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Quantum nondemolition measurement of photon-number distribution for a weak cavity field with resonant atoms*

Zheng Shi-Biao(郑仕标)

Department of Electronic Science and Applied Physics, Fuzhou University, Fuzhou 350002, China

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We propose a quantum nondemolition measurement of the photon-number distribution for a weak cavity field with no more than two photons. The scheme is based on the resonant interaction of atoms with the cavity field, and thus the required interaction time is much shorter than that using dispersive interaction. This is important in view of decoherence. Our scheme can also be used to generate even and odd coherent states for a weak cavity field with resonant atoms.

Keywords: quantum nondemolition measurement, photon-number distribution, resonant atom
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In order to measure a quantum observable of a system, one should couple it to a meter or probe, which will disturb the system in an unpredictable manner. If the disturbance feeds back into the measured observable and affects its dynamics, the outcome of further measurements of the same observable becomes unpredictable. Quantum nondemolition (QND) measurement method is designed to avoid the back-action resulting from the coupling of the system and the meter. The back-evasion condition is fulfilled if the QND variable commutes with the interaction Hamiltonian for the system and meter.

In an optical regime QND schemes have been proposed by using nonlinear interaction of signal and probe fields,[1] where a quadrature phase measurement of the probe beam gives information on the photon number of the signal beam. Such QND schemes have been realized in experiment.[2] In the microwave domain, a QND method has been proposed for measuring the photon-number in a cavity,[3] where the field is coupled to quasi-resonant Rydberg atoms. The photon-number is measured by the detection of the atomic phase shifts and velocities of atoms passing through the cavity. Holland *et al*[4] has proposed another QND measurement for photon-number statistics of a cavity field by monitoring the deflection of atoms after dispersive interactions with the field. Recently, QND schemes have been presented for measuring phonons[5] and the motional energy of an atom confined in a trap.[6] In a very recent paper, Nogues *et al* reported a QND measurement for a cavity field containing zero or one photon.[7] The scheme of Ref.[7] is to use resonant atom-field interaction, instead of dispersive interaction. Therefore, the needed interaction time is greatly shortened, which is important in view of decoherence. However, the scheme cannot be used to measure a cavity field with two or more photons. In this paper we propose a QND measurement of the photon-number distribution for a cavity field. Our scheme also uses resonant atom-field interaction. However, our scheme is applicable to a cavity field containing no more than two photons. Our scheme can also be used to generate even and odd coherent states for a weak cavity field with resonant atoms.

We assume the density operator of the cavity field is

$$\hat{\rho}_{\rm f} = \sum_{m,n=0}^{2} \rho_{mn} |m\rangle \langle n| . \qquad (1)$$

We consider the interaction of a ladder-type three-level atom with the cavity mode. The atomic states

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are denoted by $|i\rangle$, $|g\rangle$, and $|e\rangle$. The transition frequency between the states $|g\rangle$ and $|e\rangle$ is resonant with the cavity frequency, while the transition frequency between the states $|i\rangle$ and $|g\rangle$ is highly detuned from the cavity frequency, and thus the state $|i\rangle$ is not affected during the atom-cavity interaction. In the interaction picture the Hamiltonian for the system is given by

$$\hat{H}_{\rm I} = g(\hat{a}^{+}\,|g\rangle\,\langle e| + \hat{a}\,|e\rangle\,\langle g|), \tag{2}$$

where g is the atom-field coupling constant.

The atom is initially prepared in the superposition of $|i\rangle$ and $|g\rangle$

$$|\phi_a\rangle = \frac{1}{\sqrt{2}}(|i\rangle + |g\rangle). \tag{3}$$

For the vacuum state $|0\rangle$ the atom does not undergo any transition. For the Fock state $|n\rangle$ $(n \neq 0)$ the evolution of the system is

$$\begin{aligned}|\psi_n(\tau)\rangle =& \frac{1}{\sqrt{2}}[|i\rangle\,|n\rangle + \cos(\sqrt{n}g\tau)\,|g\rangle\,|n\rangle \\ & - {\rm i}\sin(\sqrt{n}g\tau)\,|e\rangle\,|n-1\rangle].\end{aligned} \tag{4}$$

Choose the interaction time appropriately so that

$$\cos(g\tau) \simeq 1, \;\; \cos(\sqrt{2}g\tau) \simeq -1. \tag{5}$$

In this case we obtain the following transformations

$$\frac{1}{\sqrt{2}}(|i\rangle + |g\rangle)\,|0\rangle \rightarrow \frac{1}{\sqrt{2}}(|i\rangle + |g\rangle)\,|0\rangle\,, \tag{6}$$

$$\frac{1}{\sqrt{2}}(|i\rangle + |g\rangle)\,|1\rangle \rightarrow \frac{1}{\sqrt{2}}(|i\rangle + |g\rangle)\,|1\rangle\,, \tag{7}$$

$$\frac{1}{\sqrt{2}}(|i\rangle + |g\rangle)\,|2\rangle \rightarrow \frac{1}{\sqrt{2}}(|i\rangle - |g\rangle)\,|2\rangle\,. \tag{8}$$

Thus, the photon-number does not change after the atom-field interaction. We now perform a transformation on the atom

$$|g\rangle \longrightarrow \frac{1}{\sqrt{2}}(|g\rangle - |i\rangle), \tag{9}$$

$$|i\rangle \longrightarrow \frac{1}{\sqrt{2}}(|i\rangle + |g\rangle). \tag{10}$$

Then we have

$$\frac{1}{\sqrt{2}}(|i\rangle + |g\rangle) \longrightarrow |g\rangle\,, \tag{11}$$

$$\frac{1}{\sqrt{2}}(|i\rangle - |g\rangle) \longrightarrow |i\rangle\,. \tag{12}$$

Therefore, for the vacuum and one-photon states the atom is finally in the state $|g\rangle$, while for the two-photon state the atom finally in the state $|i\rangle$. For the cavity field with the density operator initially given by Eq. (1), the evolution of the system is expressed as

$$\begin{aligned}\hat{\rho} =& \sum_{m,n=0}^{1} \rho_{mn}\,|g\rangle\,|m\rangle\,\langle n|\,\langle g| + \rho_{22}\,|i\rangle\,|2\rangle\,\langle 2|\,\langle i| \\ & + \sum_{m=0}^{1}(\rho_{m2}\,|g\rangle\,|m\rangle\,\langle 2|\,\langle i| + \rho_{2m}\,|i\rangle\,|2\rangle\,\langle m|\,\langle g|).\end{aligned} \tag{13}$$

Now a state-selective measurement is performed on the atom. The detection of the atom in the state $|i\rangle$ collapses the cavity field to the two-photon state $|2\rangle$. The probability of the cavity field reduced to $|2\rangle$ is ρ_{22}.

On the other hand, the detection of the atom in the state $|g\rangle$ collapses the cavity field to

$$\hat{\rho}'_{\rm f} = \frac{1}{\rho_{00}+\rho_{11}} \sum_{m,n=0}^{1} \rho_{mn}\,|m\rangle\,\langle n|\,. \tag{14}$$

In this case a second atom is sent through the cavity to measure the photon numbers 0 and 1.[7] The atom is also initially in the state $\frac{1}{\sqrt{2}}(|i'\rangle + |g'\rangle)$. This time the interaction is chosen in a way such that $g\tau' = \pi$. Then we obtain

$$\frac{1}{\sqrt{2}}(|i'\rangle + |g'\rangle)|0\rangle \rightarrow \frac{1}{\sqrt{2}}(|i'\rangle + |g'\rangle)|0\rangle, \tag{15}$$

$$\frac{1}{\sqrt{2}}(|i'\rangle + |g'\rangle)|1\rangle \rightarrow \frac{1}{\sqrt{2}}(|i'\rangle - |g'\rangle)|1\rangle. \tag{16}$$

Again, the photon-number does not change after the atom-field interaction. We now perform a transformation of the form of Eqs.(9) and (10). Then the system evolves to

$$\begin{aligned}\hat{\rho}' =& \frac{1}{\rho_{00}+\rho_{11}}[\rho_{00}|g'\rangle|0\rangle\langle 0|\langle g'| \\ & + \rho_{11}|i'\rangle|1\rangle\langle 1|\langle i'| + (\rho_{01}|g'\rangle|0\rangle\langle 1|\langle i'| \\ & + \rho_{10}|i'\rangle|1\rangle\langle 0|\langle g'|)].\end{aligned} \tag{17}$$

And a state-selective measurement is performed on this atom. The detection of the atom in the state $|i'\rangle$ collapses the cavity field to the one-photon state $|1\rangle$. On the other hand, the detection of the atom in the state $|g'\rangle$ reduces the cavity field to the vacuum state $|0\rangle$. The probabilities of the cavity field reduced to $|0\rangle$ and $|1\rangle$ are ρ_{00} and ρ_{11}, respectively.

In order for the scheme to be valid we should satisfy the condition of Eq.(5). Setting $g\tau = 2.07\pi$, we have $|\cos(g\tau)| \simeq 0.98$, $|\cos(\sqrt{2}g\tau)| \simeq -0.97$. Therefore, this condition can be well approximated.

We note that this idea can also be used to generate even and odd coherent states for a weak cavity field. It has been shown that such states may exhibit various nonclassical properties, such as squeezing and sub-Poissonian statistics.[8] Brune *et al*[9] have shown that such states can be generated using dispersive atom-field interaction. Liu *et al*[10] have shown that such states can be generated via the interaction of the field with a three-level atom. Here we show that such state can be generated with resonant atom-field interaction. We consider a cavity field initially in a small coherent state. The coherent state can be expanded in terms of Fock states

$$|\alpha\rangle = \mathrm{e}^{-|\alpha|^2/2}\sum_{n=0}^{\infty}\frac{\alpha^n}{\sqrt{n!}}|n\rangle. \tag{18}$$

Assume that α is small enough so that the terms with $n > 3$ are negligible. In this case the coherent state $|\alpha\rangle$ is approximately given by

$$|\alpha\rangle \simeq \mathrm{e}^{-|\alpha|^2/2}\sum_{n=0}^{3}\frac{\alpha^n}{\sqrt{n!}}|n\rangle. \tag{19}$$

We now send a resonant atom, initially in the state $|g\rangle$ through the cavity. After an interaction time τ the state of the whole system is

$$\begin{aligned}|\psi(\tau)\rangle =& \mathrm{e}^{-|\alpha|^2/2}\Big\{|g\rangle|0\rangle \\ &+\sum_{n=1}^{3}\frac{\alpha^n}{\sqrt{n!}}[\cos(g\sqrt{n}\tau)|g\rangle|n\rangle \\ &-\mathrm{i}\sin(g\tau)|e\rangle|n-1\rangle]\Big\}.\end{aligned} \tag{20}$$

The detection of the atom in the state $|g\rangle$ collapses the cavity field on to the state

$$|\psi_{\mathrm{f}}(\tau)\rangle = N\Big\{|0\rangle + \sum_{n=1}^{3}\frac{\alpha^n}{\sqrt{n!}}\cos(\sqrt{n}g\tau)|n\rangle\Big\}, \tag{21}$$

where N is a normalization factor. Choose the interaction time appropriately so that

$$\cos(g\tau) \simeq \cos(\sqrt{3}g\tau) \simeq 0, \;\; \cos(\sqrt{2}g\tau) \simeq 1. \tag{22}$$

This leads to

$$|\psi_{\mathrm{f}}(\tau)\rangle \simeq N\{|0\rangle + \frac{\alpha^2}{\sqrt{2!}}|2\rangle\}. \tag{23}$$

We can rewrite $|\psi_{\mathrm{f}}(\tau)\rangle$ as

$$|\psi_{\mathrm{f}}(\tau)\rangle \simeq |\psi_{\mathrm{e}}\rangle = \cosh^{-1/2}|\alpha|^2\{|\alpha\rangle + |-\alpha\rangle\}. \tag{24}$$

Therefore, we obtain an even coherent state for the weak cavity field.

We assume that $\alpha = 1$. Then the probability of the initial cavity field containing more than 3 photons is approximately 0.02. Thus Eq.(19) is a well approximation. Setting $g\tau = 1.46\pi$, we have

$$\begin{aligned}&\cos(g\tau) \simeq -0.125, \quad \cos(\sqrt{2}g\tau) \simeq 0.979, \\ &\cos(\sqrt{3}g\tau) \simeq -0.09.\end{aligned} \tag{25}$$

Eq. (22) can also be approximately satisfied. In this case the fidelity is

$$\begin{aligned}&|\langle\psi_{\mathrm{f}}(\tau)|\psi_{\mathrm{e}}\rangle|^2 \\ \simeq&\frac{\Big[1+\frac{1}{2}\cos(\sqrt{2}g\tau)\Big]^2}{\cosh 1\Big[1+\sum_{n=1}^{3}\frac{1}{n!}\cos^2(\sqrt{n}g\tau)\Big]} \simeq 0.96.\end{aligned} \tag{26}$$

Now we turn to the problem of generating odd coherent states of a cavity field. In order to do so we send an atom initially in the state $|e\rangle$ through the cavity, also initially in the coherent state $|\alpha\rangle$ approximately given by Eq.(19). Then after an interaction time τ, the system evolves to the state

$$\begin{aligned}|\psi(\tau)\rangle =&\mathrm{e}^{-|\alpha|^2/2}\Big\{\sum_{n=0}^{3}\frac{\alpha^n}{\sqrt{n!}}[\cos(g\sqrt{n+1}\tau)|e\rangle|n\rangle \\ &-\mathrm{i}\sin(g\tau)|g\rangle|n+1\rangle]\Big\}.\end{aligned} \tag{27}$$

The detection of the atom in the state $|e\rangle$ collapses the cavity field on to the state

$$|\psi_{\mathrm{f}}(\tau)\rangle = N\Big\{\sum_{n=0}^{3}\frac{\alpha^n}{\sqrt{n!}}\cos(g\sqrt{n+1}\tau)|n\rangle\Big\}, \tag{28}$$

where N is a normalization factor. Setting $g\tau = 1.46\pi$, we have

$$\cos(2g\tau) \simeq -0.968. \tag{29}$$

From Eqs.(25) and (29) we can approximately rewrite Eq.(28) as

$$|\psi_{\mathrm{f}}(\tau)\rangle \simeq N'\Big\{\alpha|1\rangle - \frac{\alpha^3}{\sqrt{3!}}|3\rangle\Big\}, \tag{30}$$

where N' is a normalization factor. We can rewrite $|\psi_{\mathrm{f}}(\tau)\rangle$ as

$$|\psi_{\mathrm{f}}(\tau)\rangle \simeq |\psi_{\mathrm{o}}\rangle = \sinh^{-1/2}|\alpha|^2\{|-\mathrm{i}\alpha\rangle - |\mathrm{i}\alpha\rangle\}. \tag{31}$$

Therefore, we obtain an odd coherent state for the weak cavity field. The fidelity is

$$|\langle\psi_{\rm f}(\tau)|\psi_{\rm o}\rangle|^2 \simeq \frac{\left[\cos(\sqrt{2}g\tau)+\frac{1}{3!}\cos(2g\tau)\right]^2}{\sinh 1\sum_{n=0}^{3}\frac{1}{n!}\cos^2(g\sqrt{n+1}\tau)} \simeq 0.97. \quad (32)$$

In conclusion, we have proposed a scheme for QND measurement of the photon-number for a cavity field containing no more than two photons. Compared with the schemes of Refs.[3] and [4], our scheme uses resonant atom-field interaction, instead of dispersive atom-field interaction. Therefore, the interaction time can be greatly shortened, which is important in view of decoherence. Unlike the scheme of Ref.[7], our scheme is applicable to a cavity field with no more than two photons. Furthermore, our scheme can be used to generate even and odd coherent states for a weak cavity field with resonant atom-field interaction.

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