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Metastability of Ising spin chains with nearest-neighbour and next-nearest-neighbour interactions in random fields

G. Ismail and S. Hassan

Department of Mathematics, Faculty of Science, Zagazig University, Zagazig, Egypt

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One-dimensional Ising systems in random fields (RFs) are studied taking into account the nearest-neighbour and next-nearest-neighbour interactions. We investigate two distributions of RFs: binary and Gaussian distributions. We consider four cases of the exchange couplings: ferro–ferromagnetic (F–F), ferro–antiferromagnetic (F–AF), antiferro–ferromagnetic (AF–F) and antiferro–antiferromagnetic (AF–AF). The energy minima of chains of no more than 30 spins with periodic boundary conditions are analysed exactly. We found that the average number of energy minima grows exponentially with the number of spins in both cases of RFs. The energy distributions across the corresponding energy minima are shown. The effects of RFs on both the average and density of metastable states are explained. For a weak RF, the energy distributions display a multipartitioned structure. We also discuss the frustration effect due to RFs and exchange fields. Finally, the distributions of magnetization are calculated. The absolute value of magnetization averaged over all metastable states decreases logarithmically with the number of spins.

Keywords: random field, metastable states, nearest-neighbour, next-nearest-neighbour, magnetization

PACC: 7510N, 7620

1. Introduction

Disordered one-dimensional (1D) Ising systems are considered to be one of the most important physical problems. These systems have been studied by using certain models under suitable boundary conditions, taking into account only the nearest-neighbour (nn) interactions, such as spin glass (SG), disordered ferromagnetic (DFM), random field (RF) Ising, random anisotropy (RA) and Potts spin systems. Next-nearest neighbour (nnn) interactions have not been taken into consideration. In fact, the study of the static magnetic properties of such systems is of interest in analysing some physical quantities which are necessary for revealing the behaviours of any system.[1]

Theoretical studies for the spin glass, disordered ferromagnetic and RF Ising systems have been introduced in 1D cases and it has been shown that they exhibit very complicated behaviours, especially in high dimensions. Also, the finite-temperature properties and frustration of the ground states in 1D ferromagnetic Ising systems with a binary distribution of RFs have been studied analytically for $T \to 0$.[2,3]

Experimentally, the RF Ising systems can be realized[4] by the use of diluted antiferromagnets in a uniform magnetic field. Various experimental techniques, such as neutron scattering[5] and linear birefringence[6] are currently used to probe RF Ising systems.

Numerically, several methods are of interest in studying the static and dynamic magnetic properties of finite 1D disordered Ising systems to specify the behaviours of such models. These calculations are concerned with investigating the metastable states (i.e. local energy minima), the energy distributions across their minima, the distributions of magnetization and the frustration effect, etc, which play an important role in specifying the behaviours of these systems.[1] The finite-temperature properties and frustration of the ground states of ferromagnetic Ising systems have been studied[7,8] in a binary distribution of a RF. The 1D Ising models of spin glass, DFM and RF Ising systems of finite number of spin chains have been investigated and exact physical results obtained.[9] The static properties of finite Ising spin chains have also been shown for SG, DFM and $\pm J$ systems. These static properties have been examined for the Potts model[10] of a finite number of spin chains for all these systems. The dynamic and static properties for finite RF Ising ferromagnetic systems on the Sierpenski gasket have been discussed.[11]

In the present work, we have considered finite 1D

RF Ising systems with nn and nnn interactions. The RF used here can be chosen either of binary distribution (BD) or of Gaussian distribution (GD). We have studied the two RF Ising systems with the four exchange couplings, F–F, F–AF, AF–F and AF–AF. Our aim here is to investigate the static magnetic properties of these systems, which are very important for the study of their dynamic properties in our next work.

We explain the model used here in section 2. This model is suitable as we have seen from theoretical interest of SG physics[12] and also other magnetic systems studied.[13] In section 3 we investigate the relation between the average number of energy minima $\langle n\rangle$ and the number of spins N. The dependence of $\langle n\rangle$ on the RF is explained. In section 4 we discuss the frustration effect due to the RF and the exchange couplings. The density of metastable states is analysed exactly in section 5. The distributions of energy minima $n(E)$ across these minima E in both cases of RF are described using the plotted histograms. An investigation of metastable states shows that the energy minima distributions for all our systems approach the normal distribution. For a weak RF, these distributions are discrete. Finally, in section 6 the magnetization and its distributions for all stable energy minima are considered. We found that the absolute value of magnetization averaged over all metastable states decreases logarithmically with the number of spins. The effect of the RF on this average is also investigated exactly. The distributions of magnetized minima states across the absolute values of magnetization per spin are discussed.

2. The model

We consider a linear chain of N Ising spins $S_i = \pm 1 (i = 1, \cdots, N)$ in a RF, taking into account the nn and nnn interactions. This model can be described by the following Hamiltonian

$$H = -J_1 \sum_{i=1}^{N} S_i S_{i+1} - J_2 \sum_{i=1}^{N} S_i S_{i+2} - \sum_{i=1}^{N} H_i S_i, \quad (1)$$

where J_1 and J_2 are the uniform exchange interactions between nn and nnn spins, respectively. These interactions have the following four cases: $J_1 = J_2 > 0$ (F–F), $J_1 = -J_2 > 0$ (F–AF), $-J_1 = J_2 > 0$ (AF–F) and $J_1 = J_2 < 0$ (AF–AF). H_i are local RFs which can be chosen to be in either BD or GD as follows.

(i) The BD case. The RF components H_i $(i = 1, \cdots, N)$ are equal to $\pm H_{\mathrm{R}}$ with a probability of 50% for up or down, and H_{R} is the amplitude of RF.

(ii) The GD case. The RF components H_i are Gaussian random numbers with zero mean and unit dispersion; here ωd is used to denote the width of the RF.

The suitable periodic boundary conditions, which we have used, can be written in the form

$$S_{N+i} = S_i, \quad i = 1, 2. \quad (2)$$

We investigate different amplitudes (i.e. H_{R}) of RF in the BD case and only one value of the width (e.g. ωd=0.5) in the GD case. GD is of the continuous type, while BD is of the discrete type. We considered 3000, 2000, 800, 500, 100 and 50 samples for chains with N=6, 10, 15, 20, 25 and 30 spins, respectively. We are interested in the calculation and analysis of some physical quantities which are important in the study of the disordered Ising systems from the magnetic point of view.[1,9,11]

3. Energy minima and effect of RF

For all our systems of spin chains and the same samples mentioned in section 2, in the cases of both BD and GD of the RF, we have performed an exact numerical analysis of $N \leq 30$ spin chains under the periodic boundary conditions (2). We investigated the dependence of the average number of energy minima $\langle n\rangle$ on the number of spins N. For a given sample we have scanned its spin states and selected all those which are stable against single spin reversals. We counted all these stable energy minima, and these calculations took very long time on an IBM PC Pentium 200. For example, one sample of 30 spins from both cases of our systems took about 40h to give its energy minima.

Generally, we found that the average $\langle n\rangle$ increases exponentially with the number of spins N, for both cases of RF, as shown in Figs.1 and 2. On the other hand, in the BD case, the average $\langle n\rangle$ saturates as long as we have $0 < H_{\mathrm{R}} < 2$, and also $2 < H_{\mathrm{R}} < 4$; while it saturates at unity for $H_{\mathrm{R}} > 4$. But at $H_{\mathrm{R}} = 2$ or 4, we have two singular points at which the values of $\langle n\rangle$ are very much smaller. Here, the exchange interactions J_1 and J_2 are considered to be $J_1 = J_2 = 1$, $J_1 = -J_2 = 1$, $-J_1 = J_2 = 1$ and $J_1 = J_2 = -1$ for F–F, F–AF, AF–F and AF–AF exchange couplings, respectively. Furthermore, in the BD case (as long as $0 < H_{\mathrm{R}} < 2$) for F–F and AF–F coupling systems, the increase of $\langle n\rangle$ against N is very small, but for F–AF

and AF–AF couplings this increase is very large, as shown in Fig.1.

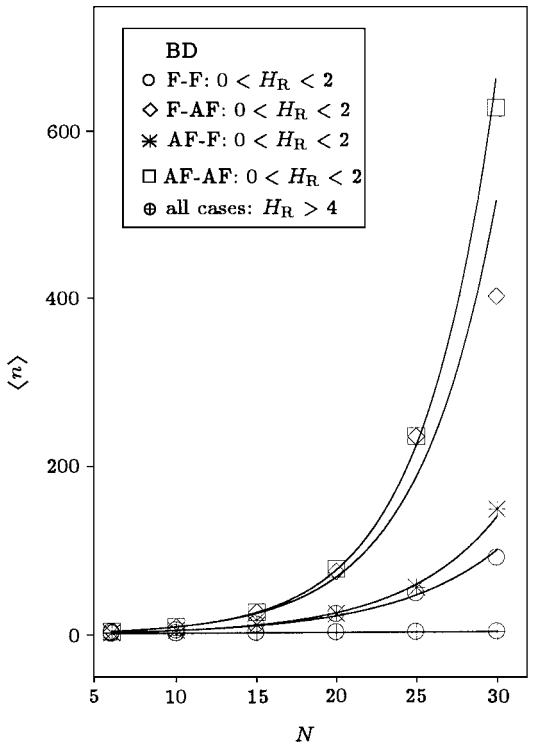


Fig.1. The average number of local energy minima $\langle n \rangle$ versus the number of spins N in the BD case (for $0 < H_R < 2$ and $H_R > 4$), for the F–F, F–AF, AF–F and AF–AF systems.

For $2 < H_R < 4$ the increase of $\langle n \rangle$ is very much smaller for all cases. In the GD case (ωd=0.5), similar results are obtained as in the BD case (for $0 < H_R < 2$), as shown in Fig.2. This exponentially growing relation can be expressed in the following form

$$\langle n \rangle = \alpha e^{\beta N}, \qquad (3)$$

where the parameters α and β are obtained through a computer fitting program as follows.

In the BD case. For $0 < H_R < 2$ we have for F–F coupling, α=0.9336 and β=0.1554; but for F–AF coupling, α=1.1437 and β=0.2036; while for AF–F coupling, α=0.7707 and β=0.1730; and for AF–AF coupling, α=0.9821 and β=0.2171.

In the GD case. We obtain α=0.8085 and β=0.1681 for F–F coupling; and α=0.8915 and β=0.2244 for F–AF coupling; while for AF–F coupling, α=0.9337 and β=0.1562; and finally for AF–AF coupling, α=0.9382 and β=0.2210.

We studied exactly the average $\langle n \rangle$ as a function of the amplitude H_R of the RF in the BD case, as shown in Fig.3, for all systems. We found that $\langle n \rangle$ saturates at a certain constant value as long as there is $0 < H_R < 2$. It saturates at a smaller constant value for $2 < H_R < 4$ (in this latter case all our systems are very close to the ordered state). While at $H_R = 2$ and 4, we have two singular points at which the values of $\langle n \rangle$ are very much smaller. This is due to the coordination number and the effect of the discrete BD of the RF. But for $H_R > 4$, i.e. for so large a RF, only one minimum state in each sample is obtained. These results show that any RF Ising system undergoes a zero-temperature phase transition from a disordered to an ordered state. In the BD of the RF, the phase transition is a first-order transition. This has been confirmed by Ogielski[14] and Ismail and Bannora[9] and also by Ismail and Salem.[11]

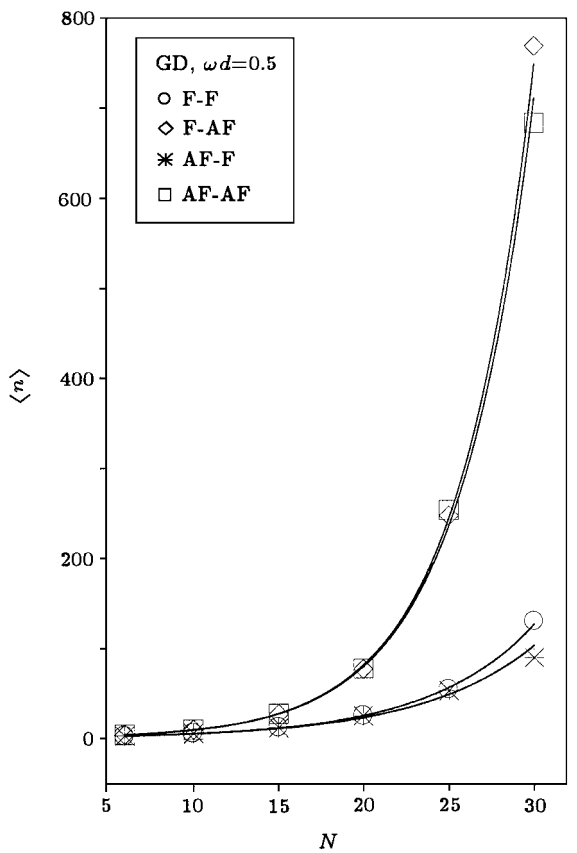


Fig.2. The average $\langle n \rangle$ versus N in the GD case (for ωd=0.5), for the F–F, F–AF, AF–F and AF–AF systems.

The general behaviours of our systems expressed by relation (3) and the effect of the RF on the number of local energy minima are very similar to those obtained in Refs.[9] and [11]. Furthermore, the behaviours of the F–F and AF–F couplings are very much the same, while in the F–AF and AF–AF cases they behave alike. We conclude that the systems with antiferromagnetic nnn interactions are more disordered than those with ferromagnetic nnn interactions.

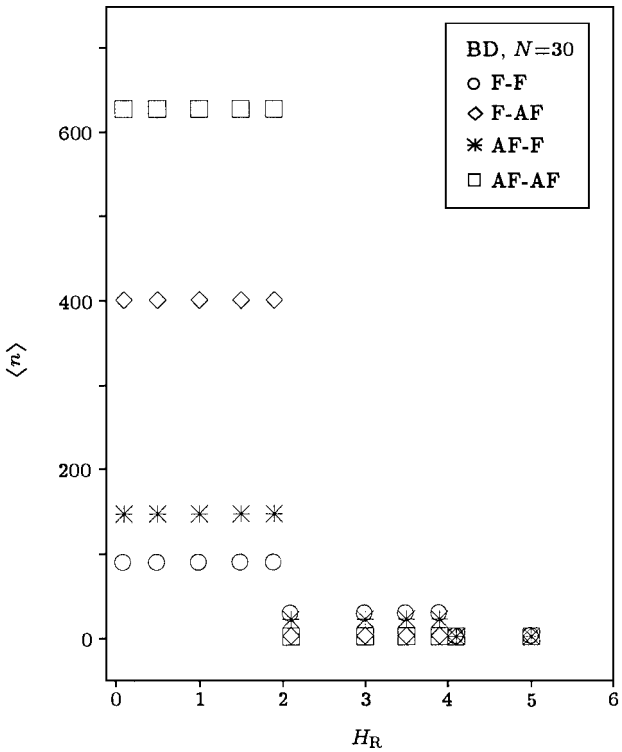


Fig.3. The average $\langle n \rangle$ versus the amplitude H_{R} in the BD case, for the F–F, F–AF, AF–F and AF–AF systems (e.g. N=30 spins).

4. Frustration phenomenon

We have discussed the frustration[9,10] of the ground state in some particular chains of no more than 30 spins for all systems, in both cases of RF. Here the frustration of the ground state (and also the higher minima states) is due to the exchange couplings J_k (k=1, 2), i.e. bonds between the nn and nnn spins, and the local RF components H_i ($i = 1, \cdots, N$).

For F–F and AF–F systems of 30 spins in the BD case, as long as there is $0 < H_{\mathrm{R}} < 2$, the true ground state has no frustrated bonds between the nn and nnn spins, since in the F–F coupling the ground state is fully aligned, and in the AF–F case its spins are anti-parallel. But there are some frustrated local RF components which are not satisfied by their spin directions for the present two systems. While as long as there is $2 < H_{\mathrm{R}} < 4$, the ground state has some frustrated bonds due to the nn and nnn spins. There are also some frustrated components of the local RF. And for $H_{\mathrm{R}} > 4$, the system has many frustrated bonds, however there is no frustration in the local RF components. Furthermore, the true ground state is not degenerate in all the above cases. Also, in the GD case, the results are very much similar to the BD case (for $0 < H_{\mathrm{R}} < 2$).

For F–AF and AF–AF interactions, in the BD case, for $0 < H_{\mathrm{R}} < 2$ and $2 < H_{\mathrm{R}} < 4$, we have some frustrated bonds and also some RF frustrations. Similar results are obtained if $H_{\mathrm{R}} > 4$ as in the F–F and AF–F cases. Also the GD case is very much like that for $0 < H_{\mathrm{R}} < 2$ in the BD case. Here, for both types of RF, the ground state is sometimes degenerate. One can say that the bonds between nnn spins are more frustrated than those between the nn spins, especially if these nnn spins are antiferromagnetic. Such frustrated bonds will increase with the increase of the RF (while the frustrated RF components will decrease with increasing RF). For so large a RF, most of these bonds between the nn and nnn spins are frustrated, but there is no frustration in the local RF components. We conclude that the bonds in the F–AF and AF–AF cases are more frustrated than those in the F–F and AF–F cases.

5. Density of metastable states

We have calculated exactly the distributions of metastable states $n(E)$ across the energy values E of these states, for the same numbers of spin chains in their corresponding samples mentioned in section 2. The four cases are investigated in both distributions of RF. For a weak RF in the BD case, i.e. at $H_{\mathrm{R}} = 0.1$, the distribution of $n(E)$ is discrete as shown in Fig.4. But for $0.1 < H_{\mathrm{R}} < 2$, semi-normal distributions have been obtained for all our systems whatever the value of N. These distributions are symmetric and continuous. In the GD case, these distributions also have the same properties as in the BD case for $H_{\mathrm{R}} = 0.5$.

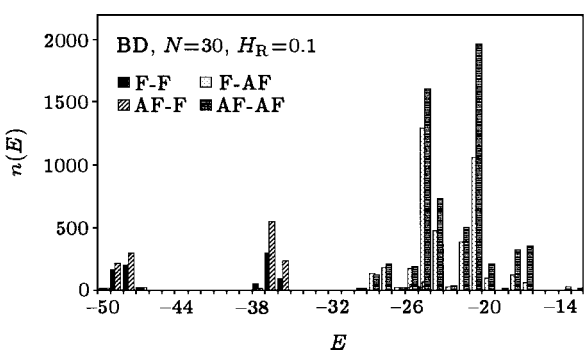


Fig.4. The distribution of local energy minima $n(E)$ across the values of these minima E for a weak RF (i.e. H_{R}=0.1) in the BD case.

For F–F and AF–F interactions, the distributions of $n(E)$ as functions of H_{R} for $0.1 < H_{\mathrm{R}} < 2$ are also investigated. These are symmetric and continuous. The width and height of the distribution increase with increasing H_{R}, since the ground energies decrease

and the number of these minima states increases, with increasing H_R. For $H_R = 0.5$ we obtain two regions for $n(E)$, each of which is continuous and symmetric as shown in Fig.5. But as $H_R > 0.5$, we have only one continuous and symmetric region whatever the value of N.

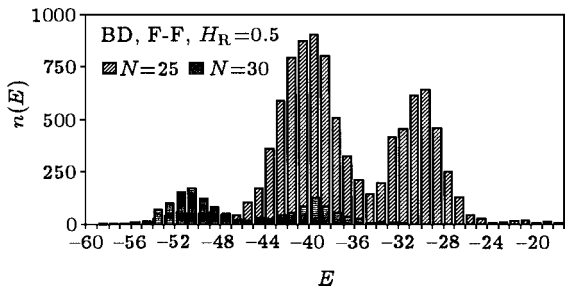


Fig.5. The distribution of $n(E)$ across E for the F–F system in the BD case (for H_R=0.5), with N=25 and 30 spins.

Also, we examine the AF–F case for H_R=1.0 and 1.5 with N=30 spins. In the GD case, similar behaviour is obtained as in the BD case for H_R=0.5. These results show that the metastable states have semi-normal distributions for the two systems, whatever the value of N. We have also discussed the distributions of $n(E)$ as functions of N in both cases of RF as shown in Fig.6. The plotted histograms are continuous, symmetric and shifted to negativity, since the ground energy decreases with increasing N. Also, $n(E)$ shows a sharp peak around its average energy. The width and the height of the distributions increase with increasing N (for $N < 20$), while this height decreases with increasing N for $N \geq 20$. These results are similar to those obtained in the study of the finite 1D SG, DFM, RF Ising and Potts spin systems with nn interactions only.[1,9,11]

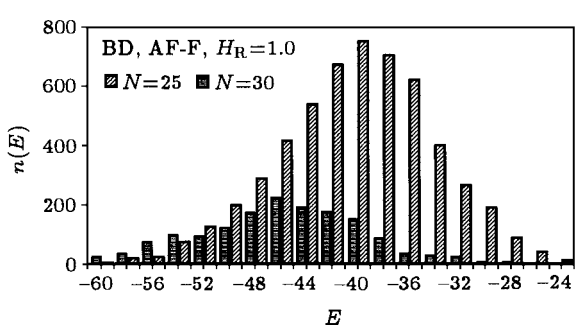


Fig.6. The distribution of $n(E)$ across E for the AF–F system in the BD case (for H_R=1.0), with N=25 and 30 spins.

For F–AF and AF–AF systems, the distributions of $n(E)$ as functions of H_R for $0.1 < H_R < 2$ are studied for $N \leq 30$. Similar results are obtained as those in the F–F and AF–F cases. But for the present two systems, the widths of the histograms are smaller and shifted to the right side (while their heights are higher) than those in the other two cases, since the number of minimum states and the corresponding energy values for the present two systems are larger than those for the other two. Furthermore, only one region is obtained here for the distribution of $n(E)$, no matter what H_R and N are, as shown in Figs.7 and 8. The GD case is very much like the BD case for H_R=0.5.

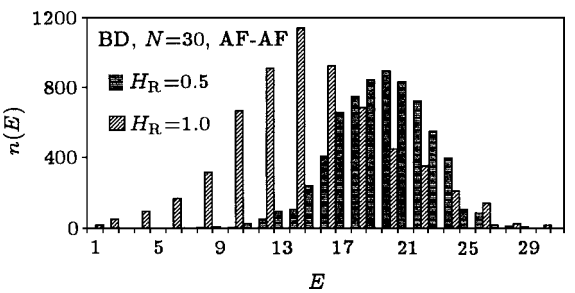


Fig.7. The distribution of $n(E)$ across E for the AF–AF system in the BD case (for H_R=0.5 and 1.0), with N=30 spins.

We have also studied the distribution of $n(E)$ as a function of N, as shown in Fig.8, similar results are obtained as those in the F–F and AF–F cases (only one region for the distributions is obtained here, no matter what N is for $H_R > 0.1$). The distribution of $n(E)$ as a function of the exchange couplings is also considered. The four systems are compared with one another for $N \leq 30$.

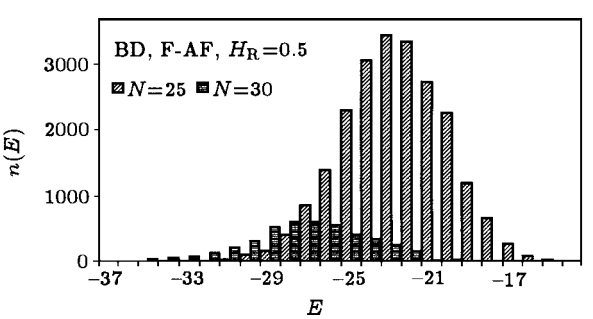


Fig.8. The distribution of $n(E)$ across E for the F–AF system in the BD case (for H_R=0.5), with N=25 and 30 spins.

We conclude that, for both cases of RF, the F–F and AF–F systems have very similar properties in the energy distributions across the energy minima, while the F–AF and AF–AF systems are very much alike, since the latter two cases are more disordered than the former two cases.

6. Magnetization

We have considered the same number of spin chains in the same corresponding samples as men-

tioned in section 2, in both BD and GD of the RF. The absolute value of magnetization $|M|$ of metastable states per spin is calculated exactly. Generally, we found that the F–F case seems to be more magnetized than the other three cases.

In the F–F case, in both distributions of RF, we have two fully magnetized minima in each sample, since the true ground state is fully aligned. While the other higher minima are either not magnetized or are weakly magnetized. Of course, these minima states are not fully aligned and they have some frustrated bonds.

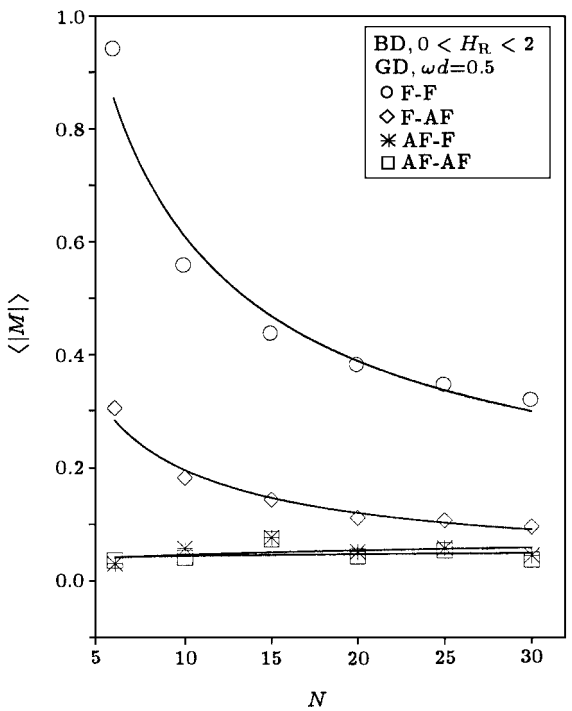


Fig.9. The average of absolute values of magnetization per spin over all metastable states $\langle |M| \rangle$ versus the number of spins N in both the BD case (for $0 < H_R < 2$) and the GD case (for ωd=0.5) of the RF, in the F–F, F–AF, AF–F and AF–AF systems.

We also studied the average of absolute values of magnetization $\langle |M| \rangle$, i.e. $|M|$ averaged over all metastable states, as functions of N. We found, in the BD case, that the average $\langle |M| \rangle$ decreases logarithmically with N and seems to depend exclusively on N. Furthermore, the average $\langle |M| \rangle$ with increasing N saturates for $0 < H_R < 2$ and $2 < H_R < 4$, and again it saturates as $H_R > 4$. But at $H_R = 2$ and 4 we have two singular points at which the values of $\langle |M| \rangle$ are very much smaller. In Fig.9 we consider the case $0 < H_R < 2$ for all systems. Similar results have been obtained in the GD case as in the BD case for $0 < H_R < 2$.

We also investigated the average $\langle |M| \rangle$ as a function of H_R in the BD case, for all our systems. We found that $\langle |M| \rangle$ saturates at a constant value (for each N) for $0 < H_R < 2$. It saturates at a higher constant value for $2 < H_R < 4$, and again for $H_R > 4$ it saturates at a smaller constant value. While at $H_R = 2$ and 4, we also have two singular points at which the values of $\langle |M| \rangle$ are very much smaller.

The distributions of magnetized minimum states $n(|M|)$ across the allowed values of absolute magnetization per spin $|M|$ are shown in the histograms plotted. In the BD case, for $0 < H_R < 2$, all the distributions coincide. The distributions of $n(|M|)$ decrease slightly with increasing $|M|$, as shown in Fig.10. But $n(|M|)$ may decrease and increase with increasing $|M|$ for $2 < H_R < 4$ no matter what the value of N. The GD case is very much like the BD case (for $0 < H_R < 2$).

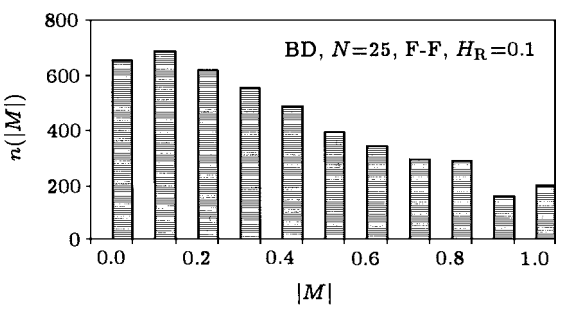


Fig.10. The distribution of the absolute value of magnetization per spin $|M|$ across the number of local energy minima $n(|M|)$ for the F–F system in the BD case of RF (for H_R=0.1), with N=25 spins.

In the F–AF, AF–F and AF–AF cases, for both types of RF, the fully magnetized state has never been reached, since the true ground state and the higher minima are not fully aligned. All these states are either not magnetized or are weakly magnetized ($0 \leq |M| \leq 0.4$). Also the average $\langle |M| \rangle$ as a function of N in the F–AF system has same behaviour as in the F–F case. But for the AF–F system, for $0 < H_R < 2$ and $2 < H_R < 4$, the average $\langle |M| \rangle$ seems to be saturated at constant values with increasing N. These values lie between 0 and 0.1. Similar results are obtained for the AF–AF case as long as $0 < H_R < 2$, and again $\langle |M| \rangle$ decreases logarithmically with N for $2 < H_R < 4$. The results in the GD case are very much like those in the BD for $0 < H_R < 2$, as shown in Fig.9. We also studied $n(|M|)$ as a function of $|M|$ through the histograms plotted for the same values of H_R and ωd as in the F–F case. We found that $n(|M|)$ decreases sharply with the increase of $|M|$, no matter what the value of N in both cases of RF, in all the

present three systems. We conclude that the properties of magnetization and its distributions in the F–F case are different from those of the other three systems.

In all our systems, for so large a RF in the BD case, i.e. for $H_R > 4$, $\langle |M| \rangle$ decreases logarithmically with N, but here it saturates in all four cases. Also, $n(|M|)$ decreases with increasing $|M|$ for any N, and we have identical distributions of $n(|M|)$ for all these systems. Here, there are no fully magnetized minimum states. One can say that, for so large a RF, the properties of magnetization and its distributions of any RF Ising system are completely independent of the exchange fields. The effects of RF on these properties are much larger than those of the exchange fields.

Our results agree with those obtained by Cieplak and Gawron,[1] and by Ismail and Bannora[10] for the SG and the corresponding DFM systems. They found that the DFM systems are magnetized more than the SG systems. Furthermore, in the DFM case there are fully magnetized minimum states, while for the SG system the fully magnetized state has never been reached. So our F–F case has similar properties of magnetization to the DFM case, but the other three cases have properties of the SG systems. Liu *et al*[15] found some difficulties in the study of nonlinear magnetization dynamics of a ferromagnet with an anisotropy in an external magnetic field. Nonlinear magnetization of the classical ferromagnet with two single-ion anisotropies in an external magnetic field[16] can provide an approximate description of various kinds of behaviour of magnetic materials, as well as the natural starting point for analysing the anomalous hydrodynamical behaviour of low-dimensional magnetic systems. Such a fascinating nonlinear dynamic problem exhibits both coherent and chaotic structures, depending on the nature of the magnetic interactions, and it is of considerable interest from the point of view of condensed-matter physics, statistical physics and soliton theory. Their results have shown that under the action of an external magnetic field nonlinear magnetization depends essentially on two parameters: the centre of the system moves with a constant velocity, while its shape changes with another constant velocity; its amplitude and width vary periodically with time, while its shape is also dependent on time and is asymmetric with respect to its centre.

Our main conclusion is that our systems are very much like SG, DFM, RF Ising and Potts spin systems. They have exponentially growing numbers of local energy minima with increasing number of spins. We can distinguish the distributions of these minima and the frustrations of the ground states between the behaviours of all these systems.

Also, our systems differ from each other merely by the magnetization. The absolute value of magnetization per spin averaged over all metastable states decreases logarithmically with the number of spins in many cases of these systems. The F–F system is magnetized more than the other three systems. On the other hand, in the F–F case we have fully magnetized minimum states, but in the other three cases the fully magnetized state has never been reached. Although we have taken the nn and nnn interactions into consideration, the effects of the RFs, on all the static properties, seem to be stronger than the effects of the exchange fields. In our future work, we will investigate the dynamic properties, the relaxation time and the susceptibility for these systems.

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