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Generalized synchronization of chaos in erbium-doped dual-ring lasers

Zhang Sheng-Hai(张胜海)† and Shen Ke(沈 柯)

Department of Physics, Changchun Institute of Optics and Fine Mechanics, Changchun 130022, China

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We investigate chaotic synchronization in the generalized sense in unidirectionally coupled erbium-doped fibre dual-ring lasers. Numerical simulation shows that no matter whether the two different erbium-doped fibre dual-ring lasers are chaotic or not before coupling, they show generalized synchronization with a suitable unidirectional coupling coefficient under which the maximum condition Lyapunov exponent is negative. We also use the auxiliary system approach to detect the generalized synchronization.

Keywords: generalized synchronization, chaos, erbium-doped fibre dual-ring laser, maximum condition Lyapunov exponent

PACC: 0545, 4255N

1. Introduction

Chaotic synchronization has attracted much attention in recent years for its potential application. Several authors have studied the synchronization of the chaotic signal in the context of electronic circuits,[1−3] secure communication[4,5] and laser dynamics.[6−11] The synchronized chaotic oscillations mostly studied in the literature are those in which two coupled systems evolve according to exactly the same dynamics.[12−15] However, there are many coupling systems with different dynamics; that is, the dynamics of the response system is different from that of the driving system, even as far as the dimensions are different for the two systems. The identical synchronization dose not exist in this situation, however there is perhaps a transformation $y = \phi(x)$ (y represents the variables of the response system and x those of the driving system) which is unknown and complex. We call this generalized synchronization.[16] Abarbanel *et al* presented an auxiliary system approach to detect the generalized synchronization of chaos,[17] and other authors investigated the generalized synchronization of chaos in different YAG lasers.[18,19] Erbium-doped fibre lasers appear very attractive for long-haul communications over optical fibres as the lasing wavelength of about 1.55μm is near the minimum attenuation and dispersion point of the standard single-mode optical fibre. Van Wiggeren and Roy[20] were the first to realize secure communication experimentally by use of chaos of erbium-doped lasers. Our group has investigated the inverse synchronization of chaos in erbium-doped fibre dual-ring lasers via mutual coupling.[21] In this paper, we investigate the generalized synchronization of chaos in two different erbium-doped fibre dual-ring lasers. We find that two different erbium-doped fibre dual-ring lasers can be synchronized with each other in the generalized sense through directional coupling only if the maximum condition Lyapunov exponent (MCLE) is negative, which is detected by the auxiliary system approach.

2. A scheme of generalized synchronization

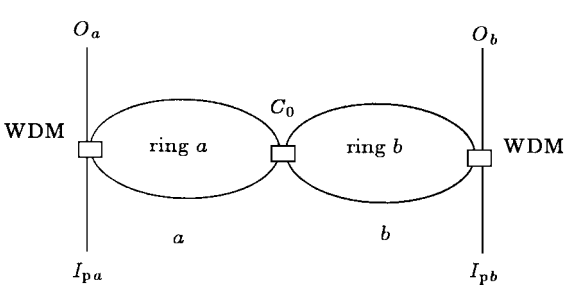


Fig.1. Dual-ring erbium-doped laser system: C_0 is the coupler; WDM is the wavelength division multiplexer; I_{pa} and I_{pb} are the pumping intensities.

The fibre dual-ring laser system is shown in Fig.1. Both ring a and ring b contain a wavelength division

†E-mail:zhangshenghai999@sohu.com

multiplexer (WDM) and ring a and ring b are coupled to each other by the coupler C_0. I_{pa} and I_{pb} are the pumping intensities of ring a and ring b, respectively. As is known,[22] the erbium-doped fibre dual-ring laser shows periodic, chaotic or developed chaotic states under different conditions.

In our scheme, one erbium-doped fibre dual-ring laser (the driven system S) is driven by a signal from the other laser (the driving system M) through a unidirectional coupler, and M is not affected by S. The dynamic equations of the whole system keep their original form[22] as follows

for system M

$$\frac{\mathrm{d}E_{aM}}{\mathrm{d}\tau} = -k_{aM}(E_{aM} + \eta_{0M}E_{bM}) + g_{aM}E_{aM}D_{aM}, \tag{1}$$

$$\frac{\mathrm{d}E_{bM}}{\mathrm{d}\tau} = -k_{bM}(E_{bM} - \eta_{0M}E_{aM}) + g_{bM}E_{bM}D_{bM}, \tag{2}$$

$$\frac{\mathrm{d}D_{aM}}{\mathrm{d}\tau} = -(1 + I_{paM} + E_{aM}^2)D_{aM} + I_{paM} - 1, \tag{3}$$

$$\frac{\mathrm{d}D_{bM}}{\mathrm{d}\tau} = -(1 + I_{pbM} + E_{bM}^2)D_{bM} + I_{pbM} - 1; \tag{4}$$

for system S

$$\frac{\mathrm{d}E_{aS}}{\mathrm{d}\tau} = -k_{aS}(E_{aS} + \eta_{0S}E_{bS} - \varepsilon E_{aM}) + g_{aS}E_{aS}D_{aS}, \tag{5}$$

$$\frac{\mathrm{d}E_{bS}}{\mathrm{d}\tau} = -k_{bS}(E_{bS} - \eta_{0S}E_{aS}) + g_{bS}E_{bS}D_{bS}, \tag{6}$$

$$\frac{\mathrm{d}D_{aS}}{\mathrm{d}\tau} = -(1 + I_{paS} + E_{aS}^2)D_{aS} + I_{paS} - 1, \tag{7}$$

$$\frac{\mathrm{d}D_{bS}}{\mathrm{d}\tau} = -(1 + I_{pbS} + E_{bS}^2)D_{bS} + I_{pbS} - 1. \tag{8}$$

Here E_a and E_b are the lasing fields and D_a and D_b are the population inversions in ring a and ring b, respectively; k_a and k_b are the products of τ_2 and decay rate and g_a and g_b are the products of τ_2 and gain of coefficient of lasing field of ring a and ring b, respectively. $\tau = \gamma t$, in an erbium-doped fibre laser, $\gamma = 1/\tau_2$, and τ_2 is the decay time of the lasing upper level and is around 10ms. In this paper, we take τ_2=10ms, η_0 is the coupling coefficient of the coupler C_0, and ε is the unidirectional coupling coefficient from M to S. The subscripts M and S denote the driving system M and driven system S.

3. Results and discussion

We first calculate the MCLE of system S as a function of ε in three cases: (I) M is chaotic and S is periodic; (II) M is periodic and S is chaotic; (III) M and S are both chaotic before S is driven by M.

The parameters in S are the same as the corresponding parameters in M except for g_{bM} and g_{bS}. They are as follows:[18] $k_{aM} = k_{bM} = 1000$; $g_{aM} = 10500$, $\eta_{0M} = 0.2$; $I_{paM} = I_{pbM} = 4$. g_{bM} and g_{bS} are different in the three cases: (I) $g_{bM} = 4700$, $g_{bS} = 4500$; (II) $g_b = 4500$, g_{bS}=4700; (III) g_{bM}=4700, $g_{bS} = 4800$. We obtain the MCLE in the three cases in the range of $0.02 < \varepsilon < 0.07$.

From Fig.2, the MCLE of S almost decreases monotonically as ε increases and becomes negative when $\varepsilon > 0.025$ for case (I). For case (II), the driving signal is periodic, and the MCLE shows much ripple as ε increases and becomes negative when $\varepsilon > 0.05$. For case (III), the MCLE also almost decreases monotonically as ε increases and becomes negative when $\varepsilon > 0.042$. From the three cases we can find that the MCLE is negative with a suitable value of ε. Meanwhile, it is not necessary to have a very large value, which can be realized using a coupler experimentally. S should synchronize to M in a generalized sense when MCLE is negative.

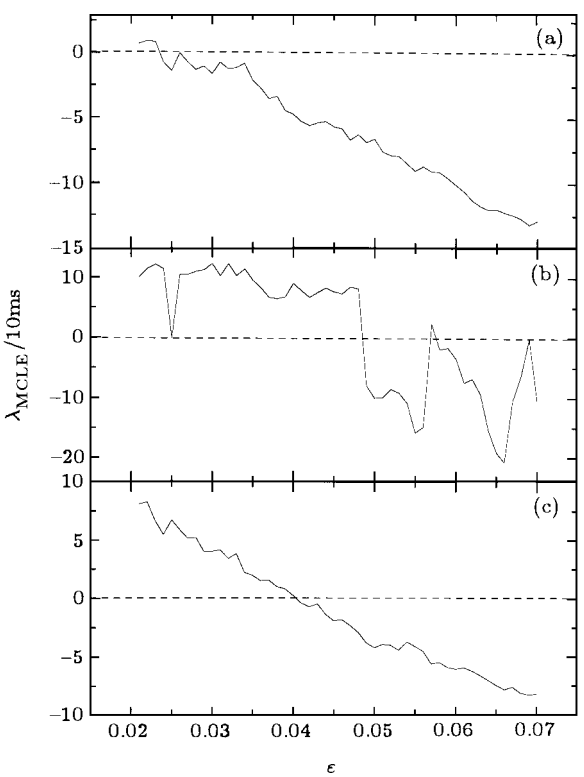


Fig.2. MCLE as a function of directional coupling coefficient ε in the three cases. The parameters in M and S are: k_a and k_b, 1000; g_a, 10500; η_0, 0.2; I_{pa} and I_{pb}, 4. (a) g_{bM}, 4700; g_{bS}, 4500. (b) g_{bM}, 4500; g_{bS}, 4700. (c) g_{bM}, 4700; g_{bS}, 4800.

We use an auxiliary system approach[18] to detect S as having generalized synchronization to M with a

negative MCLE. In this approach, it is necessary to add another driven system (we call this system A) which is exactly the same as S. Hence, system A has the same dynamic equation as S, with all parameters and variables denoted by a subscript A. A and S are simultaneously driven by the same signal from M.

In the calculation, we take $\varepsilon = 0.05$ for all three cases. The other parameters are the same as those in Fig.2, corresponding to the respective cases. The MCLE of S is negative for all three cases when $\varepsilon = 0.05$. Figure 3 shows the case (I). From Fig.3(a) one can find that M is chaotic. S becomes chaotic if it is driven by M, which can be seen in Fig.3(b). The MCLE is negative and S is not a chaotic system. The chaotic output from S is the result of being driven by M. From Fig.3(c) we cannot find a regular relation between S and M and the relationship is complex. From Fig.3(d), one can find $E_{aS} = E_{aA}$ which is always true after the transient process dies out; other corresponding variables are also the same. The states of S and A are the same after the transient process dies out even though the initial values of A and S are different. In other words, their initial conditions are irrelevant. Hence, the state of S is solely determined by M itself and a relationship exists between S and M, $y = \phi(x)$, which is not known. S is synchronized in a generalized sense in case (I).

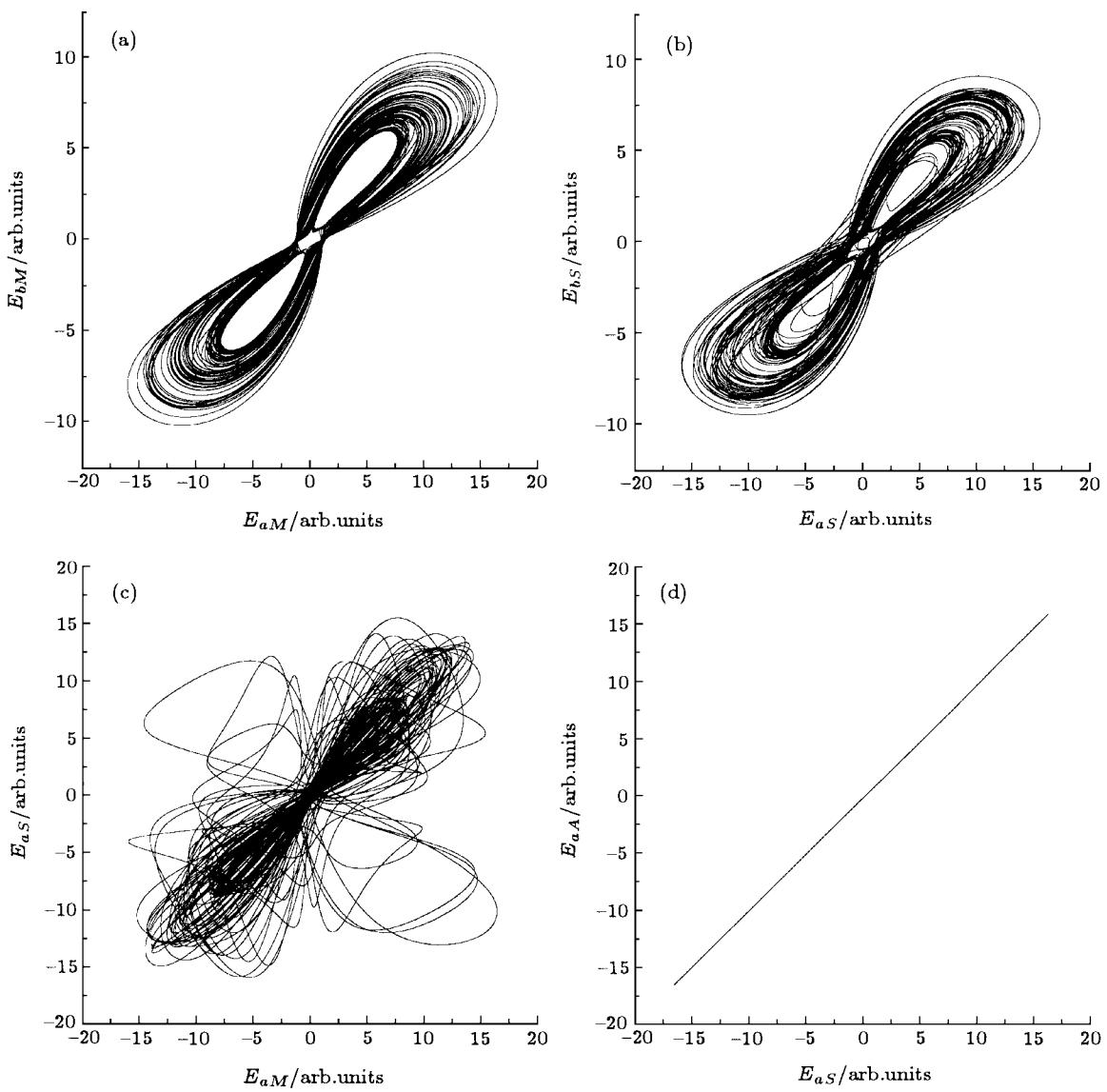


Fig.3. Generalized synchronization of M and S for case (I). $\varepsilon = 0.05$, and the other parameters are the same as those in Fig.2(a). (a) The projection on plane $E_{aM} - E_{bM}$ of system M, (b) the projection on plane $E_{aS} - E_{bS}$ of system S, (c) the relation between E_{aM} and E_{aS}, (d) the relation between E_{aS} and E_{aA}.

The cases (II) and (III) are shown in Figs.4 and 5. One can find that S also synchronizes to M in the generalized sense, which is the same as in case (I). Especially in case (II), S changes to the periodic state from the original chaotic state. However, the period is different from that of M. For case (III), though S is chaotic originally, the signal from S driven by M is different from the original signal and is determined by M.

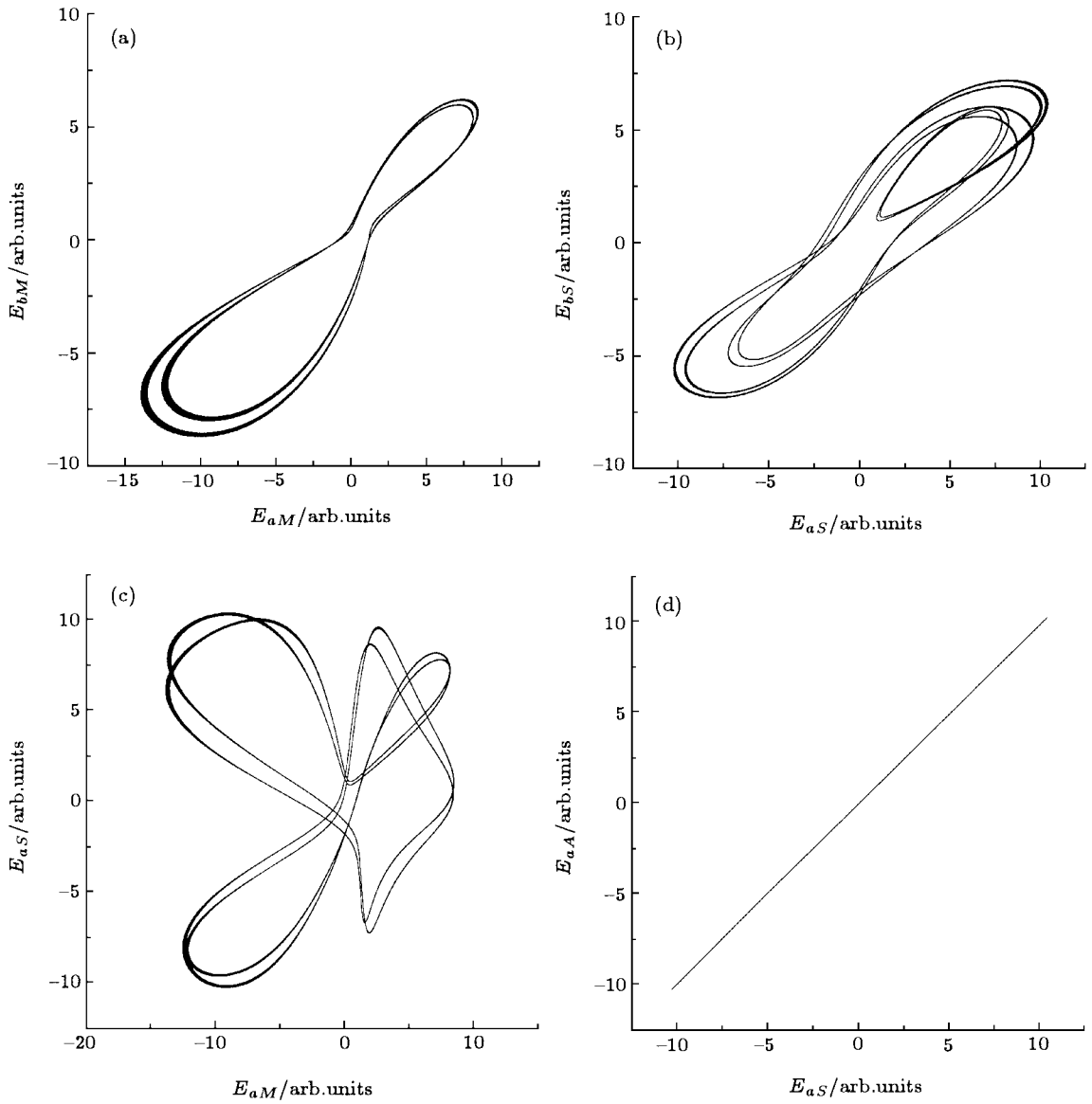


Fig.4. Same as in Fig.3 but for case (II).

From the above, whether M and S are chaotic or not originally, S and M can have generalized synchronization only if the MCLE is negative. If the condition Lyapunov exponents of the driven system are all negative when the driven system is driven by the driving system, the driven system would break away from its original state, whether it is chaotic or periodic, and enter into a new state which is formed when the driven system is affected by the driving signal. The driven system would return to its original chaotic or periodic state with the termination of the driving signal. Hence, the driving system determines the state of the driven system. In other words, there is a definite relationship between driven and driving systems: $y = \phi(x)$. If there is a positive Lyapunov exponent at least, the relationship between driven and

driving systems is not definite, though the driving signal also affects the driven system, and they could not reach generalized synchronization. One erbium-doped fibre dual-ring laser acting as a driving system can determine the state of the other erbium-doped fibre dual-ring laser through unidirectional coupling with a suitable coupling coefficient, the driven and driving lasers reaching generalized synchronization. The generalized synchronization does not require the strict condition of identical synchronization.

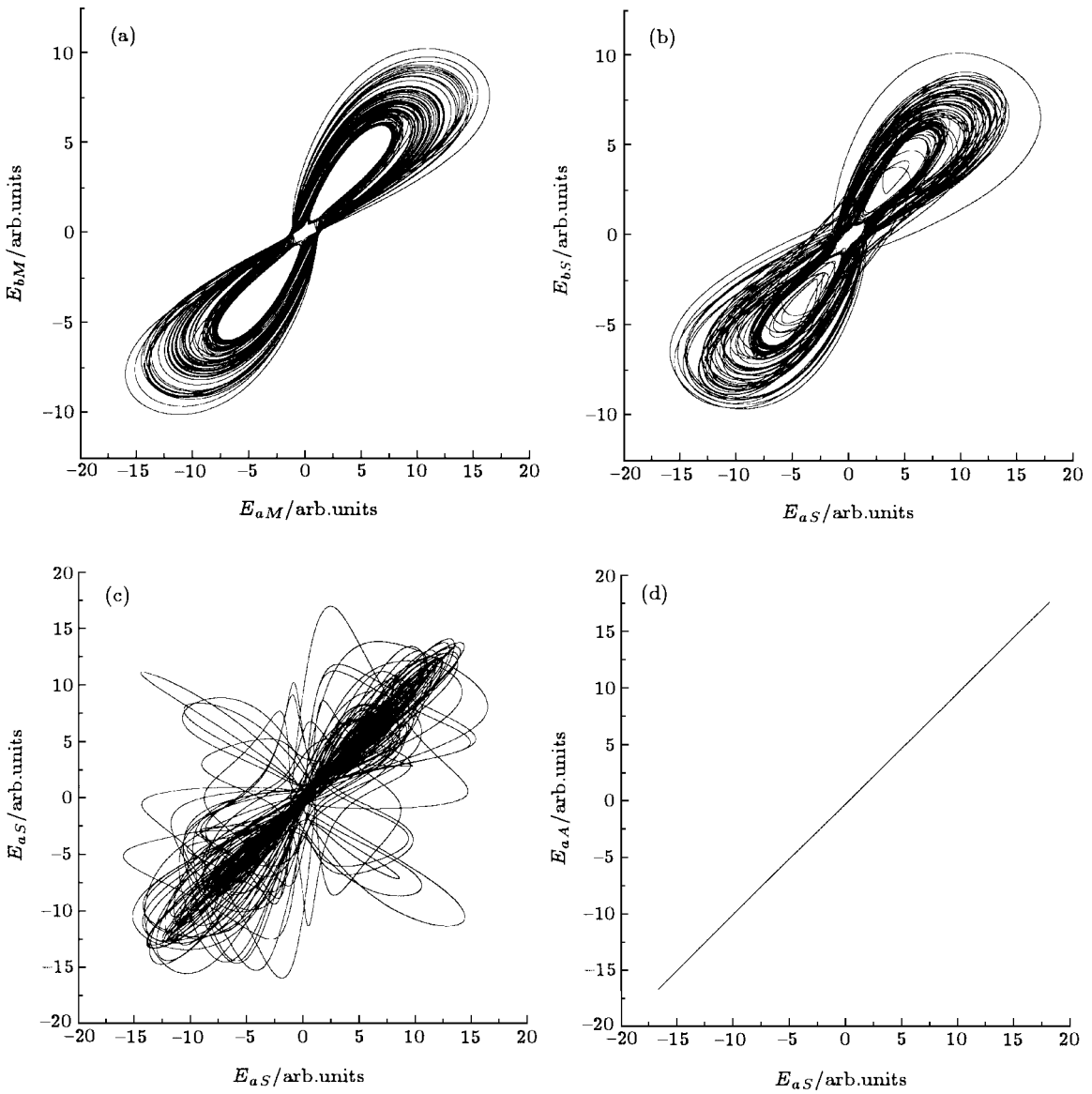


Fig.5. Same as in Fig.3 but for case (III).

4. Conclusion

The driven dual-ring erbium-doped fibre laser can have generalized synchronization with a different dual-ring erbium-doped fibre laser as a driving system if the MCLE of the driven system is negative. This can be realized by adjusting the unidirectional coupling coefficient. Whether two lasers have generalized synchronization can be detected by the auxiliary system approach. Of course, in our numerical simulation, the initial values of S and A are in the same attractive basin. This generalized synchronization is more easily realized than identical synchronization and is common. The state of the driven system is determined by the driving system under certain conditions. Hence, several identical systems can be identically synchronized by being driven by one system under the same conditions, which is very useful.

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