Vol 12 No 10, October 2003
1009-1963/2003/12(10)/1105-04

Chinese Physics

© 2003 Chin. Phys. Soc.
and IOP Publishing Ltd

Modulated stochastic multiresonance in single-mode laser system without input periodic signal*

Liang Gui-Yun(梁贵云)[a], Cao Li(曹 力)[a)b)], and Wu Da-Jin(吴大进)[a)b)]

[a)] *State Key Laboratory of Laser Technology, Huazhong University of Science and Technology, Wuhan* 430074, *China*

[b)] *China Centre of Advanced Science and Technology (World Laboratory), Beijing* 100080, *China*

(Received 14 March 2003; revised manuscript received 29 April 2003)

The stochastic resonance phenomenon in a single-mode laser system driven by multiplicative and additive Gaussian white noises without external periodic force is studied. We find that there are multiple extrema (maximum) in the curve of the mean output laser intensity versus the logarithm of multiplicative noise level. This phenomenon reveals that the mean output laser intensity can be amplified at several values of the multiplicative noise intensity, whose peaks are likely modulated by a sinusoidal function.

Keywords: stochastic multiresonance, modulated stochastic multiresonance, single-mode laser, multiplicative and additive Gaussian white noises

PACC: 4250, 0540, 0250

1. Introduction

Stochastic resonance (SR) is a nonlinear phenomenon that has been attracting considerable interest due to its potential technological applications in optimizing the transmission of information, such as the output signal-to-noise ratio (SNR) and amplification factor (η) in nonlinear dynamical systems.[1−7] Broadly speaking, SR means that the performances of system, such as the response to weak periodic signals, can be enhanced and optimized at certain nonzero noise level. This phenomenon is rather counterintuitive. In a sense, the noise plays a constructive role since an optimized amount is responsible for the enhancement of the response of system to a periodic signal. The mechanism of SR requires three basic ingredients acknowledged conventionally:[8−11] (i) an energetic activation barrier or, more generally, a form of threshold; (ii) a weak coherent input (such as a periodic signal); (iii) a source of noise that is inherent in the system, or that adds to the coherent input. Since the concept of SR was originally put forward in the seminal papers by Benzi and collaborators, much more research has been done in both theory and experiment. It presently created a buzz in a large variety of different classes of systems: monotable systems,[12,13] system with excitable dynamics,[14] noisy non-dynamical systems,[15] systems without an external force,[16] and systems without any kind of threshold.[17] Most of the works are focused upon the system driven by external periodic signal. However, in 1993, Hu *et al*[16,18] found the SR in an excitable system without external forces, which proved the external force is not a necessary ingredient to the occurrence of SR.[8] Thus, it is believed that the occurrence of SR is the intrinsic dynamic property of the nonlinear system. Later works are mainly focused upon the SR that has a single maximum in the curve of the output SNR versus the noise level. By studying the system exerted by the external periodic force, in 1997, Vilar *et al*[19] observed the SR at different values of noise intensity, which is firstly termed as stochastic multiresonance by them. To date, few works has reported the occurrence of stochastic multiresonance in the nonlinear system without external periodic force.

In this paper, by investigating the single-mode laser system driven by multiplicative (pump) and additive (quantum) noise which are white in the case of absence of external periodic force, we detect that the curves of the mean output laser intensity $< I >$, the variance $\lambda_2(0)$, and the skewness $\lambda_3(0)$ versus the logarithm of multiplicative noise level $\lg P^{'}$ exhibit

*Project supported by the National Natural Science Foundation of China (Grant No 10275025).

the stochastic multiresonance phenomenon for higher noise level. It is interesting that in our system, the peaks of extrema is modulated by a sinusoidal function, which is entirely different from that found by Vilar.

2. Stationary intensity distribution of single-mode laser

When only the intensity fluctuation is concerned, the field-amplitude Langevin equation of a single-mode laser is[20]

$$\dot{x} = a_0 x - Ax^3 + \frac{P}{x} + x\xi(t) + \eta(t), \qquad (1)$$

where a_0 is the net gain coefficient, and A denotes the self-saturation coefficient. $x = \sqrt{I}$, I is the laser intensity. The multiplicative noise $\xi(t)$ and additive noise $\eta(t)$ are Gaussian white noises with zero mean and without correlation between them,

$$< \xi(t) >=< \eta(t) >= 0,$$
$$< \xi(t)\xi(t') >= 2P'\delta(t-t'),$$
$$< \eta(t)\eta(t') >= 2P\delta(t-t')),$$

where P' and P are the intensities of multiplicative and additive noises, respectively. Equation (1) is obtained from the corresponding complex field-amplitude Langevin equation of laser.[21] As we all known, when the complex field-amplitude of laser is expressed in form of polar coordinates: $E = x\mathrm{e}^{\mathrm{i}\phi}$, the Langevin equation of E can be changed into two coupling Langevin equations about x and ϕ. Thus, Eq.(1) is obtained when the Langevin equation of field-amplitude x is decoupled from those of phase ϕ.[20]

The Fokker–Planck equation corresponding to Eq.(1) can be obtained exactly,

$$\frac{\partial}{\partial t}Q(x,t) = -\frac{\partial}{\partial x}V(x)Q(x,t) + \frac{\partial^2}{\partial x^2}G(x)Q(x,t), \quad (2)$$

where

$$V(x) = a_0 x - Ax^3 + \frac{P}{x} + P'x,$$
$$G(x) = P'x^2 + P.$$

The stationary probability distribution of field amplitude is obtained from Eq.(2),

$$Q_{\mathrm{st}}(x) = N(P'x^2 + P)^{\alpha} x \exp[-\beta x^2], \qquad (3)$$

where

$$\beta = \frac{A}{2P'}, \qquad \alpha = \beta\left(\frac{a_0}{A} + \frac{P}{P'}\right) - 1.$$

Due to $x = \sqrt{I}$, according to the Jacobian relation between the laser intensity I and laser field-amplitude x, the stationary intensity distribution (SID) is determined by

$$P_{\mathrm{st}}(I) = Q_{\mathrm{st}}(x)\frac{\mathrm{d}x}{\mathrm{d}I}. \qquad (4)$$

Thus, the SID can be derived from the stationary probability distribution of field amplitude,

$$P_{\mathrm{st}}(I) = N_0(P'I + P)^{\alpha}\exp[-\beta I]. \qquad (5)$$

Here, the normalization constant N_0 is determined by

$$\int_0^{\infty} P_{\mathrm{st}}(I)\mathrm{d}I = 1.$$

3. The mean, variance and skewness of the laser intensity

The nth moment of the laser intensity I is given by the following integral:

$$< I^n >= \int_0^{\infty} I^n P_{\mathrm{st}}(I)\mathrm{d}I. \qquad (6)$$

The mean of the laser intensity $< I >$ can be obtained when $n = 1$ in Eq.(6).

The normalized variance $\lambda_2(0)$ and the skewness $\lambda_3(0)$ of the laser intensity are defined as

$$\lambda_2(0) = \frac{< I^2 >}{< I >^2} - 1,$$
$$\lambda_3(0) = \frac{< I^3 >}{< I >^3} - 3\lambda_2(0) - 1. \qquad (7)$$

4. Conclusions

i) We depict the curve of mean output laser intensity $< I >$ as the function of logarithm of the multiplicative noise level P' in Fig.1. When $\lg P' > 15$, the curve exhibits dozens of maxima at different noise intensity. Such a high noise level is not imaginable in the past, however, it can reach such a level in present technique. Moreover, because the noise is a multiplicative one, it can be easily controlled in the experiment. The multiple maxima in the $< I >$-$\lg P'$ curve differs from that occurs in Ref.[19] in some aspects. On one hand, the occurrence of stochastic multiresonance here is not excited through the assistance of external periodic force, but completely induced by noises. On the other hand, we can see that the peaks of the

$< I >$-$\lg P'$ curve are modulated by a periodic sinusoidal function. Therefore, we term this phenomenon as the modulated stochastic multiresonance (MSMR). The multiple maxima appear at a constant frequency. In a sense, the modulated stochastic multiresonance is much like an oscillation which oscillates around the value of mean output laser intensity $< I > \approx 1$.

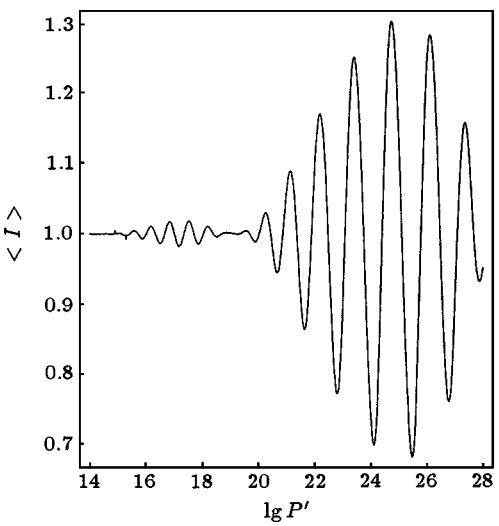


Fig.1. Mean intensity $< I >$ versus the logarithm of the multiplicative noise intensity $\lg P'$. The values of the parameters: $a_0 = 0$, $A = 1$, $P = 0.1$.

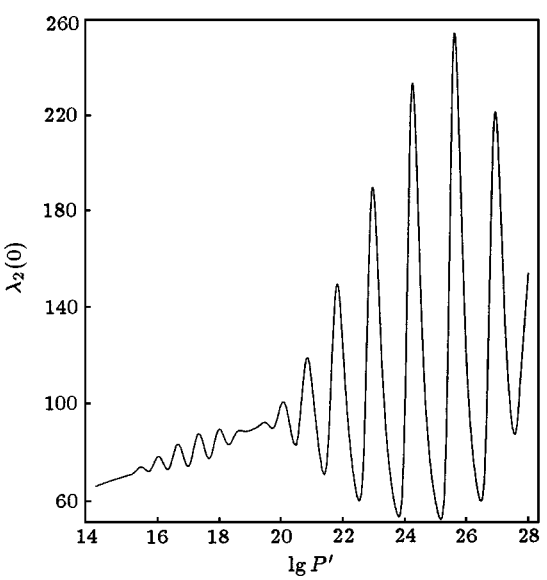


Fig.2. Normalized variance $\lambda_2(0)$ versus the logarithm of the multiplicative noise intensity $\lg P'$. The values of the parameters: $a_0 = 0$, $A = 1$, $P = 0.1$.

ii) The phenomenon of the MSMR occurs not only in the mean output laser intensity $< I >$, but also in the variance $\lambda_2(0)$ and the skewness $\lambda_3(0)$, which are usually used to describe the statistical properties of fluctuation in the laser system. The MSMR in the variance $\lambda_2(0)$ and the skewness $\lambda_3(0)$ is similar to that in the mean output laser intensity $< I >$, as can be seen from Fig.2 and Fig.3. However, in Fig.2 and Fig.3, the oscillation does not oscillate around a horizontal line. That is to say, the trend of the fluctuation always increases with increasing multiplicative noise level.

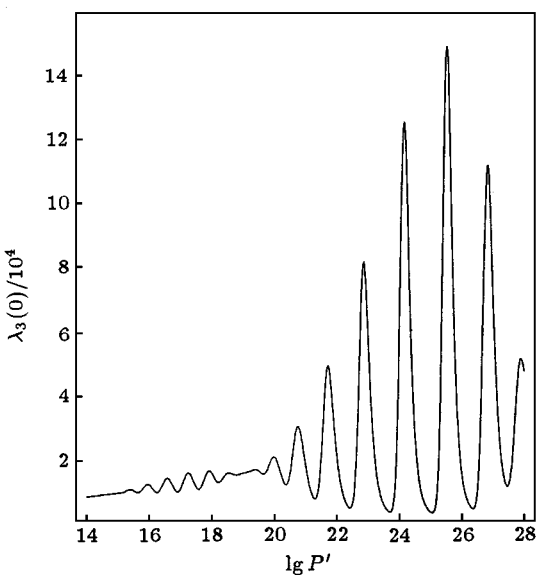


Fig.3. Normalized skewness $\lambda_3(0)$ versus the logarithm of the multiplicative noise intensity $\lg P'$. The values of the parameters: $a_0 = 0$, $A = 1$, $P = 0.1$.

In summary, we believe that the multiplicative noise plays a constructive role in the occurrence of MSMR of single-mode laser system. When the multiplicative noise arrives at the level of $\lg P' > 15$, the MSMR stimulated by the intrinsic dynamics of the nonlinear laser system appears. As viewed from physics, the phenomenon that the multiple maxima appear at a constant frequency comes from the interaction between the high noise level and the intrinsic dynamics of the nonlinear system. In a sense, it plays the role of a periodic external signal.

References

[1] Liu Z H, Lai Y C and Nachman A 2002 *Phys. Lett.* A **297** 75

[2] Zaikin A A, Kurths J and Schimansky-Geier L 2000 *Phys. Rev. Lett.* **85** 227

[3] Tessone C J, Wio H S and Hänggi P 2000 *Phys. Rev.* E **62** 4623

[4] Wang J, Cao L and Wu D J 2002 *Chin. Phys. Lett.* **19** 1416
[5] Zhang L Y, Cao L, Wu D J and Wang J 2003 *Chin. Phys. Lett.* **20** 25
[6] Zhang L, Cao L and Wu D J 2003 *Chin. Phys.* **12** 33
[7] Luo S Q, Zhu S Q and Gao W J 2001 *Chin. Phys.* **10** 1011
[8] Gammaitoni L *et al* 1998 *Rev. Mod. Phys.* **70** 223
[9] Zhou T, Moss F and Hänggi P 1990 *Phys. Rev.* A **42** 3161
[10] Jia Y, Yu S N and Li J R 2000 *Phys. Rev.* E **62** 1869
[11] Inchiosa M E *et al* 1999 *Phys. Lett.* A **252** 20
[12] Stocks N, Stei N and McClintock P 1993 *J. Phys. A: Math. Gen.* 26 385
[13] Vilar J M G and Rubi J M 1996 *Phys. Rev. Lett.* **77** 2863
[14] Wiesenfeld K *et al* 1994 *Phys. Rev. Lett.* **72** 2125
[15] Gingl Z, Kiss L and Moss F 1995 *Europhys. Lett.* **29** 191
[16] Hu G, Ditainger T, Ning C N and Haken H 1993 *Phys. Rev. Lett.* **71** 807
[17] Bezrukov S and Vodyanoy I 1999 *Unsolved Problems of Noise and Fluctuations*(AIP Conf. C.) (New York: AIP) p169
[18] Pikovsky A S and Kurths J 1997 *Phys. Rev. Lett.* **78** 775
[19] Vilar J M G and Rubi J M 1997 *Phys. Rev. Lett.* **78** 2882
[20] Zhu S Q 1992 *Phys. Rev.* A **45** 4969
Zhu S Q 1993 *Phys. Rev.* A **47** 2405
[21] Hernandez-Garcia E, Toral R and Miguel M S 1990 *Phys. Rev.* A **42** 6823