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Quantum teleportation of entangled squeezed vacuum states*

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An optical scheme for probabilistic teleporting entangled squeezed vacuum states (SVS) is proposed. In this scheme, the teleported state is a bipartite entangled SVS, and the quantum channel is a tripartite entangled SVS. The process of the teleportation is achieved by using a 50/50 symmetric beamsplitter and photon detectors with the help of classical information.

Keywords: quantum teleportation, squeezed vacuum state, beamsplitter, detection of photon
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1. Introduction

Quantum teleportation, first introduced by Bennett *et al*,[1] has become one of the cores in the rapidly developing field of quantum information theory in recent years. It is a process in which an unknown quantum state is transmitted from a system at the sender's (Alice's) location to another identical system at the receiver's (Bob's) location through a quantum channel with the help of some classical information. The original proposal[1,2] for quantum teleportation was focused on teleporting quantum states of a system with a finite-dimensional (discrete variable) state space, such as the two polarizations of a photon or the discrete levels of an atom. Since the proposals were made, a number of experimentally feasible schemes have been suggested for the teleportation of two-level atomic states[3−9] and field states.[10−13] Discrete-variable teleportation was first realized experimentally by using a polarization entangled state of photons as quantum channel by the Innsbruch group.[14]

Since the experimental demonstration of quantum teleportation of coherent states,[15] much interest has arisen in teleporting continuous variable quantum states. In the Caltech teleportation experiment,[15] an unknown coherent state was transmitted from a sender to a receiver, the state used for the teleportation was a two-mode squeezed state of the form

$$|\Psi_r\rangle_{1,2} = \frac{1}{\cosh r}\sum_{n=0}^{\infty}(\tanh r)^n|n\rangle_1|n\rangle_2, \qquad (1)$$

where $|n\rangle$ denotes a state with n photons and r is a real parameter. In the proposal of teleportation presented by van Enk *et al*,[16] the state transported was an arbitrary coherent superposition state of two coherent states $|\alpha\rangle$ and $-|\alpha\rangle$, and the quantum channel was an entangled coherent state

$$|\Psi_\alpha\rangle_{1,2} = \frac{1}{\sqrt{N_\alpha}}(|\alpha\rangle_1|\alpha\rangle_2 + \mathrm{e}^{\mathrm{i}\theta}|-\alpha\rangle_1|-\alpha\rangle_2), \qquad (2)$$

where $|\alpha\rangle = \exp(-\mid\alpha\mid^2/2)\exp(\alpha\hat{a}^\dagger)|0\rangle$ is a coherent state.

All these schemes are for the teleportation of single-qubit state. In many potential applications of quantum information processing, people needs to handle the system of many-qubit states. Therefore, researchers suggested many schemes for teleporting the multiqubit entangled states.[17−24] In this paper, we present a scheme for the teleportation of a two-particle (two-qubit) entangled squeezed vacuum state (SVS) by using a 50/50 symmetric beamsplitter and photodetectors. In our scheme, a tripartite entangled SVS is used as the quantum channel for realizing the teleportation.

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2. The function of a beam splitter

We first briefly review the function of a beamsplitter. The lossless symmetric 50/50 beamsplitter is described by

$$\hat{B}_{12} = \exp\left[\mathrm{i}\frac{\pi}{4}(\hat{a}_1^\dagger \hat{a}_2 + \hat{a}_2^\dagger \hat{a}_1)\right], \tag{3}$$

where $\hat{a}_i$ and $\hat{a}_i^\dagger$ (i denote 1,2) are the bosonic annihilation and creation operators of system i, respectively. It transforms the coherent state $|\alpha\rangle_1|\beta\rangle_2$ as

$$\hat{B}_{12}|\alpha\rangle_1|\beta\rangle_2 = |(\alpha+\mathrm{i}\beta)/\sqrt{2}\rangle_1|(\beta+\mathrm{i}\alpha)/\sqrt{2}\rangle_2. \tag{4}$$

This transformation plays an important role in the realization of quantum information processing involving coherent states. It is more intricate to transform SVSs using beam splitters. Before presenting our quantum teleportation scheme, we first analyse the action of the beamsplitter on SVSs. Let $\hat{a}_1$ and $\hat{a}_2$ denote the annihilation operators for the two light beams entering the two input ports of the beamsplitter and let $\hat{A}_1$ and $\hat{A}_2$ denote the annihilation operators for the two light beams leaving the two output ports of the 50/50 beamsplitter. The boundary conditions at the surface of the beamsplitter lead to the well-known input-output relations

$$\hat{A}_1 = \frac{1}{\sqrt{2}}(\hat{a}_1 + \mathrm{i}\hat{a}_2),$$
$$\hat{A}_2 = \frac{1}{\sqrt{2}}(\hat{a}_2 + \mathrm{i}\hat{a}_1). \tag{5}$$

We assume that the two input light beams of the beamsplitter are in SVSs

$$|\Psi\rangle_{\mathrm{in}} = \hat{S}_1(\xi_1)\hat{S}_2(\xi_2)|0,0\rangle, \tag{6}$$

where $\xi = r\mathrm{e}^{\mathrm{i}\varphi}$ is any complex number with squeezing amplitude r and squeezing angle φ and

$$\hat{S}_i(\xi_i) = \exp\left(-\frac{\xi_i}{2}\hat{a}_i^{\dagger 2} + \frac{\xi_i^*}{2}\hat{a}_i^2\right) \tag{7}$$

are the single-mode squeezing operators for mode i (i=1, 2). After interacting with the beamsplitter, the state vector of the output light beams becomes

$$\begin{aligned}|\Psi\rangle_{\mathrm{out}} =& \exp\Big[-\frac{1}{4}(\xi_1-\xi_2)\hat{A}_1^{\dagger 2} + \frac{1}{4}(\xi_1^*-\xi_2^*)\hat{A}_1^2 \\ &+ \frac{1}{4}(\xi_1-\xi_2)\hat{A}_2^{\dagger 2} - \frac{1}{4}(\xi_1^*-\xi_2^*)\hat{A}_2^2 \\ &- \frac{\mathrm{i}}{2}(\xi_1+\xi_2)\hat{A}_1^\dagger \hat{A}_2^\dagger - \frac{\mathrm{i}}{2}(\xi_1^*+\xi_2^*)\hat{A}_1\hat{A}_2\Big]|0,0\rangle.\end{aligned} \tag{8}$$

Now let us consider the following two cases which play a key role in our teleportation scheme.

Case 1 The two input light beams have the same squeezing amplitude and phase, i.e., $r_1 = r_2 = r$ and $\varphi_1 = \varphi_2 = \varphi$. In this case, from Eq.(8) it is easy to see that the state of output light beams is simply a two-mode SVS

$$\begin{aligned}|\Psi\rangle_{\mathrm{out}} =& \exp[-r\mathrm{e}^{\mathrm{i}(\varphi+\pi/2)}\hat{A}_1^\dagger \hat{A}_2^\dagger \\ &+ r\mathrm{e}^{-\mathrm{i}(\varphi+\pi/2)}\hat{A}_1\hat{A}_2]|0,0\rangle.\end{aligned} \tag{9}$$

Case 2 The two input light beams have the same squeezing amplitude but a phase difference of π, i.e., $r_1 = r_2 = r$ and $\varphi_2 - \varphi_1 = \pi$. In this case, the output state of the beamsplitter becomes a direct product of two single-mode SVSs

$$\begin{aligned}|\Psi\rangle_{\mathrm{out}} =& \exp\left(-\frac{1}{2}r\mathrm{e}^{\mathrm{i}\varphi}\hat{A}_1^{\dagger 2} + \frac{1}{2}r\mathrm{e}^{-\mathrm{i}\varphi}\hat{A}_1^2\right) \\ &\times \exp\left(-\frac{1}{2}r\mathrm{e}^{\mathrm{i}(\varphi+\pi)}\hat{A}_2^{\dagger 2}\right. \\ &\left.+ \frac{1}{2}r\mathrm{e}^{-\mathrm{i}(\varphi+\pi)}\hat{A}_2^2\right)|0,0\rangle.\end{aligned} \tag{10}$$

3. The scheme of teleportation

We consider the teleportation of the following two-partite entangled SVS:

$$|\Phi\rangle_{12} = \frac{1}{\sqrt{N_\Phi}}(\alpha|\xi\rangle_1|\xi\rangle_2 + \beta|-\xi\rangle_1|-\xi\rangle_2), \tag{11}$$

where α and β are any complex numbers and N_Φ is the normalization factor given by

$$N_\Phi = |\alpha|^2 + |\beta|^2 + 2k_\xi^2\mathrm{Re}(\alpha\beta^*) \tag{12}$$

with $k_\xi = \langle\xi|-\xi\rangle = \sqrt{\mathrm{sech}(2r)}$ being the overlap of the two SVSs $|\xi\rangle$ and $|-\xi\rangle$. In order to teleport our state $|\Phi\rangle_{12}$, we at least need a tripartite entangled state as a quantum channel. We consider the following quantum channel:

$$|\Psi\rangle_{345} = \frac{1}{\sqrt{N_\Psi}}(|\xi\rangle_3|\xi\rangle_4|\xi\rangle_5 - |-\xi\rangle_3|-\xi\rangle_4|-\xi\rangle_5), \tag{13}$$

where $N_\Psi = 2(1-k_\xi^3)$ is the normalization factor. In the above Eqs.(11) and (13), $|\xi\rangle$ and $|-\xi\rangle$ are two equal-amplitude and opposite-phase single-mode SVSs of a harmonic oscillator, and the subscript $i(i = 1,2,3,4,5)$ labels the modes. The preparation of the entangled SVSs given by Eqs.(11) and (13) has been discussed in Ref.[25,26]. Entanglement concentration

of entangled SVSs has been discussed in Ref.[27]. An expansion of single-mode SVS in terms of Fork states is

$$|\xi\rangle = \sqrt{\mathrm{sech}r}\sum_{n=0}^{\infty}\frac{\sqrt{(2n)!}}{n!}\left(-\frac{1}{2}\mathrm{e}^{\mathrm{i}\varphi}\tanh r\right)^n|2n\rangle. \quad (14)$$

Now Alice wishes to teleport the entangled SVS $|\Phi\rangle_{12}$ to the remote partner Bob. After sharing the quantum channel $|\Psi\rangle_{345}$ with Bob, firstly Alice mixes her part of the entangled state (mode 3) with the state to be teleported. Then the initial state of the whole system consisting of modes $1, 2, 3, 4$, and 5 is given by

$$\begin{aligned}|\Psi\rangle_{12345} =&|\Phi\rangle_{12}|\Psi\rangle_{345}\\ =&\frac{1}{\sqrt{N_\Phi N_\Psi}}(\alpha|\xi\rangle_1|\xi\rangle_2|\xi\rangle_3|\xi\rangle_4|\xi\rangle_5\\ &-\alpha|\xi\rangle_1|\xi\rangle_2|-\xi\rangle_3|-\xi\rangle_4|-\xi\rangle_5\\ &+\beta|-\xi\rangle_1|-\xi\rangle_2|\xi\rangle_3|\xi\rangle_4|\xi\rangle_5\\ &-\beta|-\xi\rangle_1|-\xi\rangle_2|-\xi\rangle_3|-\xi\rangle_4|-\xi\rangle_5).\end{aligned} \quad (15)$$

Note that at this time the modes $1, 2$ and 3 are located at the sender's side and modes 4 and 5 are at the receiver's side.

Secondly, Alice lets modes 2 and 3 enter the input ports of the beamsplitter, and transmits mode 1 to Clara (the third observer is introduced for convenience, in principle, she is not needed). After interacting with the beamsplitter, the state of the whole system becomes

$$\begin{aligned}|\Psi\rangle_{12345} =&\frac{1}{\sqrt{N_\Phi N_\Psi}}[\alpha|\xi\rangle_1\hat{S}_{23}(\xi)|0,0\rangle_{23}|\xi\rangle_4|\xi\rangle_5\\ &-\alpha|\xi\rangle_1|\xi\rangle_2|-\xi\rangle_3|-\xi\rangle_4|-\xi\rangle_5\\ &+\beta|-\xi\rangle_1|-\xi\rangle_2|\xi\rangle_3|\xi\rangle_4|\xi\rangle_5\\ &-\beta|-\xi\rangle_1\hat{S}_{23}(-\xi)|0,0\rangle_{23}|-\xi\rangle_4|-\xi\rangle_5],\end{aligned} \quad (16)$$

where the two-mode squeezed operator is given by

$$\hat{S}_{23}(\xi) = \exp(-\xi\hat{A}_2^\dagger\hat{A}_3^\dagger + \xi^*\hat{A}_3\hat{A}_2). \quad (17)$$

Thirdly, Alice performs photon number measurement on modes 2 and 3 on her side. From Eq.(16) it can be seen that for modes 2 and 3, the first and fourth terms on the right-hand side is the two-mode SVS in which each mode has the same photon number with both even and odd photons in the photon number-state representation, while the second and third terms is the single-mode SVS which contains only even-number of photons in their number-state representation. When the results of measurements are odd number of photons, from Eq.(16) it can be seen that modes 1, 4 and 5 collapse into (the unnormalized state)

$$|\Psi\rangle_{145} = \alpha|\xi\rangle_1|\xi\rangle_4|\xi\rangle_5 + \beta|-\xi\rangle_1|-\xi\rangle_4|-\xi\rangle_5. \quad (18)$$

Spreading the state of mode 1 in terms of Fork states, the state $|\Psi\rangle_{145}$ can be rewritten as

$$\begin{aligned}|\Psi\rangle_{145} =&\sqrt{\mathrm{sech}r}\sum_{n=0}^{\infty}\frac{\sqrt{(2n)!}}{n!}\left(-\frac{1}{2}\mathrm{e}^{\mathrm{i}\varphi}\tanh r\right)^n|2n\rangle_1\\ &\times[\alpha|\xi\rangle_4|\xi\rangle_5 + \beta(-1)^n|-\xi\rangle_4|-\xi\rangle_5],\end{aligned} \quad (19)$$

where n denotes the number of the photon pairs in mode 1.

Fourthly, Clara performs photon number measurement on her system. When the results of measurements are the even number of photon pairs, from Eq.(19) we see that modes 4 and 5 collapse into

$$|\Psi\rangle_{45} = \alpha|\xi\rangle_4|\xi\rangle_5 + \beta|-\xi\rangle_4|-\xi\rangle_5, \quad (20)$$

which indicates that Alice transmits faithfully the entangled SVS given in Eq.(11) to Bob.

4. Summary

In summary, we have presented a simple scheme to teleport a bipartite entangled SVS. In this scheme, we exploited the important features of the beamsplitter transforming SVSs, which are that if the two input light beams of the beamsplitter have the same squeezing amplitude and phase, the output state is simply a two-mode SVS; and that if the two input light beams of the beamsplitter have the same squeezing amplitude but a phase difference of π, the output state is a direct product state of two single-mode SVSs. The key factor in our teleportation scheme is photon number measurement which must be sensitive enough to measure the number of photons and determine whether the number is even or odd. In practice this is difficult, especially for large numbers of photons, but in principle, this can be done. In Ref.[20] Wang described one method to distinguish even and odd n by coupling the field to a two-level atom through dispersive interaction, which can enhance the possibility to realize this scheme.

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