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The relationship between four-dimensional $\theta = \pi$ Yang–Mills theory and the two-dimensional Wess–Zumino–Novikov–Witten model

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Used the dimensional reduction in the sense of Parisi and Sourlas, the gauge fixing term of the four-dimensional Yang–Mills field without the theta term is reduced to a two-dimensional principal chiral model. By adding the θ term ($\theta = \pi$), the two-dimensional principal chiral model changes into the two-dimensional level 1 Wess–Zumino–Novikov–Witten model. The non-trivial fixed point indicates that Yang–Mills theory at $\theta = \pi$ is a critical theory without mass gap and confinement.

Keywords: Yang–Mills field, dimensional reduction, Wess–Zumino–Novikov–Witten model
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In this paper we study the characteristics of the SU(N) Yang–Mills field at $\theta = \pi$.[1] The Lagrangian density is written as

$$\mathcal{L} = -\frac{1}{2e^2}\mathrm{tr}F_{\mu\nu}F^{\mu\nu} + \mathcal{L}_\theta, \tag{1}$$

where $F^a_{\mu\nu} = \partial_\mu A^a_\nu - \partial_\nu A^a_\mu - e\epsilon^{abc}A^b_\mu A^c_\nu$ is the SU(N) gauge field strength, and

$$\mathcal{L}_\theta = \frac{\theta}{16\pi^2}\mathrm{tr}F_{\mu\nu}{}^*F^{\mu\nu}$$

is called the theta term.

The four-dimensional Yang–Mills field without the θ term has been studied using the Parisi–Sourlas (P–S) dimensional reduction method.[2] This has shown that a version of gauge fixing term allows us to write it in the form that is both BRST and anti-BRST exact. A hidden Osp(4|2) supersymmetry was found based on the superspace formulation of BRST invariant theories.[3−12] It turns out that the hidden supersymmetry leads to the dimensional reduction in the sense of Parisi and Sourlas.[3] Then, the four-dimensional Yang–Mills field at $\theta = 0$ is reduced to the equivalent two-dimensional principal chiral model by suitable superspace embedding parameters.[2,13−15] In this paper, we show that this method is also suitable for the Yang–Mills field at $\theta = \pi$.

To deal with the gauge field, the first step is quantization. In this paper, we develop a new gauge fixing method which not only gives the same perturbative theory as the F–P method (with the ghost field and suitable gauge field propagator), but also includes non-perturbative theory. For an arbitrary function of the field $f(A_\mu)$, we introduce the Gaussian function:

$$1 = \int [\mathrm{d}f(A^g)]\exp\left[\frac{-i}{2\alpha}\int \mathrm{d}^4x\mathrm{tr}\left(f^2(A^g)\right)\right]. \tag{2}$$

From this function, it is easy to obtain the standard gauge fixing term

$$\begin{aligned} 1 &= \int [\mathrm{d}f(A^g)]\exp\left[-\frac{i}{2\alpha}\int \mathrm{d}^4x\mathrm{tr}\left(f^2(A^g)\right)\right] \qquad (3)\\ &= \int [g^{-1}\mathrm{d}g]\,\det|\frac{\delta f(A^g)}{g^{-1}\delta g}| \\ &\quad\times\exp\left[-\frac{i}{2\alpha}\int \mathrm{d}^4x\mathrm{tr}\left(f^2(A^g)\right)\right] \\ &= \int [g^{-1}\mathrm{d}g][\mathrm{d}\bar{\mathcal{C}}][\mathrm{d}\mathcal{C}][\mathrm{d}\rho]\exp[i\int \mathcal{L}_{gf}\mathrm{d}^4x], \\ \mathcal{L}_{gf} &= -\mathrm{tr}\left(\rho\partial_\mu A^g_\mu + \frac{\alpha}{2}\rho^2 + \bar{\mathcal{C}}|\frac{\delta f(A^g)}{g^{-1}\delta g}|\mathcal{C}\right) \end{aligned}$$

where $\mathcal{C}$ is the ghost field and ρ is a scalar field.

If selecting a Lorentz gauge

$$f(A^g) = \partial_\mu A^g_\mu(x), \tag{4}$$

the gauge-dependent part $\mathcal{L}_g$ would be

$$\mathcal{L}_{gf} = -\mathrm{tr}\left(\rho\partial_\mu A^g_\mu + \frac{\alpha}{2}\rho^2 + \bar{\mathcal{C}}\partial_\mu D_\mu\left[(A^g_\mu)\right]\mathcal{C}\right). \tag{5}$$

The standard formulae of BRST δ_B and anti-BRST $\bar{\delta}_B$ are

$$\delta_B A^g_\mu(x) = D_\mu\mathcal{C}(x),\ \delta_B\rho(x) = 0, \tag{6}$$

$$\begin{aligned}
&\delta_B \mathcal{C}(x) = 0,\ \delta_B \overline{\mathcal{C}}(x) = i\rho(x),\\
&\delta_B \rho(x) = \delta_B \bar{\rho}(x) = 0,\\
&\bar{\delta}_B A^g_\mu(x) = D_\mu \overline{\mathcal{C}}(x),\ \bar{\delta}_B \bar{\rho}(x) = 0,\\
&\bar{\delta}_B \mathcal{C}(x) = i\bar{\rho}(x),\ \bar{\delta}_B \overline{\mathcal{C}}(x) = 0,\\
&\bar{\delta}_B \rho(x) = \bar{\delta}_B \bar{\rho}(x) = 0,
\end{aligned} \tag{7}$$

and

$$\rho(x) + \bar{\rho}(x) = [\mathcal{C}(x), \bar{\mathcal{C}}(x)]. \tag{8}$$

Using BRST δ_B and anti-BRST $\bar{\delta}_B$, $\mathcal{L}_g$ turns into a very compact form[11,12]

$$\mathcal{L}_{gf} = i\delta_B \bar{\delta}_B \operatorname{tr}\left[\frac{1}{2}(A^g_\mu)^2 + \frac{i\alpha}{2}\bar{\mathcal{C}}\mathcal{C}\right]. \tag{9}$$

In a $(4+2)$-dimensional superspace, $\left[\frac{1}{2}(A^g_\mu)^2 + \frac{i\alpha}{2}\mathcal{C}\bar{\mathcal{C}}\right]$ can be a contravariant supervector which transforms like the supercoordinate under Osp $(4|2)$.[11,12]

Thus a $(4+2)$-dimensional superspace $\mathcal{M}$ with coordinates is written as

$$X^M = (x^\mu, \theta, \bar{\theta}) \in \mathcal{M}, \quad x \in R^4,$$

where x^μ denotes the coordinate of the four-dimensional Euclidean space, and θ and $\bar{\theta}$ are anti-Hermitian Grassmann numbers. The inner product of two vectors is defined by introducing the superspace (covarious) metric tensor η_{MN} with components

$$\eta_{\mu\nu} = \delta_{\mu\nu}, \quad \eta_{\theta\bar{\theta}} = -\eta_{\bar{\theta}\theta} = -2/\gamma, \quad \text{others} = 0.$$

The connection one-form (superspace vector potential) $\mathcal{A}(X)$ and its curvature (superspace field strength) $\mathcal{F}(X)$ in the superspace are

$$\begin{aligned}
\mathcal{A}(X) =& \mathcal{A}_M(X)\mathrm{d}X^M \\
=& \mathcal{A}_\mu(x,\theta,\bar{\theta})dx^\mu + \mathcal{A}_\theta(x,\theta,\bar{\theta})d\theta \\
&+ \mathcal{A}_{\bar{\theta}}(x,\theta,\bar{\theta})d\bar{\theta},\\
\mathcal{F}(X) =& \mathrm{D}\mathcal{A}(X) + \frac{1}{2}[\mathcal{A}(X), \mathcal{A}(X)],
\end{aligned} \tag{10}$$

where D is the exterior differential in the superspace

$$D = d + \delta + \bar{\delta} = \frac{\partial}{\partial x^\mu}dx^\mu + \frac{\partial}{\partial\theta}\mathrm{d}\theta + \frac{\partial}{\partial\bar{\theta}}\mathrm{d}\bar{\theta}.$$

It is convenient to regard the gauge vector potential $A^g_\mu(x)$ and the ghost fields $\mathcal{C}(x)$ and $\bar{\mathcal{C}}(x)$ as the boson parts and fermion parts of the superspace vector potential $\mathcal{A}(X)$, respectively

$$A^g_\mu(x) \to \mathcal{A}(X), \tag{11}$$

$$\mathcal{C}(x) \to \mathcal{A}_\theta(X),$$

$$\bar{\mathcal{C}}(x) \to \mathcal{A}_{\bar{\theta}}(X).$$

Because the ghost fields have no geometric meaning, superspace embedding should be under a certain condition: the horizontal condition.[11,12] Setting

$$\mathcal{F}(X) = \frac{1}{2}\mathcal{F}_{\mu\nu}(X)\mathrm{d}x^\mu \mathrm{d}x^\nu, \tag{12}$$

we have the horizontal condition

$$\mathcal{F}_{M\theta}(X) = \mathcal{F}_{M\bar{\theta}}(X) = 0. \tag{13}$$

From the horizontal condition, the dependence of the superfield $A_M(x,\theta,\bar{\theta})$ on θ and $\bar{\theta}$ is determined as follows:

$$\begin{aligned}
\partial_\theta \mathcal{A}_\mu(X) =& \partial_\mu \mathcal{A}_\theta(X) \\
&- i[\mathcal{A}_\mu(X), \mathcal{A}_\theta(X)],\\
\partial_\theta \mathcal{A}_\theta(X) =& i\frac{1}{2}[\mathcal{A}_\theta(X), \mathcal{A}_\theta(X)],\\
\partial_{\bar{\theta}} \mathcal{A}_\mu(X) =& \partial_\mu \mathcal{A}_{\bar{\theta}}(X) - i[\mathcal{A}_\mu(X), \mathcal{A}_{\bar{\theta}}(X)],\\
\partial_{\bar{\theta}} \mathcal{A}_{\bar{\theta}}(X) =& i\frac{1}{2}[\mathcal{A}_{\bar{\theta}}(X), \mathcal{A}_{\bar{\theta}}(X)],\\
\partial_\theta \mathcal{A}_{\bar{\theta}}(X) + \partial_{\bar{\theta}} \mathcal{A}_\theta(X) =& -\{\mathcal{A}_\theta(X), \mathcal{A}_{\bar{\theta}}(X)\}.
\end{aligned} \tag{14}$$

For the components which cannot be determined by the horizontal condition alone, we use the following identification

$$\partial_\theta \mathcal{A}_{\bar{\theta}}(x,0,0) = i\Theta(x),$$

$$\partial_{\bar{\theta}} \mathcal{A}_\theta(x,0,0) = i\bar{\Theta}(x). \tag{15}$$

This corresponds to $\mathcal{F}_{\theta\bar{\theta}} = 0$ and gives

$$i\Theta(x) + i\bar{\Theta}(x) + \{\mathcal{A}_\theta(x), \mathcal{A}_{\bar{\theta}}(x)\} = 0. \tag{16}$$

From these results, it turns out that the derivatives in the directions of θ and $\bar{\theta}$ give respectively the BRST and the anti-BRST transformations

$$\frac{\partial}{\partial\theta} = \mathrm{s}\delta_B, \quad \frac{\partial}{\partial\bar{\theta}} = \mathrm{s}\bar{\delta}_B. \tag{17}$$

At the same time, if we require that

$$\mathcal{A}_\mu(X)|_{\theta=0,\bar{\theta}=0} = \mathcal{A}_\mu(x,0,0) = A^g_\mu(x), \tag{18}$$

we have 'the superspace embedding relation'

$$\mathcal{A}_\theta(x) = \mathrm{s}\mathcal{C}(x), \mathcal{A}_{\bar{\theta}}(x) = \mathrm{s}\bar{\mathcal{C}}(x), \tag{19}$$

$$\Theta(x) = \mathrm{s}^2\rho(x), \bar{\Theta}(x) = \mathrm{s}^2\bar{\rho}(x),$$

where s is a c number.

Then the relation between s and γ in the superspace is obtained as

$$\left(\frac{4\mathrm{s}^2}{\gamma}\right) = i\alpha. \tag{20}$$

The operator

$$\mathcal{O}(x) = i\mathrm{tr}\left(\frac{1}{2}(A_\mu^g(x))^2 + \frac{i\alpha}{2}\bar{\mathcal{C}}(x)\mathcal{C}(x)\right) \tag{21}$$

has the corresponding superfield given by

$$\begin{aligned}\mathcal{O}(X) =& \frac{i}{2}\mathrm{tr}\Big((\mathcal{A}_\mu^g(X))^2 + (2/\gamma) \\ & \times [\mathcal{A}_\theta(X)\mathcal{A}_{\bar\theta}(X) + \mathcal{A}_{\bar\theta}(X)\mathcal{A}_\theta(X)]\Big) \\ =& \frac{i}{2}\mathrm{tr}\Big(\eta_{NM}\mathcal{A}^M(X)\mathcal{A}^N(X)\Big),\end{aligned} \tag{22}$$

where

$$\begin{aligned}\mathcal{A}^g(X) =& g(X)\mathcal{A}(X)g^\dagger(X) \\ & + \frac{i}{\mathrm{e}}g^\dagger(X)\mathrm{d}x^M\partial_M g(X).\end{aligned} \tag{23}$$

For the Grassmann number, the integration $\int \mathrm{d}\theta$ ($\int \mathrm{d}\bar\theta$) is equivalent to the differentiation $\dfrac{\mathrm{d}}{\mathrm{d}\theta}$ ($\dfrac{\mathrm{d}}{\mathrm{d}\bar\theta}$). Hence the BRST δ_B and anti-BRST δ_B transformations have the following correspondence

$$\mathrm{s}\delta_B \leftrightarrow \frac{\mathrm{d}}{\mathrm{d}\theta} \leftrightarrow \int \mathrm{d}\theta, \tag{24}$$

$$\mathrm{s}\bar\delta_B \leftrightarrow \frac{\mathrm{d}}{\mathrm{d}\bar\theta} \leftrightarrow \int \mathrm{d}\bar\theta.$$

This implies

$$\begin{aligned}\int \mathrm{d}\theta\mathrm{d}\bar\theta\mathcal{O}(x,\theta,\bar\theta) =& -\frac{\partial}{\partial\theta}\frac{\partial}{\partial\bar\theta}\mathcal{O}(x,\theta,\bar\theta) \\ =& -\mathrm{s}^2\bar\delta_B\delta_B\mathcal{O}(x) = \mathrm{s}^2\delta_B\bar\delta_B\mathcal{O}(x).\end{aligned} \tag{25}$$

Thus the action can be written in the manifestly superspace covariant form[11,12]

$$\begin{aligned}\mathcal{S}_{gf} =& \int \mathrm{d}^4x\left(i\delta_B\bar\delta_B\mathrm{tr}\left[\frac{1}{2}(A_\mu^g)^2 + \frac{i\alpha}{2}\bar{\mathcal{C}}\mathcal{C}\right]\right) \\ =& \frac{i}{2\mathrm{s}^2}\int \mathrm{d}^4x\int \mathrm{d}\theta\mathrm{d}\bar\theta\mathrm{tr}(\eta_{NM}\mathcal{A}^M(X)\mathcal{A}^N(X)).\end{aligned} \tag{26}$$

In our model, the operator

$$\begin{aligned}\mathcal{O}(x,\theta,\bar\theta) =& \Big[\left(g^{-1}\partial_\mu g + ieg^{-1}\mathcal{A}_\mu g\right)^2 \\ & + i\alpha\bar{\mathcal{C}}(x)\mathcal{C}(x)\Big]\end{aligned} \tag{27}$$

is not only a four-dimensional orthogonal O(4) invariant but also the symplectic group Osp(2) invariant. So one may carry out a P–S dimensional reduction for it. Let us split the four-dimensional Euclidean space into two subsets, $x = (z, \hat{x}) \in R^4$, $z \in R^2$, $\hat{x} \in R^2$. Hence, for the supersymmetric operator $\mathcal{O}(X)$, one obtains $\mathcal{O}(x,\theta,\bar\theta) = f(z,\hat{x}^2) + \dfrac{4}{\gamma}\bar\theta\theta\dfrac{d}{d\hat{x}^2}f(z,\hat{x}^2)$. The P–S dimensional reduction for a $(4+2)$-dimensional integral is written as

$$\begin{aligned}&\int \mathrm{d}^4x\int \mathrm{d}\theta\mathrm{d}\bar\theta\mathcal{O}(x,\theta,\bar\theta) \\ =& \int \mathrm{d}^2z\int \mathrm{d}^2\hat{x}\int \mathrm{d}\theta\int \mathrm{d}\bar\theta\ \frac{4}{\gamma}\bar\theta\theta\frac{\mathrm{d}}{\mathrm{d}\hat{x}^2}f(z,\hat{x}^2) \\ =& \frac{-4}{\gamma}\int \mathrm{d}^2z\int \mathrm{d}^2\hat{x}\frac{\mathrm{d}}{\mathrm{d}\hat{x}^2}f(z,\hat{x}^2) \\ =& \frac{4\pi}{\gamma}\int d^2z\mathcal{O}_0((z,0),0,0),\end{aligned} \tag{28}$$

where $\mathcal{O}_0((z,0),0,0) = \mathcal{O}_0((z,\hat{x}^2=0),\theta=0,\bar\theta=0)$. Then the gauge fixing action in the superspace can be reduced to a two-dimensional sigma model

$$\begin{aligned}\mathcal{S}_{gf} =& \frac{i}{2s^2}\int \mathrm{d}^4x\int \mathrm{d}\theta\mathrm{d}\bar\theta\mathrm{tr}(\eta_{NM}\mathcal{A}^M(X)\mathcal{A}^N(X)) \\ =& \frac{i}{2s^2}\int \mathrm{d}^4x\int \mathrm{d}\theta\mathrm{d}\bar\theta\mathrm{tr} \\ & \times\left[\mathcal{A}^\mu(X)\mathcal{A}_\mu(X) + i\alpha\bar{\mathcal{C}}(X)\mathcal{C}(X)\right] \\ =& \frac{1}{2\lambda}\int \mathrm{d}^2x\,\mathrm{tr}[(g^{-1}\partial_\mu g + ieg^{-1}A_\mu g)^2 \\ & + i\alpha\bar{\mathcal{C}}(x)\mathcal{C}(x)],\end{aligned} \tag{29}$$

where $\lambda = \dfrac{\mathrm{s}^4e^2}{\pi\alpha}$. Finally the gauge fixing term in the Lorentz gauge turns into the kinetic term of a two-dimensional $SU(N)_R \times SU(N)_L$ principal chiral model.

When $\theta = \pi$, the θ term

$$\mathcal{L}_\theta = \frac{\theta}{16\pi^2}\mathrm{tr}F_{\mu\nu}{}^*F^{\mu\nu}$$

leads to a dramatic effect on the vacuum of the Yang–Mills field. From the relation

$$\begin{aligned}Q =& \int \mathrm{d}^4x\frac{1}{16\pi^2}\mathrm{tr}F_{\mu\nu}{}^*F^{\mu\nu} \\ =& \frac{1}{24\pi^2}\int \mathrm{d}z\mathrm{d}^2x\varepsilon^{\mu\nu\rho} \\ & \times \mathrm{tr}\left(g^{-1}\partial_\mu g g^{-1}\partial_\nu g g^{-1}\partial_\rho g\right),\end{aligned} \tag{30}$$

the θ term turns into the Wess–Zumino (WZ) term of the Wess–Zumino–Novikov–Witten (WZNW) model.[16–18] The WZ term is often written as $k\Gamma$, where

$$\Gamma = \frac{1}{24\pi}\int \mathrm{d}z\mathrm{d}^2x\varepsilon^{\mu\nu\rho}\mathrm{tr}\left(g^{-1}\partial_\mu g g^{-1}\partial_\nu g g^{-1}\partial_\rho g\right). \tag{31}$$

The coefficient k is known as the level, and for compact groups it must be an integer because the different possible extensions of $g(x)$ to the ball yield a possible ambiguity of $2n\pi$ in Γ. The WZNW term changes the equations of motion and beta function, but only by terms involving $g(x)$, is the variation of the integrand a total derivative in z.

Hence, the SU(N) WZNW model with level k is deformed from the original SU(N) Yang–Mills theory as

$$S(g) = S_{gf} + S_{WZ}, \tag{32}$$

$$S_{gf} = \frac{1}{2\lambda}\int \mathrm{d}^2x \mathrm{tr}[(g^{-1}\partial_\mu g + ieg^{-1}A_\mu g)^2 + i\alpha\mathcal{C}(x)\bar{\mathcal{C}}(x)],$$

$$S_{WZ} = \frac{1}{24\pi}\int \mathrm{d}z\mathrm{d}x^2\varepsilon^{\mu\nu\rho} \times \mathrm{tr}(g^{-1}\partial_\mu g g^{-1}\partial_\nu g g^{-1}\partial_\rho g).$$

This model is conformal invariant without a mass gap.

The g field is also divided into two parts: one is a slowly varying vector g_0;[19−21] another is a fast varying one $\xi(x)$ as $g = g_0\exp[i\lambda\xi(x)]$. By g_0 and $\xi(x)$, we can rewrite the action $S(g)$ as

$$S\left(g_0^{-1}\partial_\mu g_0 + ieg_0^{-1}A_\mu g_0\right) + \int \mathrm{d}^2x \mathrm{tr}\{(\partial_\mu\xi)^2 + \frac{1}{2}\left(g_0^{-1}\partial_\mu g_0 + ieg_0^{-1}A_\mu g_0\right)[\xi, \partial_\mu\xi] + \frac{\lambda}{16\pi}\varepsilon_{\mu\nu}\left(g_0^{-1}\partial_\mu g_0 + ieg_0^{-1}A_\mu g_0\right)[\xi, \partial_\nu\xi]\}. \tag{33}$$

The $\beta(\lambda)$ function of λ changes into

$$\beta(\lambda) = \Lambda\frac{\partial\lambda}{\partial\Lambda} = -\frac{\mathrm{c}_v}{8\pi}\lambda^2\left[1 - \left(\frac{\lambda}{8\pi}\right)^2\right]. \tag{34}$$

A non-trivial fixed point appears at

$$\lambda = \lambda_c = \frac{\mathrm{s}_c^4 e_c^2}{\pi\alpha_c} = 8\pi, \tag{35}$$

indicating that the theory is scale invariant without asymptotic freedom.

Renormalization is to integrate the 'fast modes' of a field with large momenta and to renormalize the function of the remaining 'slow modes' of the field. The intermediate momentum scale Λ' ($\Lambda \gg \Lambda' > 0$) is introduced to separate the 'fast modes' and the 'slow modes'. Here Λ is the momentum cut-off. Because the gauge field is connected to the gauge modes, $A_\mu^g = g^{-1}\partial_\mu g + ieg^{-1}A_\mu g$, the momentum scales Λ'_{A_μ} and Λ'_g are the same: $\Lambda'_{A_\mu} = \Lambda'_g$.

On the other hand, the relation

$$\Lambda'_g/\Lambda = \Lambda'_{A_\mu}/\Lambda \tag{36}$$

shows that there is no energy scale for Λ'_{A_μ}. The non-trivial fixed point indicates that Yang–Mills theory at $\theta = \pi$ is a critical theory without asymptotic freedom. For the critical case, there is no mass gap[22,23]

$$m^2 \propto \left(\Lambda'_{A_\mu}\right)^2 \equiv 0, \tag{37}$$

and no confinement

$$\sigma \propto m^2 \equiv 0 \tag{38}$$

where σ is the string tension.

On the other hand, for Yang–Mills theory at $\theta = 0$, due to the existence of the mass gap effect, it indeed shows confinement:

$$\sigma \propto m^2 \propto \left(\Lambda'_{A_\mu}\right)^2 = \frac{3\pi\Lambda^2}{2}\exp\left[-\frac{1}{b_0^e e^2}\right]. \tag{39}$$

In summary, using the dimensional reduction in the sense of Parisi and Sourlas, the gauge fixing term of a four-dimensional Yang–Mills field with the θ term ($\theta = \pi$) is reduced to the two-dimensional level 1 WZNW model. The non-trivial fixed point indicates that Yang–Mills theory at $\theta = \pi$ is a critical theory without mass gap and confinement.

References

[1] Yang C N and Mills R L 1954 *Phys. Rev.* **96** 191

[2] Kou S P 2001 *Chin. Phys.* **10** 398

[3] Parisi G and Sourlas N 1979 *Phys. Rev. Lett.* **43** 744

[4] McClain B, Niemi A, Taylor C and Wijewardhana L C R 1983 *Nucl. Phys.* B **217** 430

[5] Curci G and Ferrari R 1976 *Phys. Lett.* B **63** 91

[6] Ojima I 1980 *Prog. Theor. Phys.* **64** 625

[7] Delbourgo R and Jarvis P D 1982 *J. Phys. A: Math. Gen.* **15** 611

[8] Klein A, Landau L J and Perez J F 1984 *Commun. Math. Phys.* **94** 459

[9] Klein A and Perez J F 1983 *Phys. Lett.* B **125** 473

[10] Bonora L and Tonin M 1981 *Phys. Lett.* B **98** 48

[11] Kondo K I 1998 *Phys. Rev.* D **58** 105019

[12] Kondo K I 1999 *Preprint* hep-th/9904045
Kondo K I 1999 *Preprint* hep-th/9911242
Kondo K I 1998 *Preprint* hep-th/9801024

[13] Migdal A 1975 *Zh. Eksp. Teor. Fiz.* **69** 810 (1975 *Sov. Phys. JETP* **42** 413)
[14] Migdal A 1975 *Zh. Eksp. Teor. Fiz.* **69** 1457 (1975 *Sov. Phys. JETP* **42** 743)
[15] Kogut J B 1979 *Rev. Mod. Phys.* **51** 659
[16] Wess J and Zumino B 1971 *Phys. Lett.* B **39** 95
[17] Witten E 1983 *Nucl. Phys.* B **223** 422
[18] Witten E 1994 *Comm. Math. Phys.* **92** 455
[19] Brezin E, Hikami S and Zinn-Justin J 1980 *Nucl. Phys.* B **165** 528
[20] Zinn-Justin J 1989 *Quantum Field Theory and Critical Phenomena* (Oxford: Oxford University Press)
[21] Abdalla E, Abdalla M C B and Rothe K D 1991 *Non-perturbative Methods in Two-dimensional Quantum Field Theory* (Singapore: World Scientific)
[22] Gross D J and Wilczek F 1973 *Phys. Rev. Lett.* **30** 1343
[23] Politzer H D 1973 *Phys. Rev. Lett.* **30** 1346