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Teleportation of a multiqubit state by an entangled qudit channel*

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We investigate the problem of teleportation of an M-qubit state by using an entangled qudit pair as a quantum channel and show that the teleportation of a multiparticle state can correspond to the teleportation of a multidimensional state. We also introduce a quantum-state converter composed of beamsplitter arrays, on/off-detectors and cross-Kerr couplers and demonstrate that the state conversion from an M-qubit to an N-dimensional qudit and vice versa can be implemented with this converter, where $N = 2^M$. Based on this, an experimentally feasible scheme for the teleportation of an M-qubit via an entangled N-level qudit pair channel is proposed.

Keywords: teleportation, multiqubit, qudit channel
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1. Introduction

One of the key problems in quantum communication and quantum computation is how to transmit a quantum state from one site to another and keep the received state exactly the same as that sent without necessitating the movement of an information carrier. A novel scheme proposed by Bennett *et al*[1] solved the problem in two steps. Firstly, the sender (Alice) "disassembles" information of a quantum state into two parts, one of which is sent through a quantum channel and the other of which is sent through a classical channel. Secondly, the receiver (Bob) "reconstructs" the state by information from both quantum and classical channels. Since in this process a quantum state to be transmitted is destroyed in one place and reconstructed in the other, it is termed as teleportation of a quantum state.

Quantum teleportation is a cornerstone in the developing course of quantum information and computation. It is of essential importance for many fields of quantum information processing, such as telecomputation,[2] telecloning,[3] entanglement swapping[4,5] and quantum remote control.[6] A number of experimentally feasible schemes have been suggested for the teleportation of two-level atomic states[7−14] and two-dimensional field states.[15−18] Recently, teleportation of multiparticle state have attracted much attention[23−28] for the applications mentioned above and any further applications of teleportation are inevitably involving multiparticle quantum system.[19] Though teleportation of multiparticle state is always possible by using multiple quantum channels, it is usually infeasible and uneconomical. On the other hand, N-dimensional quantum system (qudit) have also attracted a considerable research effort. Its entanglement and separability have been investigated in.[20] Several schemes for teleportation of qudit have been proposed in Refs.[21,22]. In the present paper we will propose a scheme for teleportation of an M-qubit state by using an entangled qudit pair as quantum channel. We will first describe the scheme mathematically. After that, we will introduce a quantum-state converter composed of beamsplitter arrays, on/off-detectors and cross-Kerr couplers and show how to implement the conversion from an M-

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qubit state to an N-dimensional qudit state and vice versa with this converter, where $N = 2^M$. Finally, we will demonstrate how to complete the teleportation of an M-qubit via an entangled N-level qudit pair channel.

2. Convert M-qubit state into N-level qudit state by an isomorphic mapping

Mathematically, teleportation of a multiparticle state can be corresponded to teleportation of a multidimensional state.[27,29,30] That is, for instance, teleportation of the an M-qubit state can be accomplished as follows. Firstly, the M-qubit state, which spans a Hilbert space $\mathcal{H}_a$ of $\dim(\mathcal{H}_a) = 2^M$, can be converted into another Hilbert space $\mathcal{H}_b$ spanned by an N-level system (a qudit) with $\dim(\mathcal{H}_b) = N = 2^M$. Then, teleportation of the qudit can be completed with the standard teleportation procedure for an N-dimensional state. Finally, the state of the qudit can be reconverted into a Hilbert space $\mathcal{H}_{a'}$ with $\dim(\mathcal{H}_{a'}) = 2^M$ spanned by an M-qubit on the receiver's side.

Let us assume that Alice wishes to teleport an M-qubit state $|\psi\rangle_a$ to a distant partner Bob, but they share only an entangled N-level qudit pair $|\psi\rangle_{AB}$ as quantum channel ($N = 2^M$) and a classical communication channel. To achieve her aim, Alice has to convert the M-qubit state into an N-level qudit state. This can be realized by an isomorphic mapping $\mathcal{I}(\mathcal{H}_a, \mathcal{H}_b)$ from Hilbert space $\mathcal{H}_a$ to Hilbert space $\mathcal{H}_b$. To show this explicitly, suppose the M-qubit to be teleported are in the product state

$$|\psi\rangle_a = \prod_{i=0}^{M-1} (\alpha_i |0\rangle + \beta_i |1\rangle). \tag{1}$$

It can be rewritten as a superposition state with $N = 2^M$ items by working out the product states in Eq.(1)

$$|\psi\rangle_a = \sum_{k=0}^{N-1} \gamma_k |k\rangle, \tag{2}$$

where $|k\rangle = \left|\overline{k}_{M-1} \cdots \overline{k}_1 \overline{k}_0\right\rangle$ is the decimal denotation of the an M-qubit binary state, k and $\overline{k}_i$ satisfy the relation $k = \overline{k}_{M-1} \cdot 2^{M-1} + \cdots + \overline{k}_1 \cdot 2^1 + \overline{k}_0 \cdot 2^0$; the coefficient γ_k is composed of α_i and β_i corresponding to $\overline{k}_i = 0$ and $\overline{k}_i = 1$, for $i = 0, 1, \ldots M-1$, respectively. For example, if $M = 2$, then $|0\rangle = |00\rangle, |1\rangle = |01\rangle, |2\rangle = |10\rangle, |3\rangle = |11\rangle$ and $\gamma_0 = \alpha_0\alpha_1$, $\gamma_1 = \alpha_0\beta_1, \gamma_2 = \beta_0\alpha_1$, $\gamma_3 = \beta_0\beta_1$. To teleport the M-qubit state $|\psi\rangle_a$ through an entangled N-level qudit pair channel, Alice has to convert the state $|\psi\rangle_a$ in $\mathcal{H}_a$ into another state $|\psi\rangle_b$ in an N-dimensional Hilbert space $\mathcal{H}_b$ spanned by an N-level qudit system. It can be accomplished by an isomorphic mapping, i.e., if $\{|\varphi_k\rangle\}$ is a set of orthonormal basis of the N-dimensional Hilbert space $\mathcal{H}_b$ spanned by an N-level qudit system, then an operation with outcome ω

$$Y(\omega) \sim P_{ba} = \sum_{k=0}^{N-1} |\varphi_k\rangle \langle k|, \tag{3}$$

which is the representation of an isomorphic mapping from $\mathcal{H}_a$ to $\mathcal{H}_b$, acting on the state $|\psi\rangle_a$ will realize the conversion Alice desires:

$$|\psi\rangle_b = P_{ba} |\psi\rangle_a = \sum_{k=0}^{N-1} \gamma_k |\varphi_k\rangle. \tag{4}$$

Note that though the state $|\psi\rangle_a$ in Eq.(2) and the state $|\psi\rangle_b$ in Eq.(4) have the same mathematical form, the physical systems they represent are quite different. One of them is an M-qubit system and the other is an N-level qudit system, e.g. an N-level atom or an $(N+1)$-mode single-photon system.

3. Teleportation of an M-qubit state via an entangled N-level qudit quantum channel

Now Alice is ready to use the entangled N-level qudit pair channel to teleport the state $|\psi\rangle_b$ to Bob. Suppose the channel is

$$|\psi\rangle_{AB} = \frac{1}{\sqrt{N}} \sum_{m=0}^{N-1} |\varphi_m\rangle_A |\varphi_m\rangle_B, \tag{5}$$

where both $|\varphi_m\rangle_A$ and $|\varphi_m\rangle_B$ are orthonormal basis of the N-dimensional Hilbert space spanned by an N-level qudit system. The total state of the system b and A, B can be written as

$$|\Psi\rangle_{bAB} = \frac{1}{\sqrt{N}} \sum_{k=0}^{N-1} \sum_{m=0}^{N-1} \gamma_k |\varphi_k\rangle_b |\varphi_m\rangle_A |\varphi_m\rangle_B, \tag{6}$$

with the system b, A in Alice's lab and system B in Bob's lab. Then, following the original proposal of Bennett *et al*,[1] a generalized Bell state measurement is performed on the system b, A by Alice. The eigenstates of the measurement can be taken as

$$|\Phi_{nm}\rangle_{bA} = \frac{1}{\sqrt{N}} \sum_{j=0}^{N-1} \mathrm{e}^{2\pi i j n/N} |\varphi_j\rangle_b |\varphi_j \oplus \varphi_m\rangle_A, \tag{7}$$

where the symbol $\oplus$ denotes addition modulo N. After the measurement, the state in Bob's lab collapses to the result

$$|\psi\rangle_B = \sum_{j=0}^{N-1} \gamma_j \mathrm{e}^{-2\pi i j n/N} |\varphi_j \oplus \varphi_m\rangle_B . \tag{8}$$

According to the information of n, m sent by Alice through a classical communication channel, Bob performs on system B the unitary transformation

$$U_{nm} = \sum_{k=0}^{N-1} \mathrm{e}^{2\pi i k n/N} |\varphi_k\rangle \langle\varphi_k \oplus \varphi_m| . \tag{9}$$

This transformation bring system B to the original state of system b

$$|\psi\rangle_B = \sum_{j=0}^{N-1} \gamma_j |\varphi_j\rangle . \tag{10}$$

Finely, an operation

$$P^{\dagger}_{Ba'} = \sum_{k=0}^{N-1} |k\rangle \langle\varphi_k| \tag{11}$$

is acted on the state $|\psi\rangle_B$. The operation $P^{\dagger}_{Ba'}$ represents an isomorphic mapping from Hilbert space $\mathcal{H}_b$ to another Hilbert space $\mathcal{H}_{a'}$ which is spanned by an M-qubit in Bob's lab. Hence the operation $P^{\dagger}_{Ba'}$ will convert the state $|\psi\rangle_B$ into the desired M-qubit state

$$|\psi\rangle_{a'} = \sum_{k=0}^{N-1} \gamma_k |k\rangle = \prod_{i=0}^{M-1} (\alpha_i |0\rangle + \beta_i |1\rangle) . \tag{12}$$

4. Implementation of M-qubit teleportation with a quantum state converter

We thus have seen that the key point of the above scheme for teleportation of an M-qubit state through an entangled N-level qudit pair channel is whether or not the isomorphic mappings $\mathcal{H}_a \to \mathcal{H}_b$ and $\mathcal{H}_b \to \mathcal{H}_{a'}$ can be realized physically. That is, to complete the teleportation, some physical actions which can accomplish the operations P_{ab} and $P^{\dagger}_{Ba'}$ have to be presented. Unfortunately, it is still a big challenge for converting quantum states between an M-qubit and an N-level qudit experimentally with presently achievable techniques. However, very recently, Clausen *et al*[31] proposed a quantum state converter composed of beam splitters and cross-Kerr couplers and show that with this device a state $|\psi\rangle_a$ in $\mathcal{H}_a$, which is spanned by the k-photon single-mode states $|k\rangle = (k!)^{-1/2} \left(a^{\dagger}\right)^k |0\rangle_a$, $k = 0, 1, \ldots, N-1$, can be converted into a state $|\psi\rangle_b$ in $\mathcal{H}_b$, which is spanned by the N-mode single-photon states $|\varphi_k\rangle = b^{\dagger}_k |0\rangle_{b_0} \cdots |0\rangle_{b_{N-1}}$. We will hire this device to demonstrate how to implement the state conversion between an M-qubit and an N-level qudit and how to complete the teleportation of an M-qubit state through an N-level qudit quantum channel.

Consider a device M composed of two $2N$-port beamsplitter arrays B, $B^{\dagger}$ and N cross-Kerr couplers K_k which realize the transformations

$$K_k = \mathrm{e}^{\mathrm{i}\kappa_k b^{\dagger}_k b_k a^{\dagger} a}, \tag{13}$$

where κ_k are the strengths of the cross-Kerr interactions, $b^{\dagger}_k, b_k, a^{\dagger}$and a are creation and annihilation operators in Hilbert spaces $\mathcal{H}_b$ and $\mathcal{H}_a$, respectively, as shown in Fig.1.[31] The action of the device is

$$M = B^{\dagger} \left(\prod_{k=0}^{N-1} K_{N-k} \right) B = V^{a^{\dagger} a}, \tag{14}$$

where

$$V = B^{\dagger} \exp\left(\mathrm{i} \sum_{k=0}^{N-1} \kappa_k b^{\dagger}_k b_k \right) B \tag{15}$$

describes a $2N$-port beam splitters arrays acting in $\mathcal{H}_b$ as a unitary operator

$$V_b = \left(P_{ab} P^{\dagger}_{ba}\right) V \left(P_{ab} P^{\dagger}_{ba}\right) = \sum_{k=0}^{N-1} \mathrm{e}^{\mathrm{i}\kappa_k} B^{\dagger} |\varphi_k\rangle \langle\varphi_k| B. \tag{16}$$

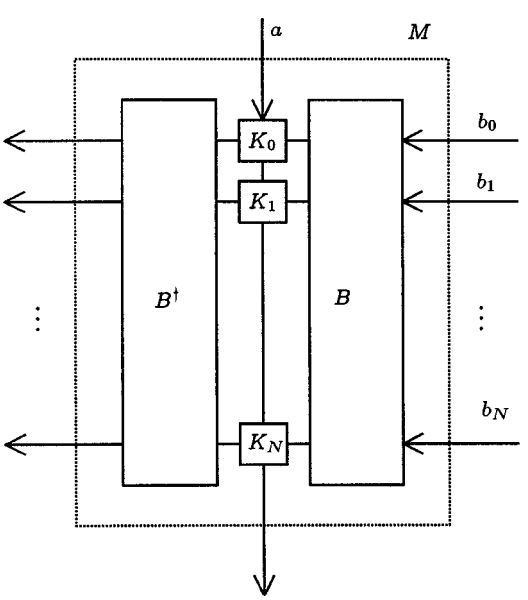


Fig.1. Quantum-state converter M, B and $B^{\dagger}$ are two $2(N+1)$-port beam splitter arrays and $K_0, \ldots, K_N$ are cross-Kerr couplers.

Combining M with N separate beam splitters U_k and choosing the parameters of the beam splitters and cross-Kerrs properly, Clausen *et al* have shown in Ref.[31] that the function of the device can be represented by a transformation operator

$$Y\left(0_{c_0},\ldots,0_{c_{N-1}},\psi\right)=\left|\langle k|\ \psi\rangle\right|_{\min}P_{ba}$$
$$=\left|\langle k|\ \psi\rangle\right|_{\min}\sum_{k=0}^{N-1}|\varphi_k\rangle\langle k|\,. \tag{17}$$

That is, if the outgoing c-modes are detected in vacuum states while the outgoing a-mode of device M is detected in a state $|\psi\rangle$, then the combined set-up shown in Fig.2(a) converts a $\mathcal{H}_a$-input state of the a-mode into a $\mathcal{H}_b$-output state of the b-modes. We have shown in the beginning that an M-qubit product state $|\psi\rangle_a$ in Eq.(1) can be expressed as a superposition state of $N=2^M$ items in Eq.(2). It can be regarded as expressing an M-photon superposition product state in a Hilbert space $\mathcal{H}_a$ spanned by the k-photon single-mode states $|k\rangle=(k!)^{-1/2}\left(a^{\dagger}\right)^k|0\rangle_a$, where $k=0,1,\ldots,N-1$. Therefore, the set-up shown in Fig.2(a) can convert it into the state $|\psi\rangle_b$ in $\mathcal{H}_b$ spanned by N-mode single-photon states $|\varphi_k\rangle=b_k^{\dagger}|0\rangle_{b_0}\cdots|0\rangle_{b_{N-1}}$, and the states $|\varphi_k\rangle$ are the binary form of the eigenstates of the N-level qudit system.

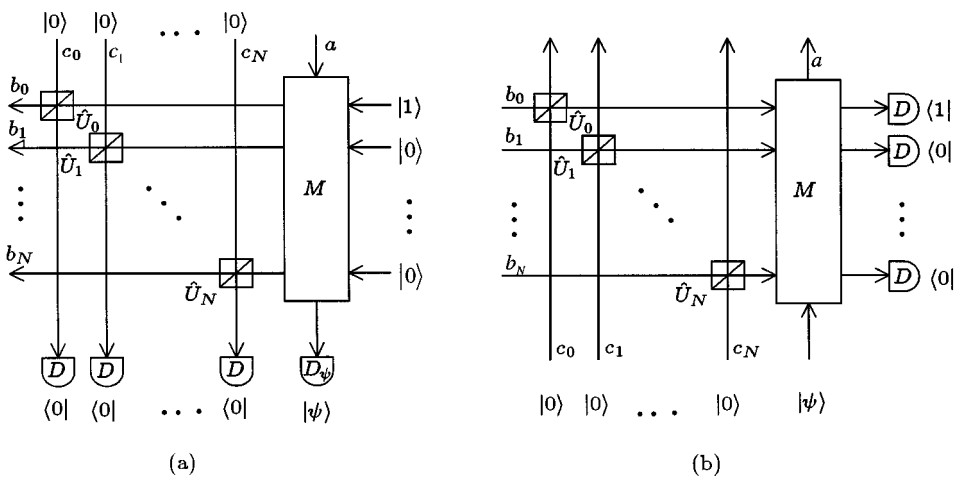


Fig.2. Quantum-state converter for (a) realizing the transformation operator Y given in Eq.(17) and (b) the adjoint $Y^{\dagger}$ given in Eq.(18).

In Fig.2(b), the device runs backwards and the transformation operator becomes

$$Y\left(1_{c_0}\right)=Y^{\dagger}\left(0_{c_0},\ldots,0_{c_{N-1}},\psi\right)=\left|\langle k|\ \psi\rangle\right|_{\min}P_{ba}^{\dagger}$$
$$=\left|\langle k|\ \psi\rangle\right|_{\min}\sum_{k=0}^{N-1}|k\rangle\langle\varphi_k|\,. \tag{18}$$

Now if the incoming a-mode is prepared in the state $|\psi\rangle$ and the incoming c-modes are prepared in vacuum states, while the state $|\varphi_0\rangle|0\rangle_{c_0}\cdots|0\rangle_{c_N}$ is measured by detecting photon presence in the outgoing b_0-channel, then a $\mathcal{H}_b$-input state of the b-modes is converted into a $\mathcal{H}_a$-output state of the a-mode. This device can be used to reconvert the state $|\psi\rangle_B$ in Eq.(10) into the state $|\psi\rangle_{a'}$ in Eq.(12).

Equipped with the quantum state converters described above, teleportation of an M-photon product state through an entangled single-photon N-mode state channel can be implemented in an experimentally feasible way. In fact, a simple set-up which combines two quantum state converters M, $M^{\dagger}$ and a set of beam splitters U_k as shown in Fig.3 can realize the following transformation operation:

$$Y\left(1_{b_0},\Phi\right)=N^{-1}P_{ba'}^{\dagger}P_{ba}. \tag{19}$$

Obviously, the operator $Y\left(1_{b_0},\Phi\right)$ can transform the superposition state $|\psi\rangle_a$ of k-photon single-mode states given in Eq.(2) in Alice's lab to the superposition state $|\psi\rangle_{a'}$ of k-photon single-mode states given in Eq.(12) in Bob's lab. The beamsplitter arrays and the left-hand converter in Fig.3 can be regarded as forming the source of entangled state with a multimode output to Alice and a single-mode output to Bob. Bob operates an optical shutter S letting his signal prepared in the output state $|\psi\rangle_{a'}$ pass only if he has received classical information from Alice confirming the detection of the states $|\Phi_P\rangle$ and $|\varphi_0\rangle$ in the respective output channels of her state converter.

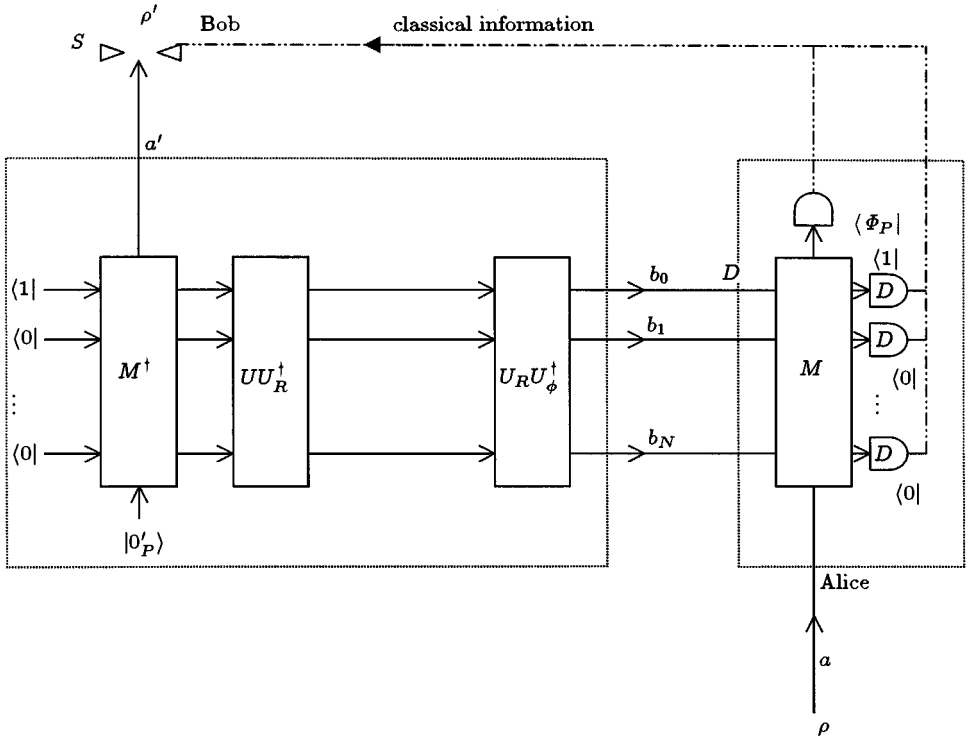


Fig.3. Scheme for quantum teleportation. Bob opens the shutter S only after having received confirmation from Alice that proper state detection has been realized.

5. Remarks and conclusions

We would like to point out that the scheme proposed above will be very useful when the qubits to be teleported are some matter systems. In that case, Alice first transforms the state of the M-qubit to a superposition state $|\psi\rangle_a$ of k-photon single-mode states given in Eq.(2). It can be accomplished by interacting the qubits with electromagnetic field, e.g., interacting two-level atoms with electromagnetic field in a single-mode cavity.[32] Then the teleportation procedures shown in Fig.3 follows. It results in the superposition state $|\psi\rangle_{a'}$ in Eq.(12) of k-photon single-mode states. Alice converts the state $|\psi\rangle_a$ into the superposition state $|\psi\rangle_b$ of single-photon N-mode states. After that, the superposition state of k-photon single-mode states can be transformed to the M-qubit state, which Alice wishes to teleport to Bob, in Bob's lab by interacting M two-level atoms with the k-photon single-mode states in a cavity.[33]

In summary, we have investigated the problem of teleportation of an M-qubit state by using an entangled qudit pair as quantum channel. We have shown that, mathematically, teleportation of a multiparticle state can correspond to teleportation of a multi-dimensional state. We have also introduced a quantum-state converter composed of beamsplitter arrays, on/off-detectors and cross-Kerr couplers and have shown that the state conversion from an M-qubit to an N-dimensional qudit and vice versa can be implemented with this converter, where $N = 2^M$. Based on this, an experimentally feasible scheme for teleportation of an M-qubit via an entangled N-level qudit pair channel has been proposed.

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