Vol 11 No 8, August 2002
1009-1963/2002/11(08)/0817-06

Chinese Physics

© 2002 Chin. Phys. Soc.
and IOP Publishing Ltd

The flavour liquid state and Hund's rule effects in spin systems with orbital degeneracy

Shi Da-Ning(施大宁)[a)b)] and Yang Zhi-Hong(杨志红)[a)]

[a)] *College of Science, Nanjing University of Aeronautics and Astronautics, Nanjing* 210016, *China*

[b)] *Province Key Laboratory of Thin Film Material, Suzhou University, Suzhou* 215006, *China*

(Received 11 March 2002; revised manuscript received 7 April 2002)

In order to understand the properties of the spin system with orbital degeneracy, we first generalize the linearized semiclassical spin wave method for the $SU(2)$ generators into the $SU(4)$ case, and then investigate the elementary excitations of the orbital-spin systems in the $SU(4)$ limit. The results show that, due to the reduction of the dimensionality of the excitations, the ordered state of the $SU(4)$ spin-orbital model is unstable. Secondly, we study the effects of the Hund interaction on the flavour liquid state of the system. Our mean-field results suggest that, for a small Hund interaction, the flavour liquid state is still stable against the generalized spin-density wave state, but with sufficient deviation from the $SU(4)$ limit, the long-range order may be attained in two-dimensional systems. Finally, the implications for the experimental observations on the material $LiNiO_2$ are discussed.

Keywords: $SU(4)$ symmetry, Hund's rule, flavour liquid state, ordered state
PACC: 7510, 7525, 7530D

The Hubbard model with a Hilbert space of only one atomic orbital per site is a popular model used to study strongly correlated electronic systems. When the correlation U is sufficiently large, the system undergoes a transition into the Mott insulator. In the large U limit, the model reduces to the familiar s=1/2 Heisenberg antiferromagnetic Hamiltonian (HAH). In spite of the success and popularity of the single orbital Hubbard model, it cannot, however, explain the magnetic behaviours of many Mott insulators, including the magnetic ordering patterns in V_2O_3 [1] and the spin liquid state in the more recently concerned material, $LiNiO_2$.[2] On the other hand, such behaviours can easily be understood if one allows for orbital degeneracy and enlarges the Hilbert space on each site.[3] Recently, direct evidence for orbital degrees of freedom and ordering in some magnetic systems has been obtained by form-factor analysis of x-ray diffraction.[4]

Castellani *et al*[5] derived the Hamiltonian describing such spin-1/2 systems with two-fold orbital degeneracy more than 20 years ago. This Hamiltonian has a rotation symmetry in $\boldsymbol{s}$-space, but in $\boldsymbol{\tau}$-space this symmetry is broken by Hund's term and the anisotropy in the orbital wavefunctions. In a previous publication,[3] together with our cooperators, we have investigated a simplified Hamiltonian, in which the Hund's term and the anisotropy in orbital wavefunctions have been neglected. Our results have shown that the full symmetry of the model is the $SU(4)$ symmetry, rather than just the spin-$SU(2)$ symmetry. In many two-dimensional (2D) lattices, such as the square and triangular lattices with nearest-neighbour (NN) coupling, the ground state of the $SU(4)$ antiferromagnetic (AF) Hamiltonian is a liquid of resonant plaquette $SU(4)$ singlet, as indicated by our variational calculation and mean-field theory.[3] Recent Monte Carlo simulations on a modified model also support this conclusion.[6] In real systems, however, the $SU(4)$ symmetry is broken by the Hund's interaction and the anisotropy of orbital wavefunctions, reducing the symmetry to just spin-$SU(2)$ symmetry. Therefore, the $SU(4)$ symmetry is at best an approximation for real systems.

Recently, there have been some arguments[7,8] on the reliability of our results.[3] It has been argued that the liquid ground state of the spin-orbital system in the $SU(4)$ limit is also based on a shaken mean-field theory in the fermion representation, similar to the previous studies for the 2D Hubbard model in high-T_c superconductors. This motivates us to re-study the ground state of the $SU(4)$ Hamiltonian by employing different techniques. Furthermore, we should investigate the stability of the flavour liquid state against the solid ground state in real systems with Hund's rule and the anisotropy of orbital wavefunctions. In this paper, starting from the $SU(4)$ Hamiltonian, we first

investigate the elementary excitations of the orbital-spin systems assuming an ordered ground state. The reduction of the dimensionality of excitations is found in the $SU(4)$ limit, which results in the instability of the assumed ordered state. Secondly, in the limit of isotropy of orbital wavefunctions, Hund's interaction is taken into account in the mean-field theory. We find that the system could have long-range order (LRO) as Hund's coupling is large enough. With a small Hund's interaction, the disordered liquid state is still stable against the ordered solid state even in the 2D unfrustrated square lattice. The implications of our results for the experiments on $LiNiO_2$ are presented.

The simplest AF quantum spin-1/2 system with two degenerate orbitals (τ-1/2) and rotational invariance in both $\boldsymbol{s}$ and $\boldsymbol{\tau}$ spaces is given by[3]

$$H_0 = J \sum_{<ij>} \left(2\boldsymbol{s}_i \cdot \boldsymbol{s}_j + \frac{1}{2}\right)\left(2\boldsymbol{\tau}_i \cdot \boldsymbol{\tau}_j + \frac{1}{2}\right) - 1, \quad (1)$$

where $J = \dfrac{4t^2}{U}$, and we set J=1. Apparently, Hamiltonian (1) has $SU(2) \times SU(2)$ symmetry, representing rotational invariance in both spin and orbital spaces and also interchange symmetry between spins and orbitals. We have shown that the full symmetry of this is actually the higher symmetry group $SU(4)$. The Hamiltonian (1) acts on a Hilbert space of four basis states at each site. Choosing these as $|\mu\rangle = |s^z, \tau^z\rangle$, we label these:

$$\begin{aligned} |1\rangle &= |1/2, 1/2\rangle, \\ |2\rangle &= |-1/2, 1/2\rangle, \\ |3\rangle &= |1/2, -1/2\rangle, \\ |4\rangle &= |-1/2, -1/2\rangle. \end{aligned} \quad (2)$$

These basis states form a fundamental representation of $SU(4)$. To obtain an insight into the physical properties, we have considered a system with a few sites and found that the ground state of Hamiltonian (1) is a unique $SU(4)$ singlet.[3] It is a singlet of spin, orbital, and the orbital-spin-crossing operator $U^{\alpha,\beta} = 4s^\alpha \tau^\beta$, a generalization of the $SU(2)$ singlet of just spin-1/2 systems. To study the nature of the ordered systems V_2O_3 and $LaMnO_3$, Castellani *et al*,[4] Ishihara *et al*,[9] and other research groups,[8] appeal to the standard mean-field decoupling $(\boldsymbol{s}_i \cdot \boldsymbol{s}_j)(\boldsymbol{\tau}_i \cdot \boldsymbol{\tau}_j) \to < \boldsymbol{s}_i \cdot \boldsymbol{s}_j > (\boldsymbol{\tau}_i \cdot \boldsymbol{\tau}_j) + (\boldsymbol{s}_i \cdot \boldsymbol{s}_j) < \boldsymbol{\tau}_i \cdot \boldsymbol{\tau}_j > - < \boldsymbol{s}_i \cdot \boldsymbol{s}_j >< \boldsymbol{\tau}_i \cdot \boldsymbol{\tau}_j >$. They investigated the elementary excitations of such orbital-spin systems, assuming that the ground state is an ordered state. However, such a decoupling would not be appropriate because, if it were a valid approximation, the low-lying excitations would consist of only two branches corresponding to spin and orbital excitations, respectively. The third spin-orbital excitation has been lost in such a mean-field decoupling theory. This contradicts the three low-lying models of the Bethe ansatz, which control low-temperature physics, as pointed out by Sutherland[10] and our research group.[11] Besides, and more importantly, such decoupling is in clear contradiction with the correlations in the $SU(4)$ limit $< s_i^\alpha s_j^\alpha > = < \tau_i^\alpha \tau_j^\alpha > = < 4 s_i^\alpha s_j^\alpha \tau_i^\alpha \tau_j^\alpha >$.[3] So, there is a manifest breakdown of such a standard mean-field decoupling when the spin and orbital degrees of freedom play a symmetric role.

What, then, is the nature of the excitations of spin systems with orbital degeneracy? Actually, this system should also be considered in $SU(4)$ space, not only in $SU(2) \times SU(2)$ space. We start with Hamiltonian (1) with $SU(4)$ generators and employ a simple approach to investigate the problem. As is known, the simple way to study this is the linearized semiclassical spin wave approach,[12] which does not involve any other approximate techniques. In the following, we generalize the method for $SU(2)$ generators (s^+, s^-, s^z) to the $SU(4)$ case. All physical excitations of the spin system with orbital degeneracy can be included in the theoretical approach. To do this, we rewrite Hamiltonian (1) in terms of $SU(4)$ generators S_m^n

$$H_0 = \sum_{<ij>} S_m^n(i) S_n^m(j), \quad (3)$$

with m, n=1,2,3,4. The conventional $SU(4)$ generators S_m^n act on the basis state $|l\rangle$ according to $S_m^n|l\rangle = \delta_{n,l}|m\rangle$, and satisfy $\sum_m S_m^m = 1$ and $(S_m^n)^+ = S_n^m$. The commutation relation of S_m^n is given by

$$[S_m^n, S_k^l] = \delta_{n,k} S_m^l - \delta_{m,l} S_k^n. \quad (4)$$

Now we consider an ordered state with four sublattices, where the sublattice A_μ is predominant in the flavour μ (μ=1,2,3,4). We assume $S_m^n(j_\mu) \approx \delta_{m,n,\mu}$ in the linearized approximation as below, where j_μ is the sublattice index of the site j. By employing the commutation relation (4), similar to the $SU(2)$ case,[12] we have

$$\begin{aligned} \dot{S}_m^n(j) =& i[H_0, S_m^n(j)] \\ =& 2i \sum_l \{S_\mu^n(j) S_m^\mu(l) - S_m^\mu(j) S_\mu^n(l)\}, \end{aligned} \quad (5)$$

where the sum over l runs all NN pairs of the site j_μ. After the linear approximation, Eq.(5) becomes

$$\dot{S}_m^n(j_\mu) = 2i \sum_{\substack{\nu \neq \mu \\ l}} \{\delta_{n,\mu} S_m^\mu(l) + \delta_{m,\nu} S_\nu^n(j_\mu) - \delta_{m,\mu} S_\mu^n(l) - \delta_{n,\nu} S_m^\nu(j_\mu)\}. \quad (6)$$

Letting $S_m^n(j_\mu) = C_\mu^{mn} e^{-i\omega t} e^{ik\cdot j_\mu}$ for $m \neq n$, and substituting it into Eq.(6), we obtain a group of linearized equations for the coefficients $C_\mu^{mn} (m \neq n = 1,2,3,4)$

$$\omega C_\mu^{mn} + 2 \sum_{l,\nu \neq \mu} \{\delta_{m,\nu} C_\mu^{\nu n} - \delta_{n\nu} C_\mu^{m\nu} + (\delta_{n,\mu} C_\nu^{m\mu} - \delta_{m\mu} C_\nu^{\mu n}) e^{ik\cdot(l_\nu - j_\mu)}\} = 0. \quad (7)$$

Considering two cases of $m = \mu, n \neq \mu$ and $m \neq \mu$, $n = \mu$, we find that in the $SU(4)$ limit the equations for each sublattice A_μ=1,2,3 and 4 are decoupled. The two self-closed equations are obtained as

$$(-\omega + 2N_c) C_\mu^{\mu n} + 2N_c \gamma_{[j][l]}(k) C_n^{\mu n} = 0,$$
$$(-\omega - 2N_c) C_{\nu n}^{\mu n} - 2N_c \gamma_{[j][l]}(k) C_\mu^{\mu n} = 0, \quad (8)$$

where N_c is the number of connecting bonds between j_μ and its NN sites of sublattice l_ν and

$$\gamma_{[j][l]}(\boldsymbol{k}) = \frac{1}{N_c} \sum_l e^{ik\cdot(l_\nu - j_\mu)}. \quad (9)$$

Therefore, we have

$$\omega(k) = 2N_c \sqrt{1 - \gamma_{[j][l]}^2(k)}. \quad (10)$$

In addition to this model of excitations, from Eq.(7), we consider the case of $\mu \neq [j], \nu \neq [j]$ and can easily obtain the local model: ω=0. In the following, we apply this theoretical result to different lattices shown in Fig.1 and discuss some related experiments.

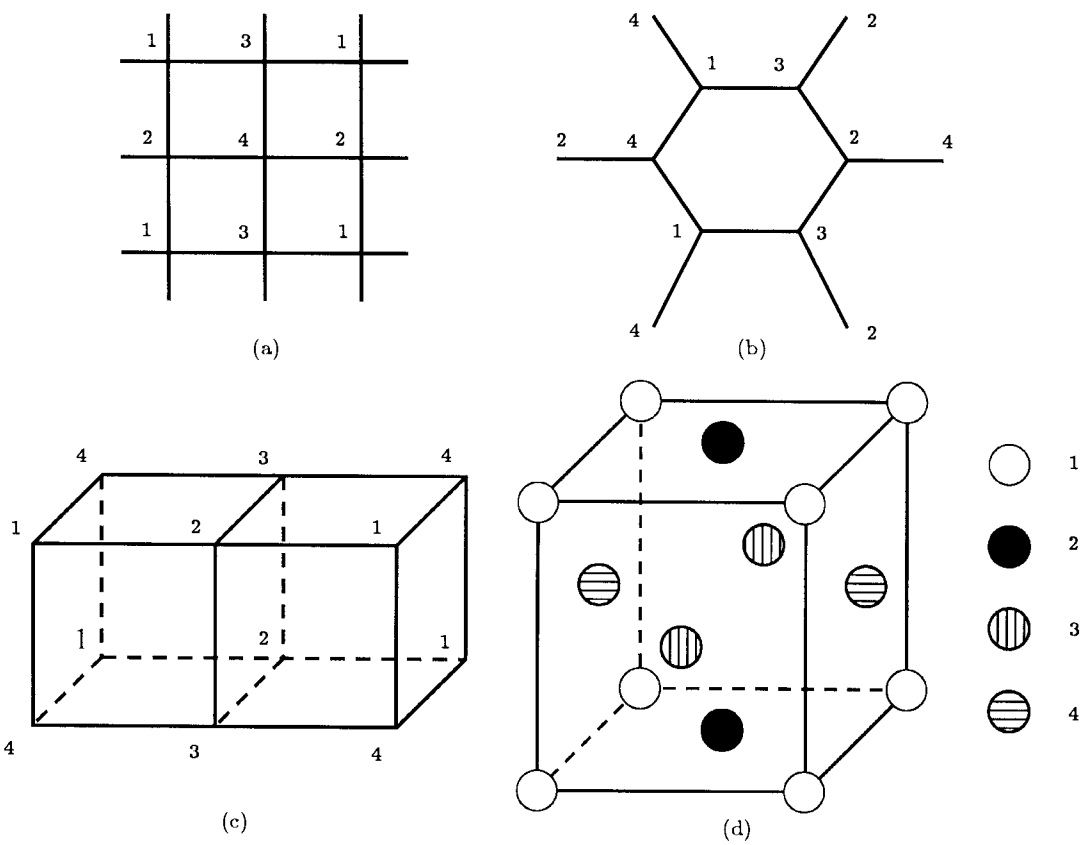


Fig.1. Four-sublattice classical ground states considered in the text: (a) square lattice; (b) honeycomb lattice; (c) cubic lattice; (d) fcc lattice.

For the AF $SU(4)$ system on the 2D square lattice, a four-sublattice classical ground state can be achieved by having any two sites connected by a bond with different flavours; one of the patterns is shown in Fig.1(a). The spin (s), orbital (τ), and spin-orbital ($s\tau$) wave dispersions are easily given by $\omega_s(\boldsymbol{k}) = 2\sqrt{1 - \cos^2 k_y}$, $\omega_\tau(\boldsymbol{k}) = 2\sqrt{1 - \cos^2 k_x}$ and $\omega_{s\tau}(\boldsymbol{k}) = 0$, respectively. We note that both the spin and the orbital excitations seem one-dimensional (1D) in spite of the underlying translationally invariant 2D ordering pattern. This means that the disordering effect due to quantum fluctuations of these 1D-like excitations should act just like those in a 1D quantum spin chain and destroy the LRO. The result obtained

here is consistent with our previous fermion mean-field result that the ground state of the AF $SU(4)$ model is a flavour liquid on many 2D lattices.[3] In other words, we can obtain the same result that the AF solid state is unstable against the flavour liquid state by employing different techniques in the $SU(4)$ limit. Besides, because of no next-nearest-neighbour coupling J' between the states 1 and 3 or 2 and 4, the flipping state from 1 to 4 on a site can be created with no cost in energy, thus $\omega_{s\tau}(\boldsymbol{k}) = 0$. As for the AF $SU(4)$ system on the honeycomb lattice, the ordering pattern in Fig.1(b) results in some modifications from that in the square lattice in the dispersion of excitations. Flavours 1 and 4 (or 2 and 3) are connected as zigzag vertical chains, and thus we have the spin-orbital wave excitation with 1D-like dispersion $\omega_{s\tau}(\boldsymbol{k}) = \sin(\sqrt{3}|k_y|/2)$. We must note that this model cannot be obtained in the standard decoupling mean-field theory.[5] Flavours 1 and 2 (or 3 and 4) are unconnected, and the spin waves are localized on a site having zero excitation energy, $\omega_s(\boldsymbol{k}) = 0$. Similarly, we also have $\omega_\tau(\boldsymbol{k}) = 0$.

In three-dimensional (3D) systems, we first consider the AF $SU(4)$ Hamiltonian in a cubic lattice in Fig.1(c). In the present case, the spin, orbital, and spin-orbital wave dispersions are given by $\omega_s(\boldsymbol{k}) = 2\sqrt{1-\cos^2 k_x}$, $\omega_\tau(\boldsymbol{k}) = 0$, and $\omega_{s\tau}(\boldsymbol{k}) = 4\sqrt{1-\frac{1}{4}(\cos k_y + \cos k_z)^2}$ respectively. It is interesting to note that, for the same reasons mentioned above in a square lattice, the ground state of the spin-orbital system may be also disordered even for a 3D cubic lattice. This result can be qualitatively used to explain the recent experiments on $LaTiO_3$, the structure of which is a 3D cubic lattice and the ground state of which shows orbital liquid.[13] To fully understand the properties of this material, of course, we should consider more physical reasons, such as Hund's rule and orbital orientations. Finally, we apply our discussions to the fcc lattice shown in Fig.1(d) and obtain three flavour wave dispersions in the AF ordered state:

$$\omega_s(\boldsymbol{k}) = 4\sqrt{1-\frac{1}{4}\left(\cos\frac{k_y+k_z}{\sqrt{2}}+\cos\frac{k_y-k_z}{\sqrt{2}}\right)^2},$$

$$\omega_\tau(\boldsymbol{k}) = 4\sqrt{1-\frac{1}{4}\left(\cos\frac{k_z+k_x}{\sqrt{2}}+\cos\frac{k_z-k_x}{\sqrt{2}}\right)^2},$$

and

$$\omega_{s\tau}(\boldsymbol{k}) = 4\sqrt{1-\frac{1}{4}\left(\cos\frac{k_x+k_y}{\sqrt{2}}+\cos\frac{k_x-k_y}{\sqrt{2}}\right)^2}.$$

Therefore, the reduction of the dimensionality of the excitations is not only found in the 2D systems but also in the 3D systems, which gives rise to the instability of the assumed ordered states.

A real system has anisotropic terms due to Hund's rule and the anisotropy of orbital wavefunctions. We have concluded that in the $SU(4)$ limit the ground state of the spin-orbital system is a flavour liquid on many 2D lattices and even in 3D lattices. In the following, we study the stability of the liquid state against the solid state if the anisotropic terms are taken into account. The full Hamiltonian including Hund's rule and the anisotropy of orbital wavefunctions is very complicated, as indicated in Ref.[5]. To simplify the calculation in the fermion mean-field theory, we neglect the anisotropy in orbital wavefunctions first, and only take Hund's interaction as a phenomenological form given in Ref.[14]

$$H' = -\frac{J_H}{8}\sum_{\alpha\beta}\boldsymbol{s}_{i\alpha}\cdot\boldsymbol{s}_{i\beta}(1-\delta_{\alpha\beta}), \tag{11}$$

where α and β are orbital indices. To illustrate the ideas mentioned above, we consider the full Hamiltonian on a square lattice, using fermion mean-field theory. The full Hamiltonian can be rewritten in the fermion representation

$$\begin{aligned} H =& H_0 + H' \\ =& -\sum_{<ij>}\sum_{\mu\nu} c^+_{i\mu}c_{j\mu}c^+_{j\nu}c_{i\nu} \\ & -\frac{J_H}{8}\sum_i\sum_{<\mu\nu>} c^+_{i\mu}c_{i\nu}c^+_{i\nu}c_{i\mu}, \end{aligned} \tag{12}$$

where μ,ν=1,2,3,4, and $<\mu\nu>$ means that as μ=1 or 2, ν=3 or 4, and vice versa. We define four sublattices 1,2,3 and 4 on the square lattice shown in Fig.2. Two parameters can be defined in the mean-field theory. Firstly, the uniform and real mean-field bond amplitude $\chi = <\chi_{ij}> = <\sum_\mu c^+_{i\mu}c_{j\mu}>$, which was previously introduced by Affleck and Marston in the t–J model[15] to describe a flavour liquid state. Secondly, the generalized spin-density wave (SDW) order parameter M, defined such that the mean occupation number for the flavour μ at site $i \subset A_\mu$, $<c^+_{i\mu}c_{i\mu}> = (1-M)/4 + M\delta_{\mu\nu}$, where A_ν=1,2,3 and 4 as indicated in Fig.2. For the spin-1/2 HAH with a single orbital per site, the uniform mean-field state was found to be unstable against the AF spin state.[16] In other words, the ground state of the spin-1/2 HAH

is the ordered state.[17] However, in spin systems with orbital degeneracy, we have pointed out that the additional orbital degrees of freedom provide a new possibility for a flavour liquid state in the $SU(4)$ limit. In other words, the ground state of H_0 is disordered.

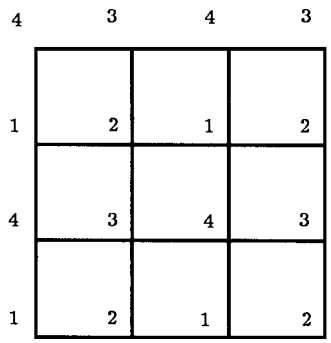


Fig.2. Illustration of the four-sublattice in the generalized SDW state on a square lattice.

To understand the effect of Hund's rule on the ground state of a real system, we decouple the full Hamiltonian (12) as follows:

$$\begin{aligned} H_{MF} =& -\sum_{<ij>}\left[\chi\sum_{\mu}c^{+}_{i\mu}c_{j\mu}+h.c.)-\chi^2\right] \\ &+\sum_{<ij>,\mu}[<n_{i\mu}>c^{+}_{j\mu}c_{j\mu}+<n_{j\mu}>c^{+}_{i\mu}c_{i\mu} \\ &-<n_{i\mu}><n_{j\mu}>] \\ &-\frac{J_H}{8}\sum_{i,<\mu\nu>}[<n_{i\mu}>c^{+}_{i\nu}c_{i\nu} \\ &+<n_{i\mu}>c^{+}_{i\mu}c_{i\mu}-<n_{i\mu}><n_{i\nu}>]. \end{aligned} \tag{13}$$

In the mean-field theory, the Hamiltonian (13) in the momentum space can be written as

$$\begin{aligned} H_{MF} =& \sum_{k,\mu}\eta^{+}_{k\mu}\tilde{M}_k\eta_{k\mu}+2N_s\chi^2-N_s(1-M^2)/2 \\ &+N_s(1+M^2)J_H/8, \end{aligned} \tag{14}$$

where $\eta^{+}_{k\mu}=(c^{+}_{k+Q_1\mu},\ c^{+}_{k+Q_2\mu},\ c^{+}_{k+Q_3\mu},\ c^{+}_{k+Q_4\mu})$, and N_s is the number of sites. Q_1=(0,0), Q_2=(π,0), Q_3=(0,π) and Q_4=(π, π,) are the generalized SDW wave vectors corresponding to the four sublattices 1, 2, 3 and 4. We have

$$\tilde{M}_k=\begin{bmatrix} 1-2\chi(c_x+c_y) & 0 & -M & J_H \\ 0 & 1-2\chi(c_x+c_y) & J_H & -M \\ -M & J_H & 1+2\chi(c_x+c_y) & 0 \\ J_H & -M & 0 & 1+2\chi(c_x+c_y) \end{bmatrix}, \tag{15}$$

with $c_x=\cos k_x$ and $c_y=\cos k_y$. The quasi-particle energies can be obtained by diagonalizing the matrix (15):

$$\begin{aligned} \xi^2_k(\pm) =& J^2_H+M^2+4\chi^2(c^2_x+c^2_y) \\ &\pm 2\sqrt{J^2_HM^2+4J^2_H\chi^2c^2_x+16\chi^2c^2_xc^2_y}. \end{aligned} \tag{16}$$

So, the total energy of the system is

$$\begin{aligned} \varepsilon =& -4{\sum_{k}}''[J^2_H+M^2+4\chi^2(c^2_x+c^2_y) \\ &\pm 2\sqrt{J^2_HM^2+4J^2_H\chi^2c^2_x+16\chi^2c^2_xc^2_y}]^2 \\ &+2N_s\chi^2-N_s(1-M^2)/2 \\ &+N_s(1+M^2)J_H/8. \end{aligned} \tag{17}$$

The important difference between the present model and the t–J model of Affleck and Marston is that here one fermion per site implies that each flavour of fermions is 1/4 filled, while in their study each flavour of fermions is close to 1/2 filled, the case relevant to the cuprates. Therefore, the restriction of k-summation over half of the Brillouin zone must be satisfied: $1/N_s\sum''_k f(\boldsymbol{k})$=1/4, where $f(\boldsymbol{k})$ is the Fermi function. For the ground state, minimizing the total energy (17) with respect to the parameters M and χ, we have the self-consistent equations

$$\begin{aligned} \chi =& \frac{4}{N_s}{\sum_{k}}''\frac{\chi[c^2_x+c^2_y]+[J^2_H\chi c^2_c+8\chi^3c^2_xc^2_y]/g(k)}{|\xi_k(+)|} \\ &\times\theta(c_x+c_y-E_F), \end{aligned} \tag{18}$$

$$\begin{aligned} & M+\frac{J_HM}{4} \\ =& \frac{4}{N_s}{\sum_{k}}''\frac{M+J^2_HM/g(k)}{|\xi_k(+)|}\theta(c_x+c_y-E_F), \end{aligned} \tag{19}$$

where $g(k)=2\sqrt{J^2_HM^2+4J^2_H\chi^2c^2_x+16\chi^2c^2_xc^2_y}$. In the $SU(4)$ limit ($J_H=0$), we find that the condition for the $M\neq 0$ solution is

$$\begin{aligned} F =& \frac{1}{N_s}{\sum_{k}}''\frac{1}{\cos k_x+\cos k_y} \\ &\times\theta(\cos k_x+\cos k_y-E_F)>\chi/2, \end{aligned} \tag{20}$$

$$\chi = \frac{2}{N_s}\sum_{k}{}''(\cos k_x + \cos k_y) \times \theta(\cos k_x + \cos k_y - E_F), \quad (21)$$

and E_F is determined by $1/N_s \sum_k'' \theta(\cos k_x + \cos k_y - E_F)=1/4$. The numerical results show that F=0.2086 and χ=0.6534. Therefore, it is clear that the uniform bond state is stable against the generalized SDW order state. This means that the ground state of the spin system with orbital degeneracy in the $SU(4)$ limit is disordered. With Hund's rule and in the case of T=0, we have consistently solved Eqs.(20) and (21), and obtained that, as J_H <0.24, there is no LRO in the ground state of the system. This indicates that the system is still disordered as Hund's rule is smaller than the critical value. As Hund's interaction J_H >0.24, the generalized SDW order parameter M appears and the uniform bond state order parameter χ decreases with J_H. The numerical results are shown in Fig.3. Therefore, we conclude that, for a small Hund's interaction, the flavour liquid state is still stable against the solid state, But, with sufficiently large deviation from the $SU(4)$ limit, the LRO may be attained in 2D systems.

Finally, we compare our results with the experimental results for $LiNiO_2$. The experiments[2] reveal that the compound shows ferromagnetic spin–spin correlation at a high temperature. But the measurements of magnetic susceptibility, specific heat, muon spin rotation (μSR) and nuclear magnetic resonance (NMR) at low temperatures show no long-range ordering of spins. Also, μSR shows that Ni spins remain fluctuating even at 20mK.[2] The NMR and specific heat measurements have ruled out the possibility of the spin glass phase. This evidence strongly indicates a spin liquid ground state in $LiNiO_2$. Furthermore, we know that there exists a moderate Hund's interaction in this material, which leads to the deviation from the $SU(4)$ limit. It is clear that our studies may qualitatively serve as a simple theory to illustrate the role of the orbital degeneracy responsible for the experimentally observed spin liquid properties in $LiNiO_2$.

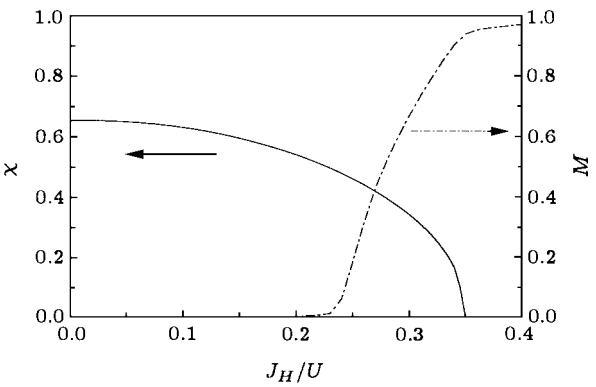


Fig.3. Variation of the mean-field parameters χ and M with Hund's interaction.

Acknowledgments

SHI thanks Professors Y.Q. Li and F.C. Zhang for useful discussions.

References

[1] Bao W, Broholm C, Aeppli G, Carter S A, Dai P, Rosenbaum T F, Honig J M, Metcalf P and Trevio S F 1998 *Phys. Rev.* B **58** 12727 and references therein

[2] Reynaud F, Mertz D, Celestini F, Debierre J M, Ghorayeb A M, Simon P, Stepanov A, Voiron J and Delmas C 2001 *Phys. Rev. Lett.* **86** 3638
Kitaoka Y, Kobayashi T, Koda A, Wakabayashi H, Niino Y, Yamakage H, Taguchi S, Amaya K, Yamamura K, Takano M, Hirano A and Kanno R 1998 *J. Phys. Soc. Jpn.* **67** 3703

[3] Li Y Q, Ma M, Shi D N and Zhang F C 1998 *Phys. Rev. Lett.* **81** 3527
Joshi A, Ma M, Mila F, Shi D N and Zhang F C 1999 *Phys. Rev.* B **60** 6584

[4] Murakami Y, Kawada H, Kawata H, Tanaka M, Arima T, Moritomo Y and Tokura Y 1998 *Phys. Rev. Lett.* **80** 1932

[5] Castellani C, Natoli C R and Ranninger J 1978 *Phys. Rev.* B **18** 4945

[6] Bossche M V, Zhang F C and Mila F 2000 *Eur. Phys. J.* B **17** 367
Frischmuth B, Mila F and Troyer M 1999 *Phys. Rev. Lett.* **82** 835
Santoro G, Sorella S, Guidoni L, Parola A and Tosatti E 1999 *Phys. Rev. Lett.* **83** 3065

[7] Auerbach A, private communication

[8] For example, Zasinas E, Sushkov O P and Oitmaa J 2001 *Phys. Rev.* B **64** 184431-1

[9] Ishihara S, Inoue J and Maekawa S 1997 *Phys. Rev.* B **55** 8208

[10] Sutherland B 1975 *Phys. Rev.* B **12** 3795

[11] Li Y Q, Ma M, Shi D N and Zhang F C 1999 *Phys. Rev.* B **60** 12781

[12] Feynman R P *Statistical Mechanics* (London: Benjamin Cummings) p206

[13] Keimer B, Casa D, Ivanov A, Lynn J W, Zimmermann M V, Hill J P, Gibbs D, Taguchi Y and Tokura Y 2000 *Phys. Rev. Lett.* **85** 3946
Khaliullin G and Maekawa S 2000 *Phys. Rev. Lett.* **85** 3950

[14] Brink J V, Stekelenburg W, Khomskii D I, Sawatzky G A and Kugel K I 1998 *Phys. Rev.* B **58** 10276

[15] Affleck I and Marston J B 1988 *Phys. Rev.* B **37** 3774

[16] For example, Yokoyama H and Shiba H 1987 *J. Phys. Soc. Jpn.* **56** 3570

[17] Dong Z H and Feng S P 1998 *Chin. Phys.* **7** 348