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Suppressing chaos by parametric perturbation at doubled frequency of periodic perturbation*

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An analysis of the chaos suppression of a nonlinear elastic beam (NLEB) is presented. In terms of modal transformation the equation of NLEB is reduced to the Duffing equation. It is shown that the chaotic behaviour of the NLEB is sensitively dependent on the parameters of perturbations and initial conditions. By adjusting the frequency of parametric perturbation to twice that of the periodic one and the amplitude of parametric perturbation to the same as the periodic one, the chaotic region of the nonlinear elastic beam driven by periodic force can be greatly suppressed.

Keywords: suppression of chaos, threshold of chaos, nonlinear elastic beam, parametric perturbation
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The inherent irregularity of chaos phenomenon in nonlinear dynamical systems usually leads to undesired performance and even catastrophic failures, so that suppressing chaotic motion in nonlinear dynamical systems has great significance in engineering and the problem of chaos control has aroused much interest in recent years. Various novel methods of chaos control have been presented.[1−15]

Generally, a nonlinear system is affected by perturbations, such as periodic perturbation, from outside and usually these perturbations cannot be controlled. For simplicity, in this paper we suppose that the uncontrollable perturbation acting on the system is periodic. According to the Melnikov criterion, if the amplitude of the periodic perturbation is larger than the threshold of chaos, the nonlinear system will be in chaos. One method of suppressing chaotic behaviour is adding a second periodic force to the system.[16−18] Recently, Chacon proposed the method of resonant parametric perturbation for suppressing chaos of nonlinear system under the action of periodic perturbation.[19,20] In Ref.[19], Chacon added a parametric perturbation of cubic term to the Duffing–Holmes periodically forced oscillator to control chaotic behaviour of the system.

In this paper, we use the method of resonant parametric perturbation to suppress the chaos of the nonlinear elastic beam driven by a periodic perturbation. In our situation the parametric perturbation is one of linear term, which is different from the case of parametric perturbation of cubic term in Ref.[19]. We make an analytical study on the effectiveness of the method. By adjusting the amplitude of parametric perturbation to the same as that of the periodic force and frequency of the parametric perturbation to twice that of the periodic perturbation, the chaotic region is greatly suppressed. Moreover, we prove theoretically that the chaotic behaviour of the nonlinear elastic beam (NLEB) is sensitively dependent on the initial conditions and parameters of perturbations.

The nonlinear elastic beam is one of the most important structures in engineering and its chaotic behaviour has been explored by many authors.[21−23] We focus our attention on revealing the property that chaotic behaviour is sensitively dependent on the parameters of perturbations and on methods of suppressing chaotic motion of this structures. Set the small periodic force acting on the beam as $\varepsilon f \sin\frac{\pi x}{l}\sin\omega t$, and suppose the damping of the beam is also small. To control the chaos of the beam, we apply an axial

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force $p_0 + \varepsilon p \sin \Omega t$ on it. So the non-dimensionally governing equation of the motion of the beam is[24]

$$\begin{aligned} & w_{tt} + w_{xxxx} \\ =& \varepsilon w_t + \Bigg(- p_0 - \varepsilon p \sin \Omega t \\ & + \frac{1}{2l} \int_0^t w_x^2 \mathrm{d}x \Bigg) w_{xx} + \varepsilon f \sin \frac{\pi x}{l} \sin \omega t, \end{aligned} \tag{1}$$

and its one end is a fixed hinged and the other is a movable hinged so that the boundary conditions are

$$w(0) = w(l) = w_{xx}(0) = w_{xx}(l) = 0, \tag{2}$$

where $w(x,t)$ is the transverse displacement of the beam, $\varepsilon (\ll 1)$ is a small number and l is the length of the beam.

The solution of the Eq. (1) satisfying the boundary conditions (2) is

$$w(x,t) = u(t) \sin \frac{\pi x}{l}. \tag{3}$$

Substituting Eq. (3) into Eq. (1) yields

$$\ddot{u} - au + bu^3 = \varepsilon h(u, \dot{u}, t), \tag{4}$$

where

$$h(u, \dot{u}, t) = \alpha \dot{u} + \beta u \sin \Omega t + f \sin \omega t, \tag{5}$$

and $a = \frac{\pi^2}{l^2}\left(p_0 - \frac{\pi^2}{l^2}\right), b = \frac{\pi^4}{4l^4}, \beta = \frac{\pi^2}{l^2} p$. Equation (4) is the famous Duffing equation with small perturbation $h(u, \dot{u}, t)$. Its equivalent form is

$$\dot{\varphi} = \psi, \quad \dot{\psi} = a\varphi - b\varphi^3 + \varepsilon h(\varphi, \psi, t), \tag{6}$$

where $\varphi = u, \psi = \dot{\varphi} = \dot{u}$. The unperturbed system of Duffing Eq. (6) is

$$\dot{\varphi} = \psi, \quad \dot{\psi} = a\varphi - b\varphi^3. \tag{7}$$

It is a Hamiltonian system. There are different solutions of Eq. (7) related to different Hamiltonians. Our interest is in the well known homoclinic orbit of it

$$\begin{aligned} \varphi_0(t) &= \sqrt{\frac{2a}{b}} \mathrm{sech}(\sqrt{a}t), \\ \psi_0(t) &= -a\sqrt{\frac{2}{b}} \mathrm{sech}(\sqrt{a}t) \tanh(\sqrt{a}t). \end{aligned} \tag{8}$$

Suppose that the perturbation $h(\varphi, \psi, t)$ contributes a small correction $\varepsilon \varphi_1(t)$ to the solution $\varphi(t)$ of Eq. (6), namely

$$\varphi(t) = \varphi_0(t) + \varepsilon \varphi_1(t). \tag{9}$$

Inserting Eq. (9) into Eq. (6) yields the first-order equation

$$\dot{\varphi}_1 = \psi_1, \quad \dot{\psi}_1 = a\varphi_1 - 3b\varphi_0^2 \varphi_1 + h(\varphi_0, \psi_0, t). \tag{10}$$

Combining Eq. (8) with (10) and using the direct perturbation technique,[25,26] we obtain the general correct solution near the homoclinic orbit

$$\begin{aligned} \varphi_1(t) =& \theta_2 \int_{A_1}^t \theta_1 h(\varphi_0, \psi_0, t) \mathrm{d}t - \theta_1 \int_{A_2}^t \theta_2 h(\varphi_0, \psi_0, t) \mathrm{d}t, \\ \psi_1(t) =& \dot{\theta}_2 \int_{A_1}^t \theta_1 h(\varphi_0, \psi_0, t) \mathrm{d}t - \dot{\theta}_1 \int_{A_2}^t \theta_2 h(\varphi_0, \psi_0, t) \mathrm{d}t, \end{aligned} \tag{11}$$

where A_1 and A_2 are arbitrary constants adjusted by the initial conditions, θ_1 and θ_2 are two fundamental solutions of Eq. (10) for $h(\varphi, \psi, t) = 0$, which can be easily derived as

$$\theta_1 = \dot{\varphi}_0(t) = -a\sqrt{\frac{2}{b}} \mathrm{sech}(\sqrt{a}t) \tanh(\sqrt{a}t),$$

$$\begin{aligned} \theta_2 =& \theta_1 \int \theta_1^{-2} \mathrm{d}t = \theta_1 \int \frac{b}{2a^2} \frac{\mathrm{d}t}{[\mathrm{sech}(\sqrt{a}t) \tanh(\sqrt{a}t)]^2} \\ =& \frac{1}{a}\sqrt{\frac{b}{2a}} \Bigg[\mathrm{sech}(\sqrt{a}t) - \frac{3}{2}\sqrt{a}t \ \mathrm{sech}(\sqrt{a}t) \\ & \times \tanh(\sqrt{a}t) - \frac{1}{2} \sinh(\sqrt{a}t) \tanh(\sqrt{a}t) \Bigg]. \end{aligned} \tag{12}$$

Examining Eqs.(11) and (12) we find that the general solution (11) of Eq. (10) is unstable because of the infinite θ_2 as $t \to \pm\infty$, and it is stable only if the coefficient function of the divergent function θ_2 obeys

$$\lim_{t \to \pm\infty} \int \theta_1 h(\varphi_0, \psi_0, t) \mathrm{d}t = 0. \tag{13}$$

We can directly prove that the general solution (9) of Eq. (6) is sensitively dependent on the initial conditions and parameters of perturbations. For this, we set (φ', ψ') as another orbit with respect to the constants A_1' and A_2' as well as the same zeroth-order terms $\varphi_0' = \varphi_0$, $\psi_0' = \psi_0$. Combining Eqs.(9), (11) and (12) yields

$$\begin{aligned} \varphi - \varphi' =& \theta_2 \Bigg[\int_{A_1}^t \theta_1 h(\varphi_0, \psi_0, t) \mathrm{d}t \\ & - \int_{A_1'}^t \theta_1 h(\varphi_0, \psi_0, t) \mathrm{d}t \Bigg] \\ & - \theta_1 \Bigg[\int_{A_2}^t \theta_2 h(\varphi_0, \psi_0, t) \mathrm{d}t \\ & - \int_{A_2'}^t \theta_2 h(\varphi_0, \psi_0, t) \mathrm{d}t \Bigg] = A\theta_2 - B\theta_1, \end{aligned} \tag{14}$$

$$\psi - \psi' = A\dot{\theta}_2 - B\dot{\theta}_1.$$

The constants A and B are dependent on the integral constants A_1, A_1' and A_2, A_2', respectively. A and B will not be equal to zero unless $A_1 = A_1'$ and $A_2 = A_2'$ so that $\varphi = \varphi'$, $\psi = \psi'$. Some small differences between the initial condition of (φ, ψ) and that of (φ', ψ') will give rise to the differences between A_1 and A_1', A_2 and A_2', so that A and B will not be equal to zero. Consequently, the differences (14) between the two orbits will tend to infinity as $t \to \pm\infty$ because of the infinities of $\theta_2(\pm\infty)$ and $\dot{\theta}_2(\pm\infty)$. The above discussion shows that the solution $\varphi_1(t)$ such that $\varphi(t)$ depends on the initial conditions sensitively. It is well known that this sensitivity is a main characteristic of chaos, so that solution (9) with the forms of (8) and (11) is chaotic unless the condition (13) is fulfilled.

Then we prove that the solution (9) with the forms of (8) and (11) is sensitively dependent on the parameters of perturbation. To this end, we suppose the parameters undergo a little change while the unperturbed Duffing Eq. (7) as well as the initial conditions remain the same, therefore the perturbation changes from $h(\varphi, \psi, t)$ to $h'(\varphi, \psi, t)$. We set (φ, ψ) and (φ'', ψ'') as the orbits corresponding to $h(\varphi, \psi, t)$ and $h'(\varphi, \psi, t)$, respectively. With the same procedure as above, we obtain

$$\varphi - \varphi' = C\theta_2 - D\theta_1, \quad \psi - \psi' = C\dot{\theta}_2 - D\dot{\theta}_1, \quad (15)$$

where

$$C = \int_{A_1}^{t} \theta_1[h(\varphi, \psi, t) - h'(\varphi, \psi, t)]\mathrm{d}t,$$
$$D = \int_{A_2}^{t} \theta_2[h(\varphi, \psi, t) - h'(\varphi, \psi, t)]\mathrm{d}t. \quad (16)$$

Eqs.(15) and (16) indicate that the constants C and D depend on $h(\varphi, \psi, t)$ and $h'(\varphi, \psi, t)$. As long as $h(\varphi, \psi, t)$ does not equal to $h'(\varphi, \psi, t)$, C and D will not vanish. The difference between $h(\varphi, \psi, t)$ and $h'(\varphi, \psi, t)$ is caused by the change of the parameters of perturbation function $h(\varphi, \psi, t)$. Consequently, a sufficiently small change of the parameters of the perturbation function $h(\varphi, \psi, t)$ will result in the difference between the two orbits tending to infinity as $t \to \pm\infty$ because of the infiniteness of $\theta_2(\pm\infty)$. This shows that the general solution (9) of Eq. (6) sensitively depends on the parameters of perturbation. This is the theoretical foundation of controlling chaos by adjusting the parameters of the perturbation.

In view of the Melnikov criterion for the threshold of chaos, we can easily calculate the Melnikov distance of the stable and unstable manifolds as follows

$$M(t_0) = \beta I_1 \cos(\Omega t_0) + f I_2 \cos(\omega t_0) + \alpha I_3, \quad (17)$$

where

$$I_1 = \frac{\pi\Omega^2}{b}\mathrm{csch}\left(\frac{\pi\Omega}{\sqrt{2a}}\right), \quad I_2 = \pi\omega\sqrt{\frac{2}{b}}\mathrm{csch}\left(\frac{\pi\omega}{\sqrt{2a}}\right),$$
$$I_3 = \frac{2\sqrt{2}}{3}\frac{a^{3/2}}{b}. \quad (18)$$

By suitably selecting the values of parameters of perturbation, we obtain an analytic presentation of threshold of chaos. To reach this end, we adjust the amplitude β of the parametric perturbation to the same as that of force perturbation f, and alter the frequency of the parametric perturbation Ω to twice the frequency of periodic perturbation ω. Therefore, Melnikov function (17) becomes

$$\begin{aligned} M(t_0) =& f[I_1' \cos(2\omega t_0) + I_2' \cos(\omega t_0)] + \alpha I_3' \\ =& f[2I_1' \cos^2(\omega t_0) + I_2' \cos(\omega t_0) - I_1'] + \alpha I_3', \end{aligned} \quad (19)$$

where

$$I_1' = \frac{4\pi\omega^2}{b}\mathrm{csch}\left(\frac{2\pi\omega}{\sqrt{2a}}\right), \quad I_2' = I_2, \quad I_3' = I_3.$$

The condition for making $M(t_0)$ to have simple zero is that to satisfies

$$\cos(\omega t_0) = \frac{-I_2' f \pm \sqrt{I_2'^2 f^2 - 8I_1' f[I_3'\alpha - I_1' f]}}{4I_1' f}. \quad (20)$$

While the necessary and sufficient conditions for t_0 being a real number are

$$I_2'^2 f^2 - 8I_1' f[I_3'\alpha - I_1' f] \geq 0 \quad (21)$$

and

$$\frac{-I_2' f \pm \sqrt{I_2'^2 f^2 - 8I_1' f[I_3'\alpha - I_1' f]}}{4I_1' f} \leq 1. \quad (22)$$

Solving these inequalities we obtain a set of solutions of them as follows

$$\frac{f}{\alpha} \geq \frac{8I_2' I_3'}{8I_1'^2 + I_2'^2} = R_1(\omega), \quad (23)$$

$$\frac{f}{\alpha} \geq -\frac{I_3'}{I_1' + I_2'} = R_2(\omega), \quad (24)$$

$$\frac{f}{\alpha} \leq -\frac{I_3'}{I_2' - I_1'} = R_3(\omega). \quad (25)$$

It is obvious that $R_1(\omega)$ is greater than $R_2(\omega)$ (because I_1', I_2' and I_3' are all positive). Combining

inequalities (23), (24) and (25) gives the chaotic region in the parameter space for the weakly damped nonlinear elastic beam excited by the small forcing perturbation and parametric perturbation as follows

$$R_1(\omega) \leq \frac{f}{\alpha} \leq R_3(\omega). \tag{26}$$

This is consistent with the result in Ref.[19] in form. We call $R_1(\omega)$ the inferior threshold of chaos and $R_3(\omega)$ the superior threshold of chaos. Comparing the value of the area of chaotic region of Duffing oscillator under the action of force periodic and parametric perturbations with that only under the action of force periodic perturbation or parametric perturbation is very important for us to find out the effect of suppressing chaos by these methods.

The thresholds of Duffing oscillator (4) affected only by periodic or parametric perturbations are

$$R_4 = \frac{I_3}{I_2} \quad \text{for} \quad h(u, \dot{u}, t) = \alpha\dot{u} + f\sin\omega t, \tag{27}$$

$$R_4 = \frac{I_3}{I_1} \quad \text{for} \quad h(u, \dot{u}, t) = \alpha\dot{u} + \beta u\sin\omega t. \tag{28}$$

According to Eqs.(26)–(28), we can determine the chaotic regions of Duffing oscillator affected by different perturbations mentioned above, which are plotted in (a)–(c) of Fig.1, respectively. In the figure, the dark region is chaotic region and the white region is periodic region, the boundary being the chaotic threshold curve. Diagrams (a) and (b) show the division of the parametric plane $(f/\alpha, \omega)$ into two regions by the threshold curve of chaos, while in diagram (c), the $(f/\alpha, \omega)$ plane is divided into several parts by the superior threshold curve, $R_3(\omega)$, and inferior threshold curve, $R_1(\omega)$. In the latter case, the chaotic region is an island surrounded by periodic oceans. The area of this chaotic island is far smaller than that of the chaotic region in (a) or (b). This means that when the Duffing oscillator is driven jointly by periodic and parametric perturbations and if the frequencies of these two perturbations have the ratio $\Omega = 2\omega$ the chaotic region is much smaller than that of the system driven singly by a periodic perturbation or a parametric perturbation.

The above analysis shows that by suitably selecting the amplitudes of periodic and parametric perturbations and the frequencies of both perturbations, we can effectively suppress the chaotic motion by means of the joint action of periodic perturbation and parametric perturbation. So if chaotic motion occurs in the periodic driven NLEB, the chaos can be effectively suppressed by exerting an additional suitable parametric perturbation, and *vice versa*.

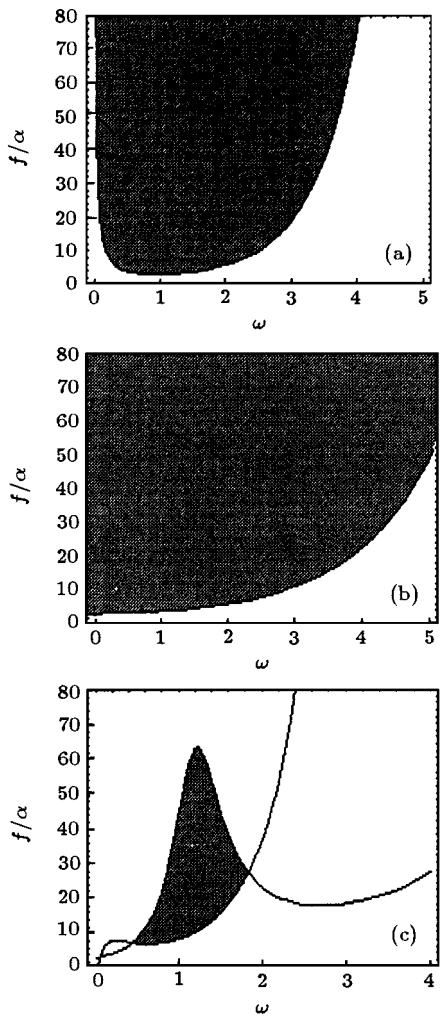


Fig.1. The chaotic regions of Eq. (6) under different perturbations in parametric space $(f/\alpha, \omega)$. $b = 1, a = \pi^2/2$. The dark area is chaotic region and white area is periodic region. The chaotic threshold curve is the separatrix of chaotic region and periodic region. (a) for $h(u, \dot{u}, t) = \alpha\dot{u} + f\sin\omega t$, (b) for $h(u, \dot{u}, t) = \alpha\dot{u} + fu\sin\omega t$ and (c) for $h(u, \dot{u}, t) = \alpha\dot{u} + fu\sin 2\omega t + f\sin\omega t$.

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