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Entropy, non-classical property of the trapped ion system driven by running-wave lasers*

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We study the dynamical behaviour of the ionic inversion in a trapped ion system in running-wave lasers, and we discuss the properties of entanglement between the ion and phonons and the statistics of the vibrational phonon field. It is shown that the appearance of the super-revivals of the ion inversion is dependent on the initial parameters of the vibrational phonon field. The Schrödinger cat and sub-Poissonian statistics of the phonon field can be also exhibited in this system for suitable initial values.

Keywords: entropy, non-classical property
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1. Introduction

Much interest has been focused on the trapped ion system due to recent advances in laser cooling and ion trapping. This system allows the preparation of nonclassical states of the vibrational motion of ions,[1−3] and has potential applications in precision spectroscopy,[4] quantum computation,[5] and quantum state reconstruction.[6,7] Although the interaction of a trapped ion with one or several laser beams is complicated, and it is difficult to study the dynamics of the trapped ion exactly, it can be greatly simplified under certain limiting conditions that may indeed be realized in experiments. For example, in the Lamb–Dicke limit and the rotating-wave approximation, Cirac *et al*[1] have shown that the dynamics of a trapped two-level ion, at the node of a standing wave, can be described by the Jaynes–Cummings model (JCM). Wu and Yang[8] and Alam *et al*[9] extended the study and predicted that an ion trapped in any position of a laser standing wave can also be described by the JCM Hamiltonian on an ionic dressed basis. Some authors,[10,11] also working within the Lamb–Dicke limit, have introduced a unitary transformation which reduces the Hamiltonian of a trapped ion at any position of a standing wave to that of the normal JCM on the bare basis of the trapped ion.

On the other hand, Meekhof *et al*[12] have reported on an experiment in which the nonclassical motional states of a trapped ion can be generated. In this experiment, the Lamb–Dicke parameter cannot be small because the higher-order terms in the expansion of the system Hamiltonian must be taken into account. This means that it is important to consider general cases without the Lamb–Dicke limit in the study of the dynamics of the trapped ion system. Recently, Moya-Cessa *et al*[13] introduced another unitary transformation which reduces the full ion-laser Hamiltonian in a running wave to that of the Jaynes–Cummings (JC) type without the Lamb–Dicke limit. This makes it possible to theoretically study the trapped ion system in general cases. However, Moya-Cessa *et al* only investigated the dynamical property of the mean phonon number in the vibrational field of the ionic centre-of-mass (CM). In this paper, we study further the dynamical behaviour of the ionic inversion, the properties of entanglement between the ion and phonons and the statistics of the vibrational phonon field. As is known, the study of these dynamical properties is important for the realization of the nonclassical state, quantum computation and quantum state reconstruction in the trapped ion system.

2. The model

We consider a single two-level ion, with excited

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and ground levels $|+\rangle$ and $|-\rangle$, trapped in a harmonic potential of frequency ν and interacting with a running-wave laser field. When the trap frequency ν is much greater than the ionic decay rate (i.e. strong confinement or strong-binding limit), the Hamiltonian of the system reads ($\hbar = 1$) [14]

$$H = \nu a^+ a + \frac{\Delta}{2}\sigma_z + \frac{\Omega}{2}[\sigma_- \exp[-\mathrm{i}\eta(a + a^+)] + \sigma_+ \exp[\mathrm{i}\eta(a + a^+)]]. \quad (1)$$

Here a^+ and a are the creation and annihilation operators of the phonons or vibrational states of the trap; $\Delta = \omega_0 - \omega_\mathrm{L}$ is the detuning between the ionic transition and laser frequency; $\sigma_+ = |+\rangle\langle -|(\sigma_- = |-\rangle\langle +|)$ is the usual electronic raising (lowering) operator; Ω is the Rabi frequency, which is proportional to the amplitude of the running-wave laser; and η is the Lamb–Dicke parameter. Considering a unitary transformation[12]

$$T = \frac{1}{\sqrt{2}} \begin{bmatrix} D^+ & D \\ -D^+ & D \end{bmatrix}, \quad (2)$$

where $D = \exp(\beta a^+ - \beta^* a)$ is Glauber's displacement operator, with $\beta = \mathrm{i}\eta/2$, then Hamiltonian (1) is transformed by T to

$$H' \equiv THT^+ = \nu a^+ a + \Omega\sigma_z - \mathrm{i}\frac{\eta\nu}{2}\left[(a^+ - a) - \mathrm{i}\frac{\Delta}{\eta\nu}\right](\sigma_+ - \sigma_-) + \frac{\nu\eta^2}{4}. \quad (3)$$

This is a JC-type Hamiltonian at any value of the Lamb–Dicke parameter η. Next, we perform the rotating-wave approximation which involves neglecting those terms in the interaction between the phonons and the ion which do not conserve energy and take $\nu = 2\Omega$. We then obtain

$$H' \approx H_\mathrm{JCM} = \nu a^+ a + \frac{1}{2}\nu\sigma_z - \mathrm{i}g(a^+\sigma_- - a\sigma_+), \quad (4)$$

where the effective coupling constant $g = \eta\nu/2$. The term $\nu\eta^2/4$ in Eq.(3) has not been taken into account.[12] In the interaction representation, then, the evolution operator of the system can be rewritten as

$$U_\mathrm{I}(t) = T^+ U_\mathrm{JCM}(t) T, \quad (5)$$

where

$$U_\mathrm{JCM}(t) = \exp\{-\mathrm{i}[-\mathrm{i}g(a^+\sigma_- - a\sigma_+)]t\} = \begin{bmatrix} \cos(V_1 t) & \frac{\sin(V_1 t)}{V_1} ga \\ -\frac{\sin(V_2 t)}{V_2} ga^+ & \cos(V_2 t) \end{bmatrix}, \quad (6)$$

with $V_1 = \sqrt{g^2 a a^+}, V_2 = \sqrt{g^2 a^+ a}$.

Assuming that the initial state of the system is $|\Psi(0)\rangle = |-\rangle|\alpha\rangle$, in which the ion is in its ground state $|-\rangle$ and the vibrational phonon field is in a coherent state $|\alpha\rangle\left(= \exp\left(-\frac{1}{2}|\alpha|^2\right)\sum_{n=0}^{\infty}\frac{\alpha^n}{\sqrt{n!}}\right)$, the state vector of the system at any time t is

$$|\Psi(t)\rangle = U_\mathrm{I}|\Psi(0)\rangle = D|A\rangle + D^+|B\rangle, \quad (7)$$

with

$$|A\rangle = \frac{1}{2}\exp\left[\frac{1}{2}(\alpha^*\beta - \alpha\beta^*)\right]\left\{\cos(V_1 t) - \cos(V_2 t) + \frac{\sin(V_2 t)}{V_2} ga^+ + \frac{\sin(V_1 t)}{V_1} ga\right\}|+, \overline{\alpha}\rangle,$$
$$|B\rangle = \frac{1}{2}\exp\left[\frac{1}{2}(\alpha^*\beta - \alpha\beta^*)\right]\left\{\cos(V_1 t) + \cos(V_2 t) - \frac{\sin(V_2 t)}{V_2} ga^+ + \frac{\sin(V_1 t)}{V_1} ga\right\}|-, \overline{\alpha}\rangle, \quad (8)$$

where $\overline{\alpha} = \alpha + \mathrm{i}\frac{\eta}{2} = |\overline{\alpha}|\mathrm{e}^{\mathrm{i}\varphi}$, and φ is the phase of the phonon field. We notice that the initial coherent state is displaced from α to $\overline{\alpha}$.

3. The ionic inversion

The inversion of a two-level trapped ion is given by

$$W(t) = \langle\Psi(t)|\sigma_z|\Psi(t)\rangle. \quad (9)$$

Using Eqs.(7) and (8), we obtain

$$\begin{aligned} W(t) =& \frac{1}{2}(\langle A|A\rangle - \langle B|B\rangle) \\ =& \frac{1}{4}\exp(-|\overline{\alpha}|^2)\sum_{n=0}^{\infty}\Big\{ -2|P_n|^2\cos(c_n t)\cos(c_{n+1}t) \\ &+ g^2(P_{n-1}^* P_{n+1} + P_{n-1}P_{n+1}^*) \\ &\times \sqrt{n(n+1)}\frac{\sin(c_n t)}{c_n}\frac{\sin(c_{n+1}t)}{c_{n+1}} \\ &+ g\sqrt{n}(P_{n-1}^* P_n + P_{n-1}P_n^*) \\ &\times \cos(c_{n+1}t)\frac{\sin(c_n t)}{c_n} \\ &- g\sqrt{n+1}(P_n^* P_{n+1} + P_n P_{n+1}^*) \\ &\times \cos(c_n t)\frac{\sin(c_{n+1}t)}{c_{n+1}}\Big\}, \end{aligned} \quad (10)$$

where $P_n = \frac{\overline{\alpha}^n}{\sqrt{n!}}$, $c_n = g\sqrt{n}$. As n is large, we have $P_{n-1} \simeq P_n \simeq P_{n+1}$. When $\overline{n} = |\overline{\alpha}|^2 \gg 1$ and $n \simeq \overline{n}$,

Eq.(10) can be rewritten as

$$W(t) \simeq -\frac{1}{2}\exp(-|\overline{\alpha}|^2)\frac{a^2}{|\overline{\alpha}|^2}\sum_{n=0}^{\infty}|P_n|^2 \times \cos[(\sqrt{n+1}+\sqrt{n})gt] - \frac{1}{2}\exp(-|\overline{\alpha}|^2)\frac{b^2}{|\overline{\alpha}|^2}\sum_{n=0}^{\infty}|P_n|^2 \times \cos[(\sqrt{n+1}-\sqrt{n})gt], \quad (11)$$

where $a = |\overline{\alpha}|\cos\varphi$, $b = |\overline{\alpha}|\sin\varphi$ and $a^2 + b^2 = |\overline{\alpha}|^2$.

The evolution of the ionic inversion $W(t)$ is presented in Fig.1 with $\Omega = \pi \times 10^6 \mathrm{s}^{-1}$ and the Lamb–Dicke parameter $\eta = 0.2$ [11] for the different phonon field $\overline{\alpha}$ (or α).

When the phonon field $\overline{\alpha} = (5.0, 0.1)$(or $\alpha = (5.0, 0)$), as in Fig.1(a), the ionic inversion $W(t)$ exhibits ordinary collapses and revivals. We can give an explanation from Eq.(11). In this case, due to $b^2 \ll a^2$ (or the phonon field phase $\varphi \to 0$), we obtain

$$W(t) \simeq -\frac{1}{2}\exp(-|\overline{\alpha}|^2)\frac{a^2}{|\overline{\alpha}|^2}\sum_{n=0}^{\infty}|P_n|^2\cos(2\sqrt{n}gt). \quad (12)$$

This is no other than the time evolution of the atomic inversion in standard JCM. The revival time $T_{\mathrm{R}} \simeq 2\pi\overline{n}^{1/2}/g$.[16]

In Fig.1(b), the condition $\overline{\alpha} = (0, 5.1)$ (or $\alpha = (0, 5)$), i.e. $b^2 \gg a^2$ (or $\varphi \to \pi/2$), is considered. We see that the ionic inversion $W(t)$ exhibits long-timescale revivals (super-revivals).[10,12,15] Similarly, this can be interpreted in terms of Eq.(11). Under the conditions considered, Eq.(11) can be written as

$$W(t) \simeq -\frac{1}{2}\exp(-|\overline{\alpha}|^2)\frac{b^2}{|\overline{\alpha}|^2}\sum_{n=0}^{\infty}|P_n|^2\cos\left(\frac{gt}{2\sqrt{n}}\right), \quad (13)$$

in which the revival time $T_{\mathrm{SR}} \simeq 8\pi\overline{n}^{3/2}/g \simeq 4\overline{n}T_{\mathrm{R}}$. For example, $T_{\mathrm{SR}} \simeq 100T_{\mathrm{R}} \gg T_{\mathrm{R}}$ for $\overline{n} \simeq 25$. That is, the time evolution of $W(t)$ exhibits super-revivals. Indeed, from Figs.1(a) and 1(b), $T_{\mathrm{R}} \simeq 0.5 \times 10^{-4}$s and $T_{\mathrm{SR}} \simeq 50 \times 10^{-4}$s.

If the phonon field phase $\varphi \to \pi/4$ (or $a \simeq b$), both terms in Eq.(11) cannot be ignored. The ionic inversion $W(t)$ shows a chaotic behaviour with no indications of revivals or any other regular structure (see Fig.1(c)). This result stems from the coexistence of the two terms in Eq.(11).

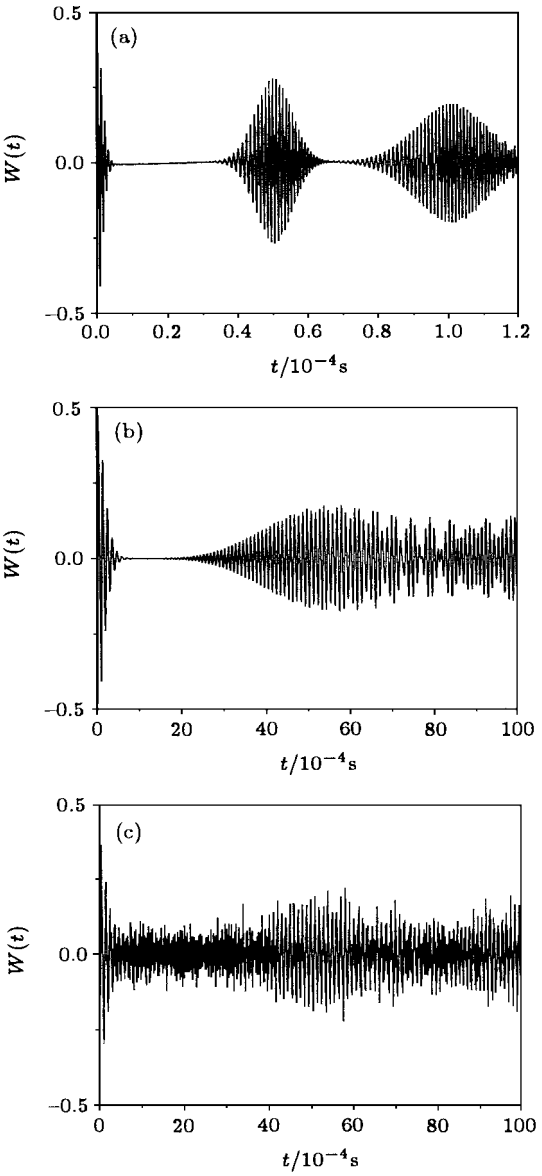


Fig.1. The time evolution of the ionic inversion $W(t)$ at $\Omega = \pi \times 10^6 \mathrm{s}^{-1}$ and $\eta = 0.2$: (a) $\overline{\alpha} = (5.0, 0.1)$ (or $\varphi \to 0$); (b) $\overline{\alpha} = (0, 5.1)$ (or $\varphi \to \pi/2$); (c) $\overline{\alpha} = (3.536, 3.536)$ (or $\varphi \to \pi/4$).

4. The ionic entropy

Quantum mechanically, the entropy is defined as

$$S(t) = -\mathrm{Tr}(\rho(t)\ln\rho(t)), \quad (14)$$

where ρ is the density operator for a given quantum system. In the trapped ion system, since for systems in which both the ion and phonon fields start from pure states, the ion and phonon entropies are precisely equal, i.e. $S_{\mathrm{i}}(t) = S_{\mathrm{p}}(t)$ (the subscripts i and p denote the ion and phonons). In the following, we only consider the time behaviour of the ion entropy $S_{\mathrm{i}}(t)$.

From Eqs.(5)–(7), we see that the density operator of the system can be written as

$$\rho(t) = |\Psi(t)\rangle\langle\Psi(t)| = T^{+}\rho' T, \tag{15}$$

and ρ' is the displaced JC density operator

$$\rho' = U_{\rm JCM} T |\Psi(0)\rangle\langle\Psi(0)| T^{+} U_{\rm JCM}^{+}. \tag{16}$$

Using the fact that

$$\mathrm{Tr}(FG) = \mathrm{Tr}(GF), \tag{17}$$

we obtain the following expression for the ionic entropy $S_{\rm i}(t)$

$$S_{\rm i}(t) = -\mathrm{Tr}(\rho_i'(t)\ln\rho_i'(t)), \tag{18}$$

where

$$\rho_i' = \frac{1}{2}\begin{bmatrix} \rho_{11}'(t) & \rho_{12}'(t) \\ \rho_{21}'(t) & \rho_{22}'(t) \end{bmatrix}, \tag{19}$$

with the elements of the matrix

$$\begin{aligned}
\rho_{11}'(t) =& 1 + \exp(-|\overline{\alpha}|^2)\sum_{n=0}^{\infty} g\sqrt{n+1}(P_n P_{n+1}^{*} \\
&+ P_{n+1}P_n^{*})\cos(c_{n+1}t)\frac{\sin(c_{n+1}t)}{c_{n+1}}, \\
\rho_{21}'(t) =& \exp(-|\overline{\alpha}|^2)\sum_{n=0}^{\infty}\Big\{|P_n|^2\cos(c_{n+1}t)\cos(c_n t) \\
&- g\sqrt{n}P_{n-1}P_n^{*}\cos(c_{n+1}t)\frac{\sin(c_n t)}{c_n} \\
&+ g\sqrt{n+1}P_n P_{n+1}^{*}\cos(c_n t)\frac{\sin(c_{n+1}t)}{c_{n+1}} \\
&- g^2\sqrt{n(n+1)}P_{n-1}P_{n+1}^{*}\frac{\sin(c_{n+1}t)}{c_{n+1}}\frac{\sin(c_n t)}{c_n}\Big\}, \\
\rho_{12}'(t) =& \rho_{21}'^{*}(t), \\
\rho_{22}'(t) =& 2 - \rho_{11}'(t),
\end{aligned} \tag{20}$$

where P_n and c_n are defined as before.

Since the trace is invariant under a similarity transformation, we can go to a basic in which the operator ρ_i' is diagonal. We rewrite Eq.(18) as

$$S_{\rm i}(t) = -\lambda_+\ln\lambda_+ - \lambda_-\ln\lambda_-, \tag{21}$$

where $\lambda_{\pm}$ can be derived from Eq.(19) in a straightforward manner

$$\lambda_{\pm} = \frac{1}{2}\{1 \pm [(\rho_{11}' - \rho_{22}')^2 + 4|\rho_{21}'|^2]^{1/2}\}. \tag{22}$$

The evolution of ionic entropy $S_{\rm i}(t)$ is presented in Fig.2 for the same parameter values as in Fig.1. In Fig.2(a), in the phonon field $\overline{\alpha} = (5, 0.1)$ (or $\varphi \to 0$), we find that the evolution of the ion entropy is the same as that of the atom entropy in the JCM. The entropy reaches a minimum value at half the revival time, which indicates that the ion and phonon subsystems roughly return to pure states. According to the dynamical properties of the JCM,[17] it is the signature of a Schrödinger cat being produced. As the phonon field $\overline{\alpha} = (0, 5.1)$ (or $\varphi \to \pi/2$) (see Fig.2(b)), it is obvious that the evolution of the entropy is completely different from Fig.2(a). The entropy does not reach its maximum value quickly. As time goes on, the entropy remains at its maximum values for a long time before some slight oscillations appear, indicating that the ion and phonons are strongly entangled.

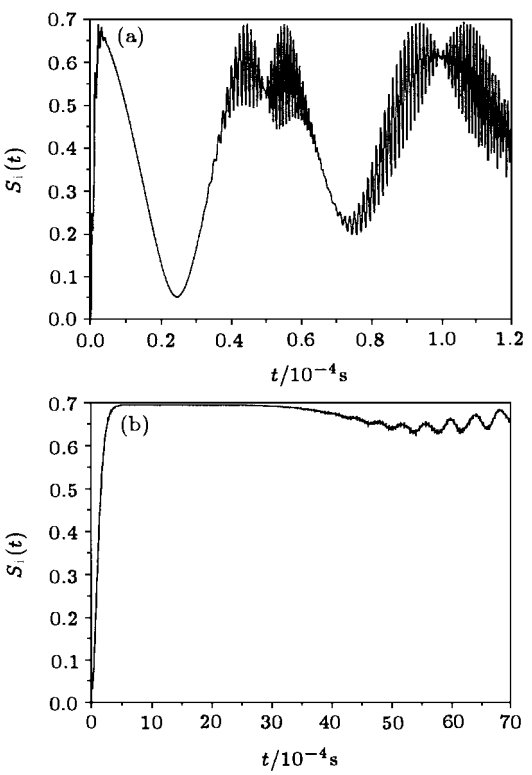


Fig.2. The time evolution of the ionic entropy $S_{\rm i}(t)$ at the same parameters as in Fig.1: (a) $\overline{\alpha} = (5.0, 0.1)$ (or $\varphi \to 0$); (b) $\overline{\alpha} = (0, 5.1)$ (or $\varphi \to \pi/2$).

5. Mandel Q factor

The Q factor

$$Q(t) = \frac{\langle(\Delta n)^2\rangle}{\langle n\rangle} - 1 \tag{23}$$

introduced by Mandel[18] is a good measure of the extent to which the photon statistics of a field are sub-Poissonian or super-Poissonian in the system of the atom and photons. When the field is Poissonian, $Q(t)$ vanishes. For sub-Poissonian fields, it is negative, and

for super-Poissonian fields, positive. Here, we use the Mandel Q factor to examine the phonon statistics of the vibrational state in an ion–phonon system.

From Eq.(23) we see that for $\Delta n = n - \langle n\rangle$ we have to compute $\langle n\rangle$ and $\langle n^2\rangle$ to obtain $Q(t)$. So we have

$$\begin{aligned}\langle n^2\rangle =&\langle A|D^+(a^+a)^2D|A\rangle + \langle B|D(a^+a)^2D^+|B\rangle\\ =&(n^2+4n|\beta|^2+|\beta|^2+|\beta|^4)(\langle A|A\rangle+\langle B|B\rangle)\\ &+[(n+2|\beta|^2\beta+\beta(n+1))\\ &\times(\langle A|a^+|A\rangle-\langle B|a^+|B\rangle)+\text{c.c.}]\\ &+[\beta^2(\langle A|a^{+2}|A\rangle+\langle B|a^{+2}|B\rangle)+\text{c.c.}],\end{aligned}\tag{24}$$

and

$$\begin{aligned}\langle n\rangle =&\langle A|D^+(a^+a)D|A\rangle + \langle B|D(a^+a)D^+|B\rangle\\ =&(n+|\beta|^2)(\langle A|A\rangle+\langle B|B\rangle)\\ &+[\beta(\langle A|a^+|A\rangle-\langle B|a^+|B\rangle)+\text{c.c.}],\end{aligned}\tag{25}$$

where

$$\begin{aligned}&\langle A|A\rangle+\langle B|B\rangle\\ =&2|P_n\cos(\sqrt{n+1})+P_{n+1}\sin(\sqrt{n+1})|^2\\ &+2|P_{n-1}\sin(\sqrt{n})-P_n\cos(\sqrt{n})|^2,\end{aligned}\tag{26}$$

$$\begin{aligned}&\langle A|a^+|A\rangle-\langle B|a^+|B\rangle\\ =&2\sqrt{n+1}(P_n^*\sin(\sqrt{n+1})-P_{n+1}^*\cos(\sqrt{n+1}))\\ &\times(P_n\cos(\sqrt{n+1})+P_{n+1}\sin(\sqrt{n+1}))\\ &+2\sqrt{n+1}\big(P_{n+1}^*\cos(\sqrt{n+2})+P_{n+2}^*\sin(\sqrt{n+2})\big)\\ &\times(P_{n-1}\sin(\sqrt{n})-P_n\cos(\sqrt{n})),\end{aligned}\tag{27}$$

$$\begin{aligned}&\langle A|a^{+2}|A\rangle+\langle B|a^{+2}|B\rangle\\ =&2\sqrt{(n+1)(n+2)}(P_n\cos(\sqrt{n+1})\\ &+P_{n+1}\sin(\sqrt{n+1}))\\ &\times(P_{n+2}^*\cos(\sqrt{n+3})+P_{n+3}^*\sin(\sqrt{n+3}))\\ &+2\sqrt{(n+1)(n+2)}(P_{n-1}\sin(\sqrt{n})-P_n\cos(\sqrt{n}))\\ &\times(P_{n+1}^*\sin(\sqrt{n+2})-P_{n+2}^*\cos(\sqrt{n+2})).\end{aligned}\tag{28}$$

The numerical results of $Q(t)$ are presented in Fig.3 for (a) $\overline{\alpha}=(5.0,0.1)$ (or $\varphi\to 0$) and (b) $\overline{\alpha}=(0,5.1)$ (or $\varphi\to\pi/2$). As can be seen from Fig.2(a), $Q(t)$ oscillates near the initial time t=0 and in the revival region ($T_{\rm R}\simeq 0.5\times10^{-4}$s), the vibrational phonon field becomes sub-Poissonian or super-Poissonian. In the collapse region, the field becomes only super-Poissonian. In Fig.3(b), we notice that the field begins to be sub-Poissonian at about half a revival time near the first revival time. The field becomes sub-Poissonian again near the second or third revival time; then it becomes super-Poissonian for the rest of its time evolution. It is shown that, for $\varphi\to\pi/2$, the time interval in which the phonon field becomes sub-Poissonian is longer than that when $\varphi\to 0$.

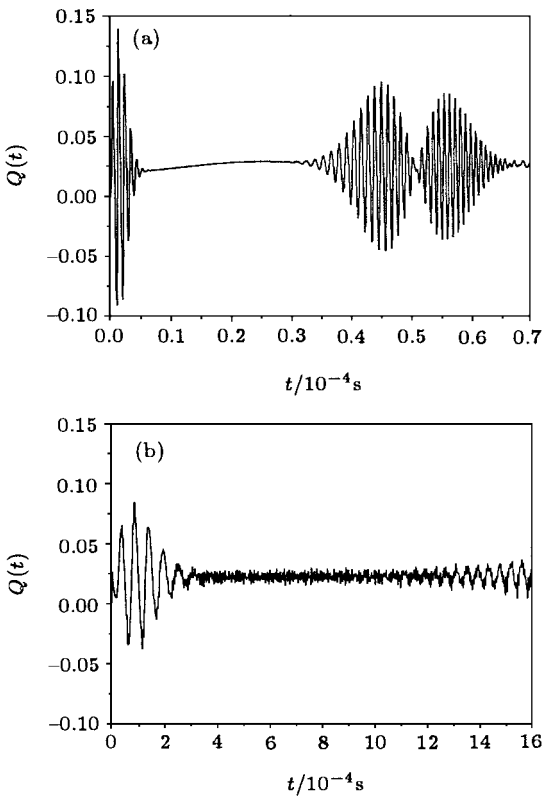


Fig.3. The time evolution of the Mandel Q factor at the same parameters as in Fig.1: (a) $\overline{\alpha}=(5.0,0.1)$ (or $\varphi\to 0$); (b) $\overline{\alpha}=(0,5.1)$ (or $\varphi\to\pi/2$).

6. Conclusion

We have studied the ion inversion, entropy and Mandel Q factor of the trapped ion system with the vibrational phonon field initially in a coherent state $|\alpha\rangle$ driven by running-wave lasers. It is shown that the dynamical properties of the system are strongly dependent on the phase φ of the initial coherent state $|\alpha\rangle$. As the phase φ tends to zero, the behaviour of the ionic inversion exhibits ordinary revivals. The Schröinger cat can be produced similarly at half of one revival time in this trapped ion system. As the phase φ tends to $\pi/2$, super-revivals occur in the time evolution of the ionic inversion at a revival time $T_{\rm SR}\simeq 4\overline{n}T_{\rm R}$; the time interval in which the vibrational phonon field remains sub-Poissonian is longer than that in the first case $\varphi\to 0$. Since dissipation in traps is much smaller than in cavities, it may be possible to experimentally observe the super-revivals of a trapped ion and the nonclassical properties of the phonon field in running-wave lasers.

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