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The effect of degenerate atomic levels on the field state dissipation in two-photon Jaynes–Cummings model

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The dissipation of the field in the two-photon Jaynes–Cummings model (JCM) with degenerate atomic levels was studied. The initial degenerate atomic state affects the field coherence loss. When the degenerate atom is initially in an equal probability superposition state, the field coherence loss is smallest. It is found that the degeneracy of the atomic level increases the period of entanglement between the atom and the field. When the degeneracy was considered, the coherence properties of the field could be affected by the reservoir qualitatively, if a nonlinear two-photon process is involved. This is different from the dissipation of one-photon JCM with degenerate atomic levels.

Keywords: two-photon JCM, degenerate atomic levels, field dissipation
PACC: 4250, 3280

1. Introduction

The interaction of a single two-level atom with a quantized electromagnetic field is a central problem in cavity quantum electrodynamics (QED). With the development of quantum information, several schemes using cavity QED have been proposed to prepare some kinds of quantum states[1−3] and to teleport an atomic state.[4−6] In these studies, the atom is a pure two-level one. If the levels of an isolated atom are degenerate in the projection of the total electronic angular momentum on the quantization axis, the degeneracy should be taken into account. Recently, the degeneracy of atomic levels has been used in quantum information. For example, the role of the degenerate Zeeman level in the electromagnetically induced transparency has been studied.[7] Duan *et al*[8] proposed a protocol via degenerate atomic levels to teleport an unknown state, where identical atoms have two degenerate ground states and two degenerate excited states. In Refs.[9]–[11], the original Jaynes–Cummings model (JCM) was generalized to the case of degenerate atomic levels.

Since the dissipation of the cavity plays an important role in the entanglement (between the atom and the field) and the decoherence of the system, the atom and the field, many authors have studied the dissipative dynamics of JCM.[11−14] On the other hand, the two-photon process often takes place in experiment. Brune *et al*[15] built a two-photon maser oscillator. In theory, the model for a two-level atom interacting with a single-mode field through a two-photon process has been widely studied by many authors.[16−18] In Ref.[14], our group found that the behaviour of the field dissipation in the two-photon process is different from that in the one-photon JCM.[12] In the one-photon JCM the cavity has practically no influence on the coherence properties of the field from the qualitative point of view. However, when a nonlinear two-photon process is involved, the coherence properties of the field can be affected by the cavity.[14] In Ref.[11], we studied the dissipative dynamics of the one-photon JCM with degenerate atomic levels. In this paper, we study the dissipative dynamics of the field when it interacts with a degenerate-level atom through the two-photon process. We find that the linear entropy is the smallest if the atom is initially in an equal probability degenerate state. The period of entanglement is increased because of the degeneracy of the atomic level, and the field coherence properties are also affected by the reservoir qualitatively.

2. The effective Hamiltonian and dissipative dynamics

Having taken into account the degeneracy of the

atomic levels, we write the full set of states of the system as

$$|n, J_\mu, m_\mu\rangle = |n\rangle \cdot |J_\mu, m_\mu\rangle, \quad n = 0, 1, \ldots,$$
$$m_\mu = -J_\mu, \ldots, J_\mu, \quad \mu = b, c, \tag{1}$$

where n is the number of photons in the cavity field, and b and c denote the upper and lower atomic levels respectively. J_b and J_c are, respectively, the values of the total electronic angular momenta of resonant levels, and m_b and m_c are, respectively, their projections on the quantization axis, the Cartesian axis z, which is directed along the polarization vector of the cavity mode.

We assume that a degenerate-level atom interacts with a single linearly polarized mode field. Hence only the atomic transition could emit a linearly polarized photon that participates in the interaction.[11] The Hamiltonian of the system may be written as ($\hbar = 1$)

$$H = \omega a^+ a + \frac{1}{2}\omega_0(n_b - n_c) + g(a^{+2}S_- + a^2 S_+), \tag{2}$$

where $\hat{a}^+$and $\hat{a}$ are the creation and annihilation operators of the photons with a frequency ω_0 in the cavity mode, and

$$n_\mu = \sum_{m_\mu=-J_\mu}^{J_\mu} |J_\mu, m_\mu\rangle\langle J_\mu, m_\mu|, \quad \mu = b, c \tag{3}$$

are the operators of the total population of resonant atomic levels b and c, ω is the frequency of the atomic transition $J_b \longrightarrow J_c$.

$$S_- = \sum_m \alpha_m |J_c, m\rangle\langle J_b, m| \tag{4}$$

is the dipole moment operator of the atomic transition $J_b \longrightarrow J_c$, where

$$\alpha_m = (-1)^{j_b - m} \begin{pmatrix} J_b & 1 & J_c \\ -m & 0 & m \end{pmatrix} \tag{5}$$

is the matrix elements defined through the Wigner $3j$-symbol.

We consider the far-off resonance limit for the atom-field interaction (dispersive interaction). The Hamiltonian takes the form

$$H_{\rm I} = \frac{\delta}{2}(n_b - n_c) + g(a^{+2}S_- + a^2 S_+), \tag{6}$$

where the detuning is $\delta = 2\omega - \omega_0$. The dipole moment operator of the atomic transitions S_- and S_+ satisfies the commutation relation $[S_+, S_-] = 2S_z$, with

$$S_z = \frac{1}{2}\sum_m \alpha_m^2(|J_b, m\rangle\langle J_b, m| - |J_c, m\rangle\langle J_c, m|). \tag{7}$$

To solve the master equation, here we take a set of unitary transformation to the Hamiltonian of Eq.(6), which is proposed in Ref.[13]. This transformation corresponds to a small rotation in the $SU(2)$ group with an operator parameter

$$H_{\rm eff} = U_2 U_1 H U_1^+ U_2^+, \tag{8}$$

where $U_1 = \exp\left(\mathrm{i}\frac{\sqrt{2}g}{\delta} p S_x\right)$, $U_2 = \exp\left(\mathrm{i}\frac{\sqrt{2}g}{\delta} q S_y\right)$, and $q = \frac{1}{\sqrt{2}}(a + a^+)$, $p = \frac{\mathrm{i}}{\sqrt{2}}(a^+ - a)$, $S_x = \frac{1}{2}(S_+ + S_-)$, $S_y = -\frac{\mathrm{i}}{2}(S_+ - S_-)$. Keeping terms up to first order in $\frac{\sqrt{2}g}{\delta} \ll 1$, we obtain the effective Hamiltonian

$$H_{\rm eff} = \frac{\delta}{2}(n_b - n_c) + \frac{g^2}{\delta}[(a^+ a + 1) \times (a^+ a + 2)R_b - a^+ a(a^+ a - 1)R_c], \tag{9}$$

where

$$R_\mu = \sum_m \alpha_m^2(|J_\mu, m\rangle\langle J_\mu, m|, \qquad \mu = b, c. \tag{10}$$

Note that the first term of Eq.(9) commutes with the effective Hamiltonian $H_{\rm eff}$. We can further simplify the effective Hamiltonian by the following transformation of the operator f

$$\tilde{f} = \exp\left[\mathrm{i}\frac{\delta}{2}(n_b - n_c)\right] f \exp\left[-\mathrm{i}\frac{\delta}{2}(n_b - n_c)\right]. \tag{11}$$

Thus, we finally obtain

$$\tilde{H}_{\rm eff} = \Omega[(a^+ a + 1)(a^+ a + 2)R_b - a^+ a(a^+ a - 1)R_c], \tag{12}$$

with $\Omega = g^2/\delta$. Hereafter, we will use the expression of Eq.(12) directly.

We assume that there is a reservoir coupled to the field in the usual way. Applying the transformation of Eq.(8) to the density matrix, at zero temperature, we have the master equation

$$\frac{\mathrm{d}\tilde{\rho}}{\mathrm{d}t} = \mathrm{i}[\tilde{\rho}, \tilde{H}_{\rm eff}] + \kappa(2a\tilde{\rho}a^+ - a^+ a\tilde{\rho} - \tilde{\rho}a^+ a), \tag{13}$$

where κ is the damping constant. We can represent the density operator as follows

$$\tilde{\rho} = \sum_{m, m', J_\alpha, J_\beta} \tilde{\rho}_{J_\alpha J_\beta m m'} |J_\alpha, m\rangle\langle J_\beta, m'|, \tag{14}$$

with $J_\alpha(J_\beta) = J_b, J_c$, where $\tilde{\rho}_{J_\alpha J_\beta mm'} = \langle J_\alpha, m|\tilde{\rho}|J_\beta, m'\rangle$. The formal solution of the master Eq.(13) is given by

$$\tilde{\rho}(t) = \mathrm{e}^{\mathcal{L}t}\rho(0), \tag{15}$$

where $\mathcal{L}$ denotes Liouvillian. The two-photon process is a nonlinear process, so it is difficult to solve Eq.(13) completely. We can only gain the expression of the diagonal matrix. Thus, we just study the properties of the subsystem field. The reduced field operator is written as $\tilde{\rho}_{\mathrm{F}} = \sum_m \langle J_b, m|\tilde{\rho}|J_b, m\rangle + \sum_m \langle J_c, m|\tilde{\rho}|J_c, m\rangle$. Employing a superoperator language, which is used in Refs.[12]–[14], and substituting Eq.(12) into Eq.(13), we obtain Liouvillians corresponding to the matrix elements $\tilde{\rho}_{J_b J_b mm}$ and $\tilde{\rho}_{J_c J_c mm}$ as

$$\begin{aligned}\mathcal{L}_{J_\mu J_\mu mm} =& 2\kappa\mathcal{F} \pm \mathrm{i}\omega_m(\mathcal{M}^2 - \mathcal{P}^2) \\ & - (\kappa + \mathrm{i}\omega_{m\mu})\mathcal{M} - (\kappa - \mathrm{i}\omega_{m\mu})\mathcal{P},\end{aligned} \tag{16}$$

where $\mu = b, c$; $\omega_m = \alpha_m^2\Omega$; $\omega_{mb} = 3\omega_m$; $\omega_{mc} = \omega_m$, and $-(+)$ corresponding to the matrix elements $\tilde{\rho}_{J_b J_b mm}$ ($\tilde{\rho}_{J_c J_c mm}$). The superoperators in Eq.(16) are defined as $\mathcal{F}\hat{\rho} = \hat{a}\hat{\rho}\hat{a}^+, \mathcal{M}\hat{\rho} = \hat{a}^+\hat{a}\hat{\rho}, \mathcal{P}\hat{\rho} = \hat{\rho}\hat{a}^+\hat{a}$. They satisfy the commutation relation $[\mathcal{F}, \mathcal{M}] = \mathcal{F}$, $[\mathcal{F}, \mathcal{P}] = \mathcal{F}$, $[\mathcal{M}, \mathcal{P}] = 0$.

In Ref.[12], the author studied the dissipation of the one-photon JCM. In order to study the loss of coherence, they assumed that the initial state of the system is $\Psi(0) = \frac{1}{\sqrt{2}}(|e\rangle + |g\rangle) \otimes |\alpha\rangle$, the atom is in an equal probability superposition of the ground and excited states, and the cavity field is in a coherent state, which is the case in the experiment.[16] In Ref.[14], for comparing the dissipation of the two-photon JCM with that of the one-photon JCM,[12] our group also studied the above state (the atom in an equal probability superposition). To include the general case, here we assume the initial state of the system to be

$$\begin{aligned}\Psi(0) = \sum_m \Bigg(& \frac{a}{\sqrt{2J_b + 1}}|J_b, m\rangle \\ & + \frac{b}{\sqrt{2J_c + 1}}|J_c, m\rangle \Bigg) \otimes |\alpha\rangle,\end{aligned} \tag{17}$$

where $|a|^2 + |b|^2 = 1$. The atom is in the degenerate excited state with a probability $|a|^2$and in the degenerate ground state with a probability $|b|^2$. The atom enters the cavity in a general superposition state and finds there a coherent field state $|\alpha\rangle$. Therefore, initially we have

$$\begin{aligned}\tilde{\rho}_{J_b J_b mm'}(0) &= \frac{|a^2|}{(2J_b + 1)}|\alpha\rangle\langle\alpha|, \\ \tilde{\rho}_{J_c J_c mm'}(0) &= \frac{|b|^2}{(2J_c + 1)}|\alpha\rangle\langle\alpha|.\end{aligned} \tag{18}$$

From Eqs.(15), (16) and (18), we finally have the density operator

$$\begin{aligned}\tilde{\rho}_{\mathrm{F}} =& \exp(-|\alpha|^2) \sum_{m,n,n'} \frac{\alpha^n \alpha^{*n'}}{\sqrt{n!n'!}} \\ & \times \Bigg\{ \frac{|a^2|}{2J_b + 1}\exp[\Gamma_{mnn'}(t) + \mathrm{i}\Theta_{bmnn'}(t)]|n\rangle\langle n'| \\ & + \frac{|b|^2}{2J_c + 1}\exp[\Gamma_{mnn'}(t) + \mathrm{i}\Theta_{cmnn'}(t)]|n\rangle\langle n'| \Bigg\},\end{aligned} \tag{19}$$

where

$$\begin{aligned}\Gamma_{mnn'}(t) =& -\kappa(n + n')t + \frac{|\alpha|^2\kappa}{\kappa^2 + \omega_m^2(n - n')^2} \\ & \times \{\kappa - \exp(-2\kappa t)[\kappa\cos 2\omega_m t(n - n') \\ & - \omega_m(n - n')\sin 2\omega_m t(n - n')]\},\end{aligned} \tag{20}$$

and

$$\begin{aligned}\Theta_{\mu mnn'}(t) =& -\omega_{m\mu}(n - n')t \pm \omega_m(n^2 - n'^2)t \\ & \pm \frac{|\alpha|^2\kappa}{\kappa^2 + \omega_m^2(n - n')^2}[\omega_m(n - n') \\ & - \exp(-2\kappa t)[\omega_m(n - n')\cos 2\omega_m t(n - n') \\ & + \kappa\sin 2\omega_m t(n - n')],\end{aligned} \tag{21}$$

with $\mu = b, c$. The plus (minus) sign is chosen in Eq.(21) when μ is equal to c (b).

In Refs.[12] and [14], the atom is in an equal probability superposition state. To gain insight into the role of the degenerate atomic level and to compare the result with that of usual JCM, we choose $a = b = 1/\sqrt{2}$, and obtain the average photon number as

$$\begin{aligned}< n > =& \mathrm{Tr}(a^+ a\tilde{\rho}_{\mathrm{F}}) \\ =& |\alpha|^2\exp[-2\kappa t + |\alpha|^2(1 - \exp(-2\kappa t))].\end{aligned} \tag{22}$$

From Eq.(22), we find that the average photon number has nothing to do with the degeneracy of the atom. In fact, one can also obtain the same expression by employing the density matrix without degeneracy in Ref.[14]. Furthermore, in usual dissipation of JCM,[12] the average photon number is still in the form of Eq.(22). We conclude that the average photon number of the field relates only to the dissipation of the field. The one-photon process, the two-photon process or the atomic degeneracy have no effect on the average photon number, which only depends on the dissipation of the cavity.

The coherence properties of this density operator as functions of time are conveniently studied by means of the linear entropy $S = 1 - \mathrm{Tr}(\tilde{\rho}^2)$. The quantity S can be regarded as a measure of the purity degree of the reduced state. For a pure state, S is zero. But for a mixture we have $0 < S \leq 1$, with the information effectively lost. From Eq.(19), we have the linear entropy of the field

$$\begin{aligned} S_{\mathrm{d}} =& 1 - \exp(-2|\alpha|^2) \sum_{mm'nn'} \frac{|\alpha|^{2(n+n')}}{n!n'!} \\ & \times \exp[\Gamma_{mnn'}(t) + \Gamma_{m'nn'}(t)] \\ & \times \left\{ \frac{|a|^4}{(2J_b+1)^2} \cos[\Theta_{bmnn'}(t) - \Theta_{bm'nn'}(t)] \right. \\ & + \frac{|b|^4}{(2J_c+1)^2} \cos[\Theta_{cmnn'}(t) - \Theta_{cm'nn'}(t)] \\ & + \frac{2|a|^2|b|^2}{(2J_b+1)(2J_c+1)} \\ & \left. \times \cos[\Theta_{bmnn'}(t) - \Theta_{cm'nn'}(t)] \right\}. \end{aligned} \tag{23}$$

In order to study the role of the degenerate atomic level, we should compare the result with the nondegeneracy case. We directly employ the expression of linear entropy without degeneracy given in Ref.[14]. In the two-photon process, we have

$$\begin{aligned} S =& 1 - \exp(-2|\alpha|^2) \sum_{nn'} \frac{|\alpha|^{2(n+n')}}{n!n'!} \\ & \times \exp[2\Gamma_{nn'}(t)] \cos^2 \frac{\Theta_{bnn'}(t) - \Theta_{cnn'}(t)}{2}, \end{aligned} \tag{24}$$

where $\Gamma_{nn'}(t)$ and $\Theta_{\mu nn'}(t)$ have the form similar to Eqs.(20) and (21), respectively. The difference is just in replacing ω_m with Ω.

The function $\Gamma_{mnn'}(t)$ embodies the effect of the reservoir because it vanishes for $k = 0$. Note that the function $\Gamma_{mnn'}(t)$ presents both in Eq.(23) and in Eq.(24). We see that no matter whether the degeneracy of the atomic level is considered or not, the coherence properties of the field can be affected by the reservoir equally. In the one-photon JCM, even if the degeneracy is considered, the field coherence properties are not affected by the reservoir qualitatively. [11] Note that the form of S_{d} is apparently different from S . We will show the difference in detail.

3. Results and discussion

Here we take that J_b and J_c both are 3/2, which has been explained in Ref.[14]. In this case, $\alpha_{1/2} = \alpha_{-1/2} = \dfrac{1}{2\sqrt{15}}$, and $\alpha_{3/2} = \alpha_{-3/2} = \dfrac{3}{2\sqrt{15}}$.

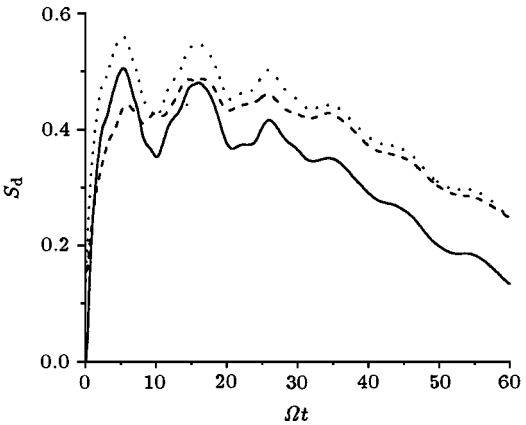


Fig.1. The evolution of the linear entropy of the field for different initial atomic degenerate state; solid line is for $a = b = 1/\sqrt{2}$, dotted line for a=0.8, b=0.6, and dashed line for a=0.6, b=0.8, with α=1, κ/ω=0.02.

Firstly, we check the effect of different initial atomic degenerate states on the coherence loss of the field. From Eq.(23), we show the evolution of the linear entropy of the field in Fig.1. The solid line with $a = b = 1/\sqrt{2}$ corresponds to the initial atomic equal probability superposition state. Another two curves are unequal probability ones. In most of the time (except in the interval $0 < \Omega t < 8.0$), the linear entropy in the solid line is the smallest. Because the linear entropy is a measure of purity degree of the state, the larger the linear entropy, the larger the degree of statistical mixture. When the atom is initially in an equal probability superposition state, the atomic coherence property is the best. Through interaction the initial coherence property of the atom will affect the field. Thus the field coherence loss is the smallest.

Then, we study the effects of degenerate atomic levels on the field dissipation. According to Eq.(23), we choose $a = b = 1/\sqrt{2}$ and plot the evolution of the field linear entropy with the degenerate atomic levels for two values of dissipation constant κ in Fig.2. From Eq.(24), we take the same two values of κ for the cases of Fig.2 and show the evolution of the field linear entropy without degenerate atomic levels in Fig.3. Because of the dissipation of the cavity, the field finally becomes a pure state (the vacuum state) and its coherence is lost completely, so both S_{d} and S reach their asymptotic value of zero. Comparing Fig.2 with Fig.3, we can see that both S_{d} and S show local minima and maxima in the envelopes. The appearance of local minima and maxima is due to the interaction between the field and the atom, corresponding to periodical entanglement and disentanglement, respectively. However, the periods of entanglement are obviously different. The periods of S_{d} are much larger than

those of S, that is to say, the degeneracy of the atomic level increases the period of entanglement. Thus, in a two-photon process, the degeneracy of the atomic level makes the entanglement survive long. This property has its advantage. Because, in the preparation of some quantum states, the condition is that the entanglement between the atom and the field can survive long enough. For example, in Ref.[19], to generate the entanglement of the atomic state, the entanglement between the first atom and the cavity field must survive long enough so that its coherence can be transferred to a second atom via interaction. At this point, the increasing long period of entanglement has some advantages.

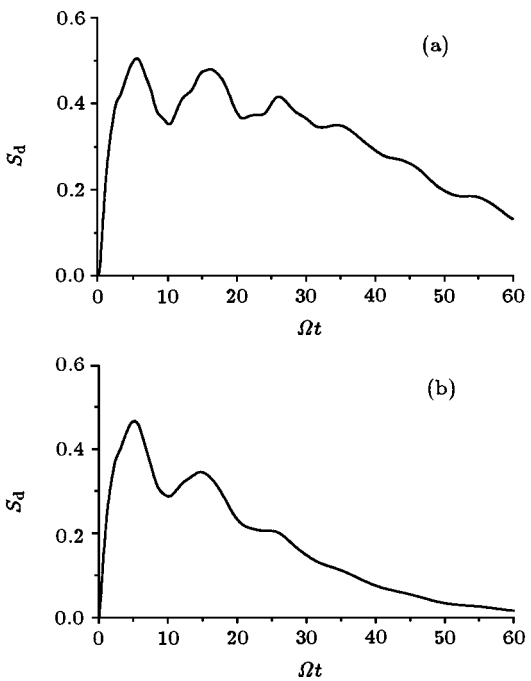


Fig.2. The linear entropy of the field as a function of Ωt for two values of dissipation constant κ, with α=1 and $a = b = 1/\sqrt{2}$, (a):κ/ω=0.02, (b):κ/ω=0.04.

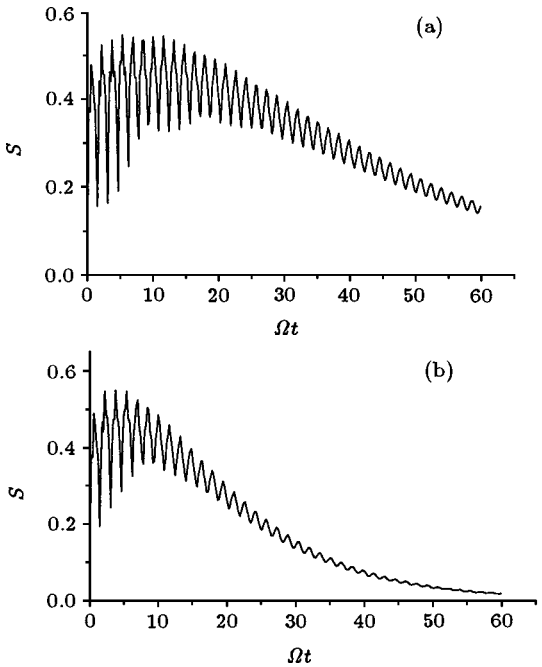


Fig.3. The evolution of field linear entropy without degenerate atomic levels for two values of dissipation constant κ, with α=1, (a): κ/ω=0.02, (b):κ/ω=0.04.

In conclusion, we have studied the effect of degenerate atomic levels on the field dissipation. Among different initial degenerate atomic states, the equal probability superposition state of the atom makes the field coherence loss smallest. When the degeneracy is considered, the coherence properties of the field can be affected by the reservoir qualitatively, if a nonlinear two-photon process is involved. This is different from the dissipation of the one-photon JCM with a degenerate atomic level.[11] It is noticeable that the period of entanglement becomes longer due to the degenerate atomic levels. Although some authors[2,6]used another method to prepare multiparticle entanglement, the longer period entanglement is also available.

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