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Preparation of entangled squeezed states and quantification of their entanglement*

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We propose a scheme for generating bipartite and multipartite entangled squeezed states via the Jaynes–Cummings model with large detuning. Bipartite entanglement of these entangled states is quantified by the concurrence. We also use the N-tangle to compute multipartite entanglement of these multipartite entangled squeezed states. Finally we discuss two limiting cases which arise from $r \to \infty$ and $r \to 0$, in which the multipartite entangled squeezed state reduces correspondingly into an N-qubit Greenberger–Horne–Zeilinger state and an N-qubit W state.

Keywords: quantum entanglement, squeezed state, Jaynes–Cummings model
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1. Introduction

Quantum entanglement, first noted by Einstein, Podolsky and Rosen (EPR)[1] and Schrödinger,[2] is one of the essential features of quantum mechanics. For decades, entanglement has been the focus of much work on the foundations of quantum mechanics, being associated particularly with quantum nonseparability and the violation of Bell's inequalities. In recent years, however, entanglement has aroused much interest in quantum information processing, for example, quantum teleportation,[3] superdense coding,[4] quantum key distribution[5] and telecoloning.[6] Recently, Giovannetti *et al* proposed the idea of positioning and clock synchronization to employ entangled and squeezed light,[7] which means that the scope of the use of entanglement is expanding continuously.

There are different types of entanglement for the light field. In the previous work, Bouwmeester *et al* studied a polarization-entangled state for their teleportation experiments.[8] Furusawa *et al* have realized experimentally unconditional quantum teleportation of an unknown coherent state by exploiting a two-mode squeezed vacuum state as an entanglement resource.[9] Recently, many other authors have investigated the properties of entangled coherent states and their use in quantum information processing.[10–14] Also, some authors have presented various methods of preparing entangled states by cavity quantum electrodynamics.[15–18] In this paper, we make use of the Jaynes–Cummings model with large detuning to prepare entangled squeezed states (ESS) with an arbitrary multipartite system. We calculate the degree of bipartite entanglement of these entangled states by exploiting concurrence.[19] We also compute the N-tangle[20] to characterize the multipartite entanglement of the tripartite and even-number ESS.

2. Preparation of the entangled squeezed states

First, we review the description of a dispersive atom–cavity field interaction. For a two-level atom and a cavity field, we assume that the atomic frequency is tuned off-resonant with the cavity frequency. The effective Hamiltonian for the dispersive atom–cavity field interaction is given by[21]

$$\hat{H}_{\rm I} = 2\frac{g^2}{\Delta}\hat{a}^+\hat{a}\hat{S}_z, \tag{1}$$

where $\hat{a}^+$ and $\hat{a}$ are the creation and annihilation operators of the cavity field, $\hat{S}_z$ is the atomic inversion operator, Δ is the detuning between the atomic transition and the cavity field, and g is the atom–cavity coupling constant. If the atom is initially prepared in a superposition of its excited state $|e\rangle$ and ground

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state $|g\rangle$, and the cavity field is initially in the squeezed vacuum state $|\xi\rangle$, the initial atom-field state is

$$|\Psi(0)\rangle_{a-f} = \frac{1}{\sqrt{2}}(|g\rangle + |e\rangle) \otimes |\xi\rangle, \tag{2}$$

where $\xi = re^{i\varphi}$ is any complex number with modulus r and argument φ. An expansion in terms of Fock states is

$$|\xi\rangle = \sqrt{\mathrm{sech}r} \sum_{n=0}^{\infty} \frac{\sqrt{(2n)!}}{n!} \left(-\frac{1}{2} e^{i\varphi} \tanh r \right)^n |2n\rangle. \tag{3}$$

Letting the atom enter the cavity, after an interaction time t', the system evolves to

$$|\Psi(t')\rangle = \frac{1}{\sqrt{2}} \left[|g\rangle|re^{i\left(\varphi + \frac{2g^2}{\Delta}t'\right)} + |e\rangle|re^{i\left(\varphi - \frac{2g^2}{\Delta}t'\right)} \right]. \tag{4}$$

With the choice $(g^2/\Delta)t' = \pi/4$, we obtain

$$|\Psi(t')\rangle_{a-f} = \frac{1}{\sqrt{2}}(|g\rangle|\xi\rangle + |e\rangle|-\xi\rangle). \tag{5}$$

According to the description above, it is easy to realize the generation of the ESS. We assume that there are two cavities (referred to as 1 and 2), which are both initially prepared in the squeezed state $|\xi\rangle$, and that the atom is initially prepared in the superposition. Thus, the initial state of the whole system is given by

$$|\Psi(0)\rangle_{a-f} = \frac{1}{\sqrt{2}}(|g\rangle + |e\rangle) \otimes |\xi\rangle_1|\xi\rangle_2. \tag{6}$$

Letting the atom enter cavities 1 and 2 in a regular order with same interaction time t', after the atom comes out from these cavities the system evolves to

$$|\Psi'\rangle_{a-f} = \frac{1}{\sqrt{2}}(|g\rangle|\xi\rangle_1|\xi\rangle_2 + |e\rangle|-\xi\rangle_1|-\xi\rangle_2). \tag{7}$$

Now we let the atom cross a classical field. The classical microwave field can be used to cause the transitions[22]

$$|e\rangle \to \frac{1}{\sqrt{2}}(|e\rangle + |g\rangle), |g\rangle \to \frac{1}{\sqrt{2}}(|g\rangle - |e\rangle), \tag{8}$$

such that the above state becomes

$$\begin{aligned} |\Psi'\rangle_{a-f} =& \frac{1}{2}|g\rangle(|\xi\rangle_1|\xi\rangle_2 + |-\xi\rangle_1|-\xi\rangle_2) \\ & - \frac{1}{2}|e\rangle(|\xi\rangle_1|\xi\rangle_2 - |-\xi\rangle_1|-\xi\rangle_2). \end{aligned} \tag{9}$$

Thereupon, we perform an atom measurement. When the atom is detected in the ground or excited states, the cavity field collapses correspondingly into

$$|\Psi'\rangle \to |\xi\rangle_1|\xi\rangle_2 + |-\xi\rangle_1|-\xi\rangle_2 \tag{10}$$

or

$$|\Psi'\rangle \to |\xi\rangle_1|\xi\rangle_2 - |-\xi\rangle_1|-\xi\rangle_2. \tag{11}$$

After normalization again, we obtain two ESS

$$|B\rangle_{12}^{\pm} = \frac{1}{\sqrt{N_{\pm}}}(|\xi\rangle_1|\xi\rangle_2 \pm |-\xi\rangle_1|-\xi\rangle_2), \tag{12}$$

where $N_{\pm} = 2(1 \pm k^2)$ are normalization constants and $k = \langle\xi|-\xi\rangle = \sqrt{\mathrm{sech}(2r)}$ is the overlap.

It is possible to generate the ESS with N subsystems. We assume that there are N cavities which are initially prepared each in the squeezed state, by employing a similar method as above. We can obtain the ESS with N subsystems

$$\begin{aligned} |G\rangle_{12\cdots N}^{\pm} =& \frac{1}{\sqrt{N'_{\pm}}}(|\xi\rangle_1|\xi\rangle_2 \cdots |\xi\rangle_N \\ & \pm |-\xi\rangle_1|-\xi\rangle_2 \cdots |-\xi\rangle_N), \end{aligned} \tag{13}$$

where $N'_{\pm} = 2(1 \pm k^N)$ are normalization constants.

3. Quantification of the entanglement

The method for preparing the ESS has been discussed. Now we quantify the entanglement for these states. First we consider that the bipartite states $|B\rangle_{12}^{\pm}$ are given by Eq.(12). It is obvious that $|\xi\rangle$ and $|-\xi\rangle$ are not orthogonal. However, the two nonorthogonal states $|\xi\rangle$ and $|-\xi\rangle$ are assumed to be linearly independent and span a two-dimensional subspace of the Hilbert space. Thus we can introduce a pair of orthogonal bases $\{|\bar{0}\rangle, |\bar{1}\rangle\}$ as $|\bar{0}\rangle = |\xi\rangle$, $|\bar{1}\rangle = (|-\xi\rangle - k|\xi\rangle)/N$ with $N = \sqrt{1-k^2}$. These play the role of the general qubit basis $\{|0\rangle, |1\rangle\}$. Under this basis we have

$$|\xi\rangle = |\bar{0}\rangle, |-\xi\rangle = N|\bar{1}\rangle + k|\bar{0}\rangle, \tag{14}$$

$$\begin{aligned} |B\rangle_{12}^{\pm} =& \frac{1}{\sqrt{N_{\pm}}}[(1 \pm k^2)|\bar{0}\rangle_1|\bar{0}\rangle_2 \pm kN|\bar{0}\rangle_1|\bar{1}\rangle_2 \\ & \pm Nk|\bar{1}\rangle_1|\bar{0}\rangle_2 \pm N^2|\bar{1}\rangle_1|\bar{1}\rangle_2]. \end{aligned} \tag{15}$$

The states $|B\rangle_{12}^{\pm}$ are the pure states, whose density matrices are $\rho_{12}^{\pm} = |B\rangle_{12}^{\pm}\langle B|$. The reduced density matrices are

$$\rho_1^{\pm} = \frac{1}{N_{\pm}} \begin{pmatrix} (k'_{\pm})^2 + k^2N^2 & kN^3 \pm k'_{\pm}kN \\ kN^3 \pm k'_{\pm}kN & k^2N^2 + N^4 \end{pmatrix}, \tag{16}$$

where $k'_{\pm} = 1 \pm k^2$. Hence the determinants of $\rho_1^{\pm}$ are $\det\rho_1^{\pm} = N^4/N_{\pm}^2$. If the state of particle 1,2 is pure,

one can show $C_{1,2}=2\sqrt{\det\rho_1}$,[23] where ρ_1 is the reduced density matrix, and $C_{1,2}$ is the concurrence, which is a kind of measure of the entanglement in its own right. The relationship between concurrence and binary entropy is[19]

$$E(c)=-\frac{1+\sqrt{1-C^2}}{2}\log\frac{1+\sqrt{1-C^2}}{2} -\frac{1-\sqrt{1-C^2}}{2}\log\frac{1-\sqrt{1-C^2}}{2}. \quad (17)$$

Therefore we obtain

$$C_{12}^{\pm}=2(N^2/N_{\pm})=\frac{1-\mathrm{sech}2r}{1\pm\mathrm{sech}2r}. \quad (18)$$

From Eq.(18) we immediately see that the relative phase plays an important role in the entanglement. $|B\rangle_{1,2}^{-}$ is a maximally entangled state; its amount of entanglement is exactly one ebit and the entanglement is independent of the parameters involved. However, for the state $|B\rangle_{1,2}^{+}$ the corresponding concurrence is

$$C_{12}^{+}=\frac{1-\mathrm{sech}2r}{1+\mathrm{sech}2r}=\tanh^2 r, \quad (19)$$

which indicates that the state $|B\rangle_{12}^{+}$ is not a maximally entangled state except in the case of $r\to\infty$.

It is more interesting to discuss the entanglement of the ESS with the multipartite subsystem. We concentrate on bipartite entanglement, the entanglement of a pair of qubits, and N-way or N-party entanglement in multipartite systems. First we consider the bipartite entanglement of the states $|G\rangle_{12\cdots N}^{\pm}$. Splitting the entire system into two subsystems A and B, the subsystem A containing any $l(1\ll l\ll N-1)$ particles and the subsystem B the remaining $N-l$ particles. We let $C_{(l,N-l)}$ denote the concurrence between the two subsystems. Similarly, we can introduce two pairs of orthogonal bases $\{|\bar{0}\rangle_A,|\bar{1}\rangle_A\}$ and $\{|\bar{0}\rangle_B,|\bar{1}\rangle_B\}$, and let

$$|\bar{0}\rangle_A=|\xi\rangle_1|\xi\rangle_2\cdots|\xi\rangle_l, \quad (20)$$

$$|\bar{1}\rangle_A=(|-\xi\rangle_1|-\xi\rangle_2\cdots|-\xi\rangle_l -k^l|\xi\rangle_1|\xi\rangle_2\cdots|\xi\rangle_l)/N_A, \quad (21)$$

$$|\bar{0}\rangle_B=|\xi\rangle_{l+1}|\xi\rangle_{l+2}\cdots|\xi\rangle_N, \quad (22)$$

$$|\bar{1}\rangle_B=(|-\xi\rangle_{l+1}|-\xi\rangle_{l+2}\cdots|-\xi\rangle_N -k^{N-l}|\xi\rangle_{l+1}|\xi\rangle_{l+2}\cdots|\xi\rangle_N)/N_B, \quad (23)$$

where $N_A=\sqrt{1-k^{2l}}$ and $N_B=\sqrt{1-k^{2(N-l)}}$. Through a calculation completely similar to the one above, we obtain the concurrence between the two subsystems:

$$C_{(l,N-l)}^{\pm}=2(N_AN_B/N'_{\pm}) =\frac{\sqrt{[1-(\mathrm{sech}2r)^l][1-(\mathrm{sech}2r)^{N-l}]}}{1\pm(\mathrm{sech}2r)^{N/2}}. \quad (24)$$

From Eq.(24) we can see that $|G\rangle_{12\cdots N}^{-}$ is a maximally entangled state if and only if N is even and $l=N/2$.

Now we consider the bipartite reduced density matrix ρ_{12} of the states $|G\rangle_{12\cdots N}^{\pm}$. Because every qubit is symmetric in the states $|G\rangle_{12\cdots N}^{\pm}$, we can use ρ_{12} to express the reduced density matrix of an arbitrary pair of qubits. For convenience, we first make a local transformation $(-1)^{\hat{a}_2^{+}\hat{a}_2/2}$ on the state $|G\rangle_{12\cdots N}^{\pm}$. This local transformation does not change the amount of entanglement in the state. Then, by tracing out systems $3,4,\cdots,N$ in the transformed states, we obtain the reduced density matrix

$$\rho_{12}^{\pm}=\frac{1}{N'_{\pm}}[|\xi\rangle_1|-\xi\rangle_{21}\langle\xi|_2\langle-\xi| +|-\xi\rangle_1|\xi\rangle_{21}\langle-\xi|_2\langle\xi| \pm k^{N-2}(|-\xi\rangle_1|\xi\rangle_{21}\langle\xi|_2\langle-\xi| +|\xi\rangle_1|-\xi\rangle_{21}\langle-\xi|_2\langle\xi|)]. \quad (25)$$

Now $\rho_{12}^{\pm}$ describe mixed states, so their concurrence is[23,19]

$$C_{12}^{\pm}=\max\{0,\lambda_1-\lambda_2-\lambda_3-\lambda_4\}, \quad (26)$$

where λ_i 's are the square roots of the eigenvalues, in decreasing order, of the non-Hermitian matrix $\rho_{12}\tilde{\rho}_{12}$ and

$$\tilde{\rho}_{12}=(\sigma_y\otimes\sigma_y)\rho_{12}(\sigma_y\otimes\sigma_y). \quad (27)$$

Substituting Eq.(14) into Eq.(25), we obtain the density matrix

$$\rho_{12}^{\pm}=\frac{1}{N'_{\pm}}\begin{pmatrix}2k^2k''_{\pm} & kNk''_{\pm} & kNk''_{\pm} & 0\\ kNk''_{\pm} & N^2 & \pm N^2k^{N-2} & 0\\ kNk''_{\pm} & \pm N^2k^{N-2} & N^2 & 0\\ 0 & 0 & 0 & 0\end{pmatrix} \quad (28)$$

in the standard basis $\{|\bar{0}\bar{0}\rangle,|\bar{0}\bar{1}\rangle,|\bar{1}\bar{0}\rangle,|\bar{1}\bar{1}\rangle\}$, where $k''_{\pm}=1\pm k^{N-2}$. Thus, we can obtain $\lambda_1^{\pm}=\frac{1}{N'_{\pm}}(1-k^2)(1+k^{N-2})$, $\lambda_2^{\pm}=\frac{1}{N'_{\pm}}(1-k^2)(1-k^{N-2})$, $\lambda_3^{\pm}=0$, $\lambda_4^{\pm}=0$ and

$$C_{12}^{\pm} = \frac{2(1-k^2)}{N'_{\pm}} k^{N-2} = \frac{1-\mathrm{sech}2r}{1 \pm (\mathrm{sech}2r)^{N/2}} (\mathrm{sech}2r)^{(N-2)/2}. \quad (29)$$

If $N = 2$, $C_{12}^{\pm}$ reduces to $\frac{1-\mathrm{sech}2r}{1 \pm \mathrm{sech}2r}$, which is the concurrence of the states $|B\rangle_{12}^{\pm}$.

In order to investigate the entanglement of the multipartite quantum system, a potential measure of a type of entanglement of a pure state of N qubits, N-tangle, has been proposed.[20] For all even N and also N=3, it is a suitable measure of N-way entanglement. However, the N-tangle is undefined for odd $N > 3$. For a pure state with even N, the N-tangle is the square of the multiqubit concurrence,[20] namely

$$\tau_{12\cdots N} = C_{12\cdots N}^2 = |\langle \Psi | \sigma_y^{\otimes N} | \Psi^* \rangle|^2. \quad (30)$$

From Eqs.(13), (14) and (30), we can obtain the N-tangle of the states $|G\rangle_{1,2,\cdots,N}^{\pm}$

$$\tau_{12\cdots N}^{\pm} = \frac{(1-\mathrm{sech}2r)^N}{[1 \pm (\mathrm{sech}2r)^{N/2}]^2}. \quad (31)$$

By employing $\tau_{123} = C_{1(23)}^2 - C_{12}^2 - C_{13}^2$,[20,23] we can show that, for $N = 3$, Eq.(31) holds.

It is interesting to consider that two limits arise from $r \to \infty$ and $r \to 0$. In the asymptotic limit $r \to \infty$, the overlap $k = \sqrt{\mathrm{sech}2r} \to 0$, and two states $|\xi\rangle \to |\bar{0}\rangle$ and $|-\xi\rangle \to |\bar{1}\rangle$ approach orthogonal. In this case, the N-tangle approaches one, and the states $|G\rangle_{12\cdots N}^{\pm}$ approach the multipartite maximally entangled states

$$|G\rangle_N^{\pm} = \frac{1}{\sqrt{2}} (|\bar{0}\rangle_1 |\bar{0}\rangle_2 \cdots |\bar{0}\rangle_N \pm |\bar{1}\rangle_1 |\bar{1}\rangle_2 \cdots |\bar{1}\rangle_N). \quad (32)$$

For $r \to 0$, the overlap $k \to 1$. From Eqs.(24) and (29), we can get the concurrence of state $|G\rangle_{12\cdots N}^{-}$ as follows

$$C_{12} = \frac{2}{N}, C_{1(23\cdots N)} = \sqrt{4(N-1)}/N, \quad (33)$$

$$C_{12}^2 + C_{13}^2 + \cdots + C_{1N}^2 = C_{1(23\cdots N)}^2 \quad (34)$$

and

$$\tau_{12\cdots N} = C_{1(23\cdots N)}^2 - C_{12}^2 - C_{13}^2 - \cdots - C_{1N}^2 = 0. \quad (35)$$

In particular, for $r \to 0$ the state $|G\rangle_{12\cdots N}^{-}$ becomes the W state,[24] which can be proved easily. Expanding $|G\rangle_{12\cdots N}^{-}$ in the Fock state, we obtain

$$|G\rangle_{12\cdots N}^{-} = \frac{(\mathrm{sech}r)^{N/2}}{\sqrt{2[1-(\mathrm{sech}2r)^{N/2}]}} \times \sum_{n_1 n_2 \cdots n_N} \frac{\sqrt{(2n_1)!(2n_2)!\cdots(2n_N)!}}{n_1! n_2! \cdots n_N!} \times \left(-\frac{1}{2} e^{i\varphi} \tanh r \right)^{n_1+n_2+\cdots+n_N} \times [1-(-1)^{n_1+\cdots+n_N}] |2n_1, 2n_2, \cdots, 2n_N\rangle. \quad (36)$$

When r approaches zero, we can prove

$$\lim_{r\to 0} \frac{\tanh r}{\sqrt{1-(\mathrm{sech}2r)^{N/2}}} = \frac{1}{\sqrt{N}}. \quad (37)$$

Therefore we see that only the terms with $n_1 + n_2 + \cdots + n_N = 1$ survive, and the resultant state is

$$|W\rangle_N = \frac{1}{\sqrt{N}} (|200\cdots0\rangle + |020\cdots0\rangle + |002\cdots0\rangle + \cdots + |000\cdots2\rangle). \quad (38)$$

Unlike the general W state class, there are two photons in every mould.

4. Summary

To summarize, on the basis of the Jaynes–Cummings model with large detuning we have proposed a scheme to produce ESS with an arbitrary number of qubits. We calculated the bipartite entanglement of these states by concurrence, and multipartite entanglement of the multipartite ESS by N-tangle. Some maximally entangled states were pointed out. For the limit $r \to \infty$, the $|G\rangle_{12\cdots N}^{\pm}$ become the N-qubit Greenberger–Horne–Zeilinger states, $(|\bar{0}^{\otimes n}\rangle \pm |\bar{1}^{\otimes n}\rangle)/\sqrt{2}$; measuring any one of the qubits in the standard basis determines the value of all of the other qubits. However, if one of the qubits is traced out, the remaining qubits are unentangled. For these states, the N-tangle is 1. We have also proved that, in the limit $r \to 0$, the state $|G\rangle_{12\cdots N}^{-}$ reduces to a two-photon W state $(|200\cdots0\rangle + |020\cdots0\rangle + |002\cdots0\rangle + \cdots + |000\cdots2\rangle)/\sqrt{N}$, in which tracing out all but two of the qubits leaves the two remaining qubits partially entangled.

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