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The soliton properties of dipole domains in superlattices*

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The formation and propagation of dipole domains in superlattices are studied both by the modified discrete drift model and by the nonlinear Schrödinger equation. The spatiotemporal distribution of the electric field and electron density are presented. The numerical results are compared with the soliton solutions of the nonlinear Schrödinger equation and analysed. It is shown that the numerical solutions agree with the soliton solutions of the nonlinear Schrödinger equation. The dipole electric-field domains in semiconductor superlattices have the properties of solitons.

Keywords: superlattice, dipole domain, soliton, nonlinear Schrödinger equation
PACC: 7220H, 7230, 0560, 0365

1. Introduction

In the study of electronic transport in semiconductor superlattices (SLs), the properties of the electric-field domains and transportation phenomena have been extensively studied[1−5] both experimentally and theoretically. Recently, a discrete drift–diffusion model of charge transportation in weakly-coupled SLs has been established and used to analyse the dipole electric-field domains successfully.[3] Based on the microscopic sequential tunnelling model, the charge monopoles, the charge dipoles, even the coexistence of both the oscillation types are possible under various contact doping.[6] Experimentally, a self-sustained high-frequency current oscillation attributed to propagating dipole domains is reported to be due to a miniband transportation mechanism.[7] We use the nonlinear Schrödinger equation (NLSE) to describe the electric-field domains resulting from the negative differential conductivity and to obtain soliton solutions.[8,9]

In this paper, the dipole domains in SLs are studied both numerically and analytically. Our numerical work is based upon the discrete drift model of charge transportation in SLs. We present a detailed numerical analysis of the dipole electric-field domains in doped weakly-coupled GaAs/AlAs SLs. Then we investigate the properties of the dipole domains making use of the NLSE. The numerical simulation results are analysed and compared with the analytical results of the NLSE. The results demonstrate that the dipole electric-field domains in SLs have the behaviours of solitons.

2. Physical model and numerical simulation

According to the discrete drift model[10] of transportation in SLs, each quantum well reaches a local equilibrium through sequential tunnelling processes in weakly-coupled SLs. The vertical transport properties in the growth direction can be described by the following equations:

$$\varepsilon_i - \varepsilon_{i-1} = \frac{e}{\epsilon}(n_i - N_D^w) \text{ (Poisson equation).} \quad (1)$$

$$\epsilon \frac{\mathrm{d}\varepsilon_i}{\mathrm{d}\tau} + \frac{e n_i \nu(\varepsilon_i)}{d} = J \text{ (Ampere equation).} \quad (2)$$

In these relations, ϵ, e and d are the average permittivity, the electron charge and the superlattice period, respectively. ε_i is the electric field, n_i is the two-dimensional (2D) (areal) electron number density (to be determined), with i=1,2,$\cdots$,N denoting the quantum well index. N_D^w is the 2D intentional doping at the wells.

To study the equations, it is convenient to render them dimensionless, using characteristic physical quantities. As the unit of the electric field $E_i = \varepsilon_i/\varepsilon_0$, we adopt $\varepsilon_0 = \Delta/ed$, with Δ being the energy separation between the first and the second electron subbands. $\nu(E_i) = \nu(\varepsilon_i)/\nu(\varepsilon_0)$ is the normalized and dimensionless effective electron velocity.

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From Eqs.(1) and (2), we obtain a set of N equations for the electric-field profiles

$$\frac{\mathrm{d}E_i}{\mathrm{d}t} = \sum_{j=1}^{N} \nu(E_j)(E_j - E_{j-1} + \mu) - N\nu(E_i)(E_i - E_{i-1} + \mu), \quad (3)$$

where $t = \tau\nu(\varepsilon_0)/Nd$ is the dimensionless tunnelling time. The characteristic charge density $n_0 = \epsilon\varepsilon_0/e$ is used to normalize the doping $\mu = N_D^w/n_0$.

The initial conditions are $E_i(0) = V_{\mathrm{dc}}/Nd\varepsilon_0$, with V_{dc} being the applied dc voltage bias and V_{dc}/Nd the average electric field in the superlattice sample. The global charge conservation[11]

$$\sigma_e + \sum_{i=1}^{N}(n_i - eN_D^w) = eN_d\left(\delta_2 + \frac{1}{2}\delta_3\right) \quad (4)$$

is adopted as the boundary condition in numerical simulations. Here N_d is the doping density in the contact layers (see Ref.[11] for a detailed discussion of other parameters).

We consider a GaAs/AlAs SL with d=16nm, N=80, μ=0.09, $E_i(0)$=1.53, $N_d = 1.23 \times 10^{24}\mathrm{m}^{-3}$. Equation (3) is solved numerically by the fourth-order Runge–Kutta method to obtain electric-field distributions through the SL region. From the electric-field profiles, the total current density and the electron density can be calculated.

The spatiotemporal distributions of the electric field are seen from Fig.1. The calculated electron density and time-dependent current oscillations are shown in Figs.2 and 3. The spatial structures of the electron density and the electric-field distributions show that the oscillation is due to the propagating dipole domains. The charge dipoles move with maintaining both the shapes and the amplitudes.

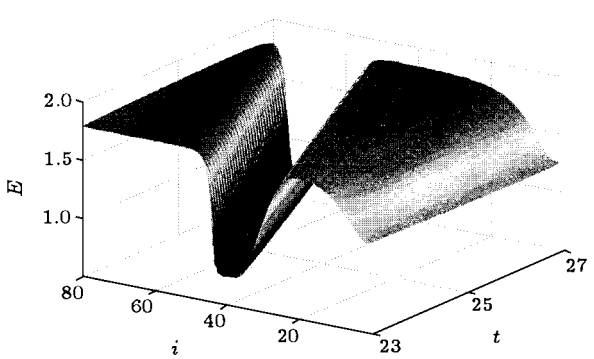


Fig.1. Spatiotemporal distributions of the electric field.

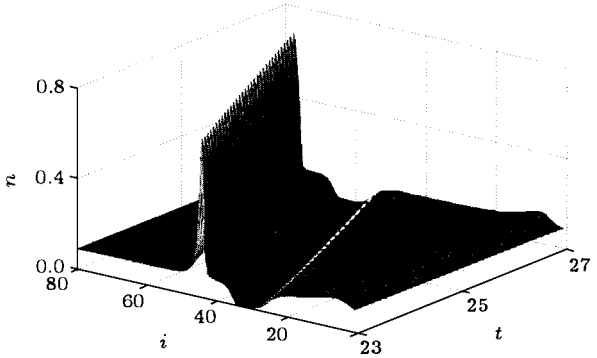


Fig.2. Spatiotemporal distributions of electron density.

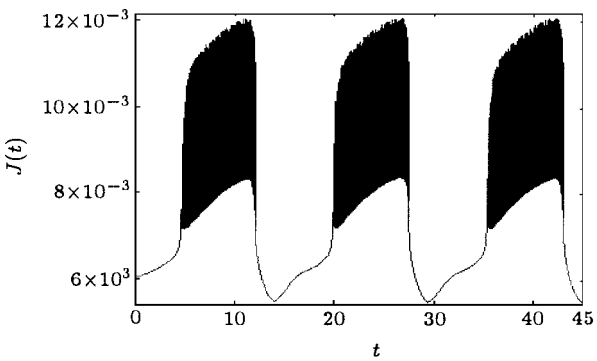


Fig.3. Temporal evolution of the current density J.

3. Nonlinear Schrödinger equation and its soliton solution

In terms of the negative differential conductivity, the formation and transportation of dipole domains in SLs can be described phenomenologically by the self-trapped potential[8,12] $\kappa|\Psi(x,t)|^2$, where $\Psi(x,t)$ is the electron wavefunction, $|\Psi(x,t)|^2$ is the electron density, and κ is a real constant.

Then the Hamiltonian is

$$\hat{H} = -\frac{\hbar^2}{2m}\nabla^2 - V(x) + eEx - \kappa|\Psi(x,t)|^2 = \hat{H}_0 - \kappa|\Psi(x,t)|^2, \quad (5)$$

where $V(x+d) = V(x)$ is the periodic SL potential with a period d, e is the electronic charge, and E is the strength of the electric field.

The corresponding Schrödinger equation is

$$i\frac{\partial}{\partial t}\Psi(x,t) - \hat{H}_0\Psi(x,t) + \kappa|\Psi(x,t)|^2\Psi(x,t) = 0. \quad (6)$$

Its formal solution can be written as $\Psi(x,t) = u(x,t)\phi(x,t)$, where $\phi(x,t)$ is the Bloch function, which satisfies the Schrödinger equation

$$i\frac{\partial}{\partial t}\phi(x,t) = \hat{H}_0\phi(x,t). \quad (7)$$

$u(x,t)$ is the envelope of the wavefunction $\phi(x,t)$.

The tight-binding approach[8] and the multiple scales method[13] show that the envelope function $u(x,t)$ is governed by the NLSE

$$i\frac{\partial u(x,t)}{\partial t} + \frac{\partial^2 u(x,t)}{\partial x^2} + b|u|^2 u(x,t) = 0, \qquad (8)$$

where $b = \kappa D_0$ is real, and D_0 is the average electron density.

The NLSE has a soliton solution

$$u(x,t) = \sqrt{\frac{-2}{b}}\,2A\,\mathrm{sech}[2A(x-4\lambda t)] \times \exp[-2i\lambda x + 4i(\lambda^2 - A^2)t]. \qquad (9)$$

Here, A is the soliton amplitude, and -4λ is its velocity. A and λ are determined by the initial conditions.

The soliton solutions imply that solitons are observed as envelopes of the Bloch functions. The envelope functions correspond to the electric-field domains in SLs. So the dipole electric-field domains in semiconductor SLs are characterized by solitons.

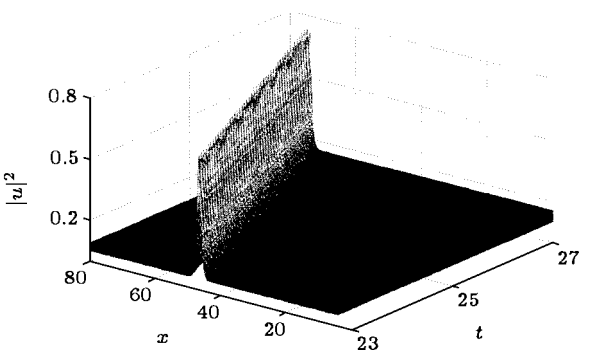


Fig.4. Soliton solutions of the envelope functions.

The soliton solutions (9) are shown in Fig.4, which correspond to the electron density $|u(x,t)|^2$. The dimensionless parameters used in our calculations are as follows: b=–22, $A = 9.5 \times 10^8$, $-4\lambda = 183.5$.

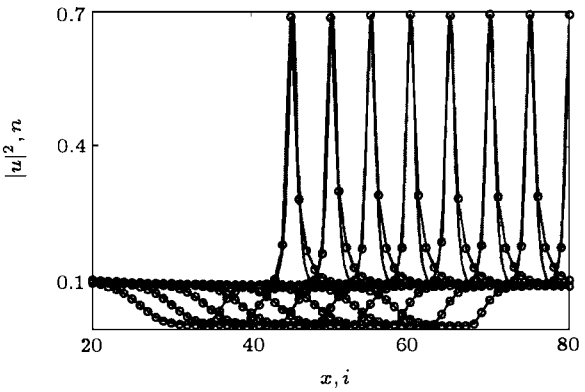


Fig.5. Comparison of the numerical solutions (circled line) with the analytical soliton solutions (solid line) at different times.

4. Analyses and conclusions

The electric field, the electron density and the current self-oscillations of the dipole electric-field domains have been studied both numerically, with the self-consistent discrete drift model, and analytically, by means of the NLSE. The numerical results show that the dipole electric-field domains exhibit soliton-like behaviours, whose electric field and electron density move with their shapes and amplitudes remaining unchanged. The spatiotemporal properties of the numerical and analytical results are in agreement with each other, as shown in Fig.5, where the circled lines indicate the numerical solutions, and the solid lines indicate the analytical soliton solutions. The numerical solutions and the analytical results demonstrate that the dipole electric-field domains in semiconductor SLs have the characteristics of solitons.

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