C_{12}}\mathrm{i}\right). \tag{35}$$

Substituting Eq.(34) into the Eqs.(29)–(33), through some derivation, omitting the rigid body motion, the analytic solutions of stresses and displacements fields of the Problem II are obtained as follows:

$$\begin{aligned} u = & \frac{2A(3C_{11} - C_{12})}{C_{11}^2 - C_{12}^2}\ln(z-\xi) \\ & + \frac{2A}{C_{11} - C_{12}}\ln(\overline{z} - \overline{\xi}) \\ & - \frac{2\overline{A}}{C_{11} - C_{12}}\frac{z + \xi - \overline{z} - \overline{\xi}}{\overline{z} - \overline{\xi}}, \end{aligned} \tag{36}$$

$$\sigma_{11} + \sigma_{22} = 4\left(\frac{A}{z-\xi} + \frac{\overline{A}}{\overline{z} - \overline{\xi}}\right), \tag{37}$$

$$\sigma_{22} - \mathrm{i}\sigma_{12} = \frac{2A}{z-\xi} + \frac{2A}{\overline{z} - \overline{\xi}} - \frac{2\overline{A}(z - \overline{z} + \overline{\xi} - \xi)}{(\overline{z} - \overline{\xi})^2}, \tag{38}$$

$$\sigma_{33} = \frac{4C_{13}}{C_{11} + C_{12}}\left(\frac{A}{z-\xi} + \frac{\overline{A}}{\overline{z} - \overline{\xi}}\right), \tag{39}$$

$$H_3 = \frac{4R_1}{C_{11} + C_{12}}\left(\frac{A}{z-\xi} + \frac{\overline{A}}{\overline{z} - \overline{\xi}}\right), \tag{40}$$

where A is given by Eq.(35).

Hence Eqs.(26) and (27) and Eqs.(36)–(40) give the analytic solutions of elastic fields of dislocation in 1D hexagonal QCs. These formulae indicate that the straight dislocation in QCs cannot be simply thought of as edge dislocation or screw dislocation, but possesses two components of dislocation simultaneously.

4. Interaction of parallel dislocations

Supposing there are two dislocations z_1 and z_2 parallel to the quasi-periodic direction inside an infinite 1D hexagonal QCs, the Burgers vectors of which are $(b_1^{(1)}, b_2^{(1)}, b_3^{(1)}, b_\perp^{(1)})$ and $(b_1^{(2)}, b_2^{(2)}, b_3^{(2)}, b_\perp^{(2)})$, respectively. Since z_1 and z_2 are arbitrary points, it is enough to consider the interaction force of z_2 on z_1. Next we will calculate each interaction force corresponding to Problem I and Problem II at first, then make a superposition.

For Problem I, from Eq.(27), the field forces at z_1 from z_2 are

$$\sigma = \frac{C_{44}b_\perp^{(2)} + R_3 b_3^{(2)}}{2\pi(z_1 - z_2)}, \qquad H = \frac{R_3 b_3^{(2)} + K_2 b_\perp^{(2)}}{2\pi(z_1 - z_2)}. \tag{41}$$

Let $g_{\rm I} = (g_\parallel, g_\perp)$ denote the interaction force of the parallel dislocations z_2 on z_1 corresponding to Problem I, from Ref.[19], we have

$$\begin{aligned}\overline{g}_\parallel =& \pi {\rm Res}\left(\sigma + \frac{C_{44}b_\perp^{(1)} + R_3 b_3^{(1)}}{2\pi(z - z_1)}\right)^2 \\ =& \sigma(C_{44}b_\perp^{(1)} + R_3 b_3^{(1)}),\end{aligned} \tag{42}$$

$$\begin{aligned}\overline{g}_\perp =& \pi {\rm Res}\left(H + \frac{R_3 b_\perp^{(1)} + K_2 b_3^{(1)}}{2\pi(z - z_1)}\right)^2 \\ =& H(R_3 b_3^{(1)} + K_2 b_\perp^{(1)}),\end{aligned} \tag{43}$$

where Res denotes the residue in complex variable function theory. Eqs.(42) and (43) are just versions of the well-known Peach–Koehler formula in 1D hexagonal QCs. Clearly, the phason field plays a role in the interaction force of the parallel dislocations.

For Problem II, from Eqs.(37)–(40), the field forces of z_2 at z_1 are

$$\sigma_{11} + \sigma_{22} = 4\left(\frac{A'}{z_1 - z_2} + \frac{\bar{A}'}{\bar{z}_1 - \bar{z}_2}\right), \tag{44}$$

$$\begin{aligned}\sigma_{22} - {\rm i}\sigma_{12} =& \frac{2A'}{z_1 - z_2} + \frac{2A'}{\bar{z}_1 - \bar{z}_2} \\ & - \frac{2\bar{A}'(z_1 - \bar{z}_1 + \bar{z}_2 - z_2)}{(\bar{z}_1 - \bar{z}_2)^2},\end{aligned} \tag{45}$$

$$\sigma_{33} = -\frac{8C_{12}C_{13}}{C_{11}^2 - C_{12}^2}\left(\frac{A'}{z_1 - z_2} + \frac{\bar{A}'}{\bar{z}_1 - \bar{z}_2}\right), \tag{46}$$

$$H_3 = -\frac{8C_{12}R_1}{C_{11}^2 - C_{12}^2}\left(\frac{A'}{z_1 - z_2} + \frac{\bar{A}'}{\bar{z}_1 - \bar{z}_2}\right), \tag{47}$$

and

$$A' = \frac{C_{11}^2 - C_{12}^2}{8\pi {\rm i}}\left(\frac{b_1^{(2)}}{2C_{11}} + \frac{b_2^{(2)}}{C_{11} - C_{12}}{\rm i}\right). \tag{48}$$

Let $F_\parallel$ denote the interaction force of dislocations z_2 on z_1 corresponding to Problem II, the classical linear elastic dislocation theory[20,21], through some derivation, yields

$$F_\parallel = b_2^{(1)}(\sigma_{22} - {\rm i}\sigma_{12}) + b_1^{(1)}(\sigma_{12} - {\rm i}\sigma_{11}). \tag{49}$$

Subtracting Eq.(45) from Eq.(44), then multiplying by $-{\rm i}$, we obtain

$$\begin{aligned}\sigma_{12} - {\rm i}\sigma_{11} =& (-2{\rm i})\left[\frac{A'}{z_1 - z_2} + \frac{A'}{\bar{z}_1 - \bar{z}_2}\right. \\ & \left. + \frac{\bar{A}'(z_1 - \bar{z}_1 + \bar{z}_2 - z_2)}{(\bar{z}_1 - \bar{z}_2)^2}\right].\end{aligned} \tag{50}$$

Substituting Eqs.(45) and (50) into the Eq.(49), $F_\parallel$ can be obtained. Hence, in physical space, the interaction force of z_2 on z_1 is as following:

$$\begin{aligned}g_\parallel + F_\parallel =& R_3 b_3^{(1)}\frac{C_{44}b_\perp^{(2)} + R_3 b_\perp^{(2)}}{2\pi(\bar{z}_1 - \bar{z}_2)} \\ & + C_{44}b_\perp^{(1)}\frac{C_{44}b_\perp^{(2)} + R_3 b_3^{(2)}}{2\pi(\bar{z}_1 - \bar{z}_2)} \\ & + 2b_2^{(1)}\left[\frac{A'}{z_1 - z_2} + \frac{A'}{\bar{z}_1 - \bar{z}_2}\right. \\ & \left. - \frac{\bar{A}'(z_1 - \bar{z}_1 + \bar{z}_2 - z_2)}{(\bar{z}_1 - \bar{z}_2)^2}\right] \\ & - 2{\rm i}b_1^{(1)}\left[\frac{A'}{z_1 - z_2} + \frac{A'}{\bar{z}_1 - \bar{z}_2}\right. \\ & \left. + \frac{\bar{A}'(z_1 - \bar{z}_1 + \bar{z}_2 - z_2)}{(\bar{z}_1 - \bar{z}_2)^2}\right].\end{aligned} \tag{51}$$

Eq. (51) indicates that the force acting on a dislocation is composed of four parts, corresponding to each component of Burgers vector. Meanwhile, the interaction force between two parallel dislocations is proportional to the components of the Burgers vectors and inversely proportional to the distance of two parallel dislocations, as in conventional crystals, but the value is related to the phason field.

5. Solution of crack problem

In this section, the problem of a semi-infinite crack throughout an elastic body of 1D hexagonal QCs along the quasi-periodic direction is investigated. Due to symmetry, it is enough only to consider the periodic plane. We postulate a crack which in complex plane $x_1 + \mathrm{i}x_2$ (namely periodic plane) occupies the negative real axis, and the tip of the crack is taken as the origin, building up the coordinate system, see Fig.1. The boundary conditions are given by

$z \to \infty$:

$$\sigma_{ij}(z) = H_{ij}(z) = 0, \qquad i,j = 1,2,3; \tag{52}$$

$z = t = (-\infty, 0)$:

$$\sigma_{22}(t) - \mathrm{i}\sigma_{21}(t) = -(\sigma_{22}^{(0)}(t) - \mathrm{i}\sigma_{21}^{(0)}(t)), \tag{53}$$

$$\sigma_{32}(t) + \mathrm{i}\sigma_{31}(t) = -(\sigma_{32}^{(0)}(t) + \mathrm{i}\sigma_{31}^{(0)}(t)), \tag{54}$$

$$H_{32}(t) + \mathrm{i}H_{31}(t) = -(H_{32}^{(0)}(t) + \mathrm{i}H_{31}^{(0)}(t)), \tag{55}$$

where $(H_{32}^{(0)}(t) + \mathrm{i}H_{31}^{(0)}(t))$ are extra general forces in the complementary orthogonal space.

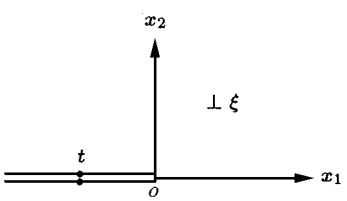


Fig.1. Interaction between a semi-infinite crack and a dislocation.

The Eqs.(52), (54) and (55) together with Problem I constitute a coupling antiplane boundary problem, and the Eqs.(52) and (53) together with Problem II a plane boundary problem. They are both Riemann–Hilbert boundary problems. The Muskhelishvili method[17,22] yields

$$\sigma(z) = -\frac{z^{-1/2}}{\pi}\int_{-\infty}^{0}\frac{[\sigma_{32}^{(0)}(t) + \mathrm{i}\sigma_{31}^{(0)}(t)](-t)^{1/2}}{(t-z)}\mathrm{d}t, \tag{56}$$

$$H(z) = -\frac{z^{-1/2}}{\pi}\int_{-\infty}^{0}\frac{[H_{32}^{(0)}(t) + \mathrm{i}H_{31}^{(0)}(t)](-t)^{1/2}}{(t-z)}\mathrm{d}t, \tag{57}$$

$$\phi'(z) = -\frac{z^{-1/2}}{2\pi}\int_{-\infty}^{0}\frac{[\sigma_{22}^{(0)}(t) - \mathrm{i}\sigma_{21}^{(0)}(t)](-t)^{1/2}}{(t-z)}\mathrm{d}t, \tag{58}$$

$$\begin{aligned}\psi'(z) =& \phi'(\bar{z}) \\ =& -\frac{z^{-1/2}}{2\pi}\int_{-\infty}^{0}\frac{[\sigma_{22}^{(0)}(t) + \mathrm{i}\sigma_{21}^{(0)}(t)](-t)^{1/2}}{(t-z)}\mathrm{d}t.\end{aligned} \tag{59}$$

Eqs.(56) and (57) give the stress field corresponding to Problem I. Substituting Eqs.(58) and (59) into the Eqs.(30)–(33), all components of stresses corresponding to Problem II are obtained.

6. Interaction between a dislocation and a crack

We now discuss the interaction between a dislocation and a crack in 1D hexagonal QCs. The configuration investigated is a semi-infinite crack throughout the elastic body of 1D hexagonal QCs along the quasi-periodic direction and an infinitely long straight dislocation parallel to the quasi-periodic direction. Like last sections, it is enough only to consider the periodic plane. Let the crack in complex plane $x_1 + \mathrm{i}x_2$ (namely periodic plane) occupy the negative real axis, and the tip of the crack be taken as the origin, the dislocation locate at point ξ and its Burgers vector be $(b_1, b_2, b_3, b_\perp)$, see Fig.1.

Since the crack surface is free, the stresses which the dislocation ξ exerts on the crack surface must be cancelled by a distribution of sources at t on the crack surface. By means of Section 3, the distribution of sources are given by

$$\sigma_{32}(t) + \mathrm{i}\sigma_{31}(t) = \frac{C_{44}b_\perp + R_3 b_3}{2\pi(t-\xi)}, \tag{60}$$

$$H_{32}(t) + \mathrm{i}H_{31}(t) = \frac{R_3 b_3 + K_2 b_\perp}{2\pi(t-\xi)}, \tag{61}$$

$$\sigma_{22}(t) - \mathrm{i}\sigma_{12}(t) = \frac{2A}{t-\xi} + \frac{2A}{t-\bar{\xi}} - \frac{2\bar{A}(\bar{\xi}-\xi)}{(t-\bar{\xi})^2}. \tag{62}$$

Substituting Eqs.(60)–(62) into the Eqs.(56)–(58) respectively, integrating by means of Cauchy residue theorem in complex variable function theory, yields

$$\sigma(z) = \frac{C_{44}b_\perp + R_3 b_3}{2\pi}\frac{1}{\xi - z}\left(\sqrt{\frac{\xi}{z}} - 1\right), \tag{63}$$

$$H(z) = \frac{R_3 b_3 + K_2 b_\perp}{2\pi}\frac{1}{\xi - z}\left(\sqrt{\frac{\xi}{z}} - 1\right), \tag{64}$$

$$\begin{aligned}\phi'(z) =& \frac{1}{\sqrt{z}}\left[\frac{A}{\sqrt{z}+\sqrt{\xi}} + \frac{A}{\sqrt{z}+\sqrt{\bar{\xi}}}\right. \\ &\left. + \frac{\bar{A}}{2(\sqrt{z}+\sqrt{\bar{\xi}})^2\sqrt{\xi}}\right].\end{aligned} \tag{65}$$

From Eq.(59), we have

$$\begin{aligned}\psi'(z) =& \frac{1}{\sqrt{z}}\left[\frac{\bar{A}}{\sqrt{z}+\sqrt{\xi}} + \frac{\bar{A}}{\sqrt{z}+\sqrt{\xi}}\right. \\ &\left. + \frac{A}{2(\sqrt{z}+\sqrt{\xi})^2\sqrt{\xi}}\right].\end{aligned} \tag{66}$$

Substituting Eqs.(65) and (66) into the Eqs.(30)–(33), stress fields of the extra distribution of sources will be obtained. Then these stress fields in turn are added to the stress fields of the dislocation obtained in Section 3, resulting in stress fields about interaction between a dislocation and a crack which are given by

$$\begin{aligned}(\sigma_{11}+\sigma_{22})_{\text{total}} =&\frac{1}{\sqrt{z}}\left[\frac{2A}{\sqrt{z}+\sqrt{\xi}}+\frac{2A}{\sqrt{z}+\sqrt{\bar{\xi}}}+\frac{\bar{A}}{(\sqrt{z}+\sqrt{\bar{\xi}})^2\sqrt{\bar{\xi}}}\right]\\&+\frac{1}{\sqrt{\bar{z}}}\left[\frac{2\bar{A}}{\sqrt{\bar{z}}+\sqrt{\bar{\xi}}}+\frac{2\bar{A}}{\sqrt{\bar{z}}+\sqrt{\xi}}+\frac{A}{(\sqrt{\bar{z}}+\sqrt{\xi})^2\sqrt{\xi}}\right]\\&+4\left(\frac{A}{z-\xi}+\frac{\bar{A}}{\bar{z}-\bar{\xi}}\right),\end{aligned}\tag{67}$$

$$\begin{aligned}(\sigma_{22}-\mathrm{i}\sigma_{12})_{\text{total}} =&\frac{1}{\sqrt{z}}\left[\frac{A}{\sqrt{z}+\sqrt{\xi}}+\frac{A}{\sqrt{z}+\sqrt{\bar{\xi}}}+\frac{\bar{A}}{2(\sqrt{z}+\sqrt{\bar{\xi}})^2\sqrt{\bar{\xi}}}\right]\\&+\frac{1}{\sqrt{\bar{z}}}\left[\frac{A}{\sqrt{\bar{z}}+\sqrt{\xi}}+\frac{A}{\sqrt{\bar{z}}+\sqrt{\bar{\xi}}}+\frac{\bar{A}}{2(\sqrt{\bar{z}}+\sqrt{\bar{\xi}})^2\sqrt{\bar{\xi}}}\right]\\&-\frac{z-\bar{z}}{2\sqrt{(\bar{z})^3}}\left[\frac{\bar{A}}{\sqrt{\bar{z}}+\sqrt{\bar{\xi}}}+\frac{\bar{A}}{\sqrt{\bar{z}}+\sqrt{\xi}}+\frac{A}{2(\sqrt{\bar{z}}+\sqrt{\xi})^2\sqrt{\xi}}\right]\\&-\frac{z-\bar{z}}{2}\left[\frac{\bar{A}}{(\sqrt{\bar{z}}+\sqrt{\bar{\xi}})^2}+\frac{\bar{A}}{(\sqrt{\bar{z}}+\sqrt{\xi})^2}+\frac{A}{(\sqrt{\bar{z}}+\sqrt{\xi})^3\sqrt{\xi}}\right]\\&+\frac{2A}{z-\xi}+\frac{2A}{\bar{z}-\bar{\xi}}-\frac{2\bar{A}(z-\bar{z}+\bar{\xi}-\xi)}{(\bar{z}-\bar{\xi})^2},\end{aligned}\tag{68}$$

$$\begin{aligned}(\sigma_{33})_{\text{total}} =&\frac{4C_{13}}{C_{11}+C_{12}}\left\{\frac{1}{\sqrt{z}}\left[\frac{A}{\sqrt{z}+\sqrt{\xi}}+\frac{A}{\sqrt{z}+\sqrt{\bar{\xi}}}+\frac{\bar{A}}{2(\sqrt{z}+\sqrt{\bar{\xi}})^2\sqrt{\bar{\xi}}}\right]\right.\\&\left.+\frac{1}{\sqrt{\bar{z}}}\left[\frac{\bar{A}}{\sqrt{\bar{z}}+\sqrt{\bar{\xi}}}+\frac{\bar{A}}{\sqrt{\bar{z}}+\sqrt{\xi}}+\frac{A}{2(\sqrt{\bar{z}}+\sqrt{\xi})^2\sqrt{\xi}}\right]+\frac{A}{z-\xi}+\frac{\bar{A}}{\bar{z}-\bar{\xi}}\right\},\end{aligned}\tag{69}$$

$$\begin{aligned}(H_3)_{\text{total}} =&\frac{4R_1}{C_{11}+C_{12}}\left\{\frac{1}{\sqrt{z}}\left[\frac{A}{\sqrt{z}+\sqrt{\xi}}+\frac{A}{\sqrt{z}+\sqrt{\bar{\xi}}}+\frac{\bar{A}}{2(\sqrt{z}+\sqrt{\bar{\xi}})^2\sqrt{\bar{\xi}}}\right]\right.\\&\left.+\frac{1}{\sqrt{\bar{z}}}\left[\frac{\bar{A}}{\sqrt{\bar{z}}+\sqrt{\bar{\xi}}}+\frac{\bar{A}}{\sqrt{\bar{z}}+\sqrt{\xi}}+\frac{A}{2(\sqrt{\bar{z}}+\sqrt{\xi})^2\sqrt{\xi}}\right]+\frac{A}{z-\xi}+\frac{\bar{A}}{\bar{z}-\bar{\xi}}\right\},\end{aligned}\tag{70}$$

$$\sigma_{\text{total}} = \frac{C_{44}b_\perp+R_3b_3}{2\pi}\frac{1}{\xi-z}\left(\sqrt{\frac{\xi}{z}}-2\right),\tag{71}$$

$$H_{\text{total}} = \frac{R_3b_3+K_2b_\perp}{2\pi}\frac{1}{\xi-z}\left(\sqrt{\frac{\xi}{z}}-2\right),\tag{72}$$

where A is given by Eq.(35).

7. Conclusions

Now we summarize the overall conclusions of this paper. The unique condition on the material is $C_{44}K_2-R_3^2\neq 0$, which can be checked by experiment.

1) The analytic solution of elastic field of a dislocation in 1D hexagonal QCs is given in the form of complex variable function. It indicates that the straight dislocation in QCs cannot be simply thought of as edge dislocation or screw dislocation, but possesses two components of dislocations simultaneously. However, the order of singularity is not changed, i.e., still one order like general crystal.

2) The interaction force between two parallel dislocations is obtained, which is proportional to the components of Burgers vectors and inversely propor-

tional to the distance of two parallel dislocations, as in conventional crystals, but the value is related to the phason field.

3) The complex stress functions of elastic field of semi-infinite crack under general loading in the material are obtained in the form of integrals. Using these complex representations, the elastic field of interaction between a dislocation and a crack is given, which offers the theoretical basis to discuss dislocation emission in the tip of crack, dislocation screening and crack shielding in 1D hexagonal QCs.

4) These theories are the development of the corresponding parts of classical elastic theory[19,23] to QCs. A version of the well-known Peach–Koehler formula in 1D hexagonal QCs is given.

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