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Shapiro effect in mesoscopic LC circuit*

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In this paper we consider the movement of an electron in the single electron tunnel process through a mesoscopic capacitor. The results show that, due to the Coulomb force, there is a threshold voltage V_t in the mesoscopic LC circuit. When the external voltage is lower than the threshold voltage, the tunnel current value is zero, and the Coulomb blockade phenomenon arises. Furthermore, considering that the mesoscopic dimension is comparable to the coherence length in which charge carriers retain the phase remembrance, a weak coupling can be produced through the proximity effect of the normal metal electrons of both electrodes of a mesoscopic capacitor. By varying the external voltage, we can observe the Shapiro current step on the current–voltage characteristic curve of a mesoscopic LC circuit.

Keywords: mesoscopic LC circuit, Coulomb blockade, single electron tunnelling effect, Shapiro current step

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1. Introduction

With the developments of nanometre techniques and microelectronics, the miniaturization of integrated circuits and components is approaching nanometre scale. When the inelastic coherence length of the charge carrier approaches the Fermi wavelength, classical physics is expected to be invalid and quantum effects will have to be considered. This has become an active problem now. Recently, the quantum fluctuations of charge and current in mesoscopic LC circuits and RLC circuits have been discussed, as well as the influence of the coupling parameters and environmental temperature on the fluctuations of charge and current.[1−15] The theoretical analysis shows that there should be some new effects in nanometre tunnel junctions with capacitance C, such as the charging energy of a single electron $e^2/2C$, which exceeds the characteristic energy k_BT of thermal fluctuations. Simple energy considerations suggest that the tunnelling of a single electron is completely blocked when the junction capacitor holds a charge less than e/2. And large amounts of experimental data seem to support the theoretical ideas of the Coulomb blockade of tunnelling.[16,17] Undoubtedly, quantum effects of a mesoscopic circuit must be considered in the tunnelling process of electrons where capacitors are considered as tunnel junctions. We have studied the quantum effect caused by a single electron crossing a tunnel, and the results show that there is a Coulomb gap because of the effect of the Coulomb force.[18] That is to say, when the external voltage is lower than a certain threshold voltage V_t, the tunnel current vanishes because of the effect of the Coulomb force; when the external voltage is much higher than the threshold voltage V_t, the tunnel current is directly proportional to the external voltage. With an external constant current source, the circuit will be in a state of single electron tunnel oscillation (SET oscillation). The theoretical analysis and the experimental results indicate that the movement of an electron in a mesoscopic capacitor is actually the process of a single electron crossing a tunnel. In this paper, we consider that the mesoscopic dimension is in a scale that the charge carrier can keep the phase memory, and then a weak coupling can be produced through the proximity effect of the normal metal electrons of the two electrodes of a mesoscopic capacitor. By varying the external voltage, we can observe the Shapiro current step in the current–voltage characteristic curve of a mesoscopic LC circuit.

2. Coulomb gap

From an atomistic viewpoint, the capacitance of a mesoscopic capacitor is defined by the voltage change

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for ΔV [19,20]

$$e\Delta V = \mu(N+\Delta N) - \mu(N),$$

where $\mu(N)$ is the chemical potential of an N particle system. In the ground state, the total energy of an N particle system is

$$E(N) = E_p(N) + \frac{(eN)^2}{2C} - eN\phi_{ext},$$

where $E_p(N)$ is the total kinetic energy of N electrons, and ϕ_{ext} is the external source. The chemical potential for an N particle system is equal to the ground-state total energy change of the system when taking away or adding in an electron:

$$\mu(N) = E(N) - E(N-1) \cong \frac{e^2}{C}\left(N - \frac{1}{2}\right) - e\phi_{ext}.$$

In the case of single electron tunnelling (the transfer of electrons by tunnelling through the junction of a capacitor), the total energy change is

$$\begin{aligned}&[E(N+1) - E(N)] - [E(N) - E(N-1)]\\ =&\mu(N+1) - \mu(N) = \frac{e^2}{C}.\end{aligned}$$

So, with the capacitor as an N particle system, the minimum energy for reducing or adding an electron is

$$\Delta = \frac{e^2}{2C}, \tag{1}$$

which is caused by the Coulomb force of the system. We call this the Coulomb gap. The above discussion indicates that there must be an original threshold voltage $V_t = C^{-1}e$ to conquer the Coulomb force when the electrons transfer through the junction of the capacitor. Also, if the external voltage is lower than the original threshold voltage, the current crossing the junction equals zero, and thus the Coulomb blockade phenomenon appears.

3. Single electron tunnelling current

Consider a voltage-biased single LC mesoscopic circuit system consisting of a tunnel junction (C) connected to an external circuit, and the junction resistance is R_t. For an actual tunnelling junction of a capacitor, we may take it as a parallel capacitor and resistor. So, the current through the junction includes the single electron tunnelling current I_n and the displacement current I_d through the ideal capacitor. If the normal metal materials are the same in both electrodes of the capacitor, when a negative voltage is added to the left metal electrode, the electron energy at the left side will be increased by eV. Considering that the tunnelling probability is unrelated to the electron energy, according to the stationary tunnel model, we can obtain the single electron tunnelling current I_n

$$I_n = AN_1(0)N_2(0)\int [f(E-eV) - f(E)]dE, \tag{2}$$

where $N_1(0)$ and $N_2(0)$ refer to the electron density of states near the Fermi level of the right and left sides, respectively. The coefficient A is related to the geometrical character of the metal electrode and the tunnelling probability of the transfer of electrons by tunnelling through the junctions. We consider that the dimension of the junction is longer than the phase coherence length of the electron, and we may obtain from Eq.(2) approximately

$$I_{n0} = AN_1(0)N_2(0)eV. \tag{3}$$

This indicates that the current and voltage of the tunnelling junction C satisfy Ohm's law, in which the relation between current and voltage is linear. There is no Coulomb blockade phenomenon.

However, under the mesoscopic dimension, because of the effect of the Coulomb force, the density of state of the electron around the Fermi level will vary. If both sides of the metal material are identical, the electrons in both sides have equivalent gaps $\Delta_1 = \Delta_2 = \Delta = e^2/2C$. So, when the dimension of a mesoscopic capacitor tunnel junction is comparable to the phase coherence length, the single tunnel current I_n in Eq.(2) should be

$$\begin{aligned}I_n =& AN_1(0)N_2(0)\int \frac{|E-eV|}{\sqrt{(E-eV)^2-\Delta^2}}\frac{|E|}{\sqrt{E^2-\Delta^2}}\\ &\times\{f(E-eV) - f(E)\}dE.\end{aligned} \tag{4}$$

Here $N_1(0)$ and $N_2(0)$ are the density of states when the junction dimension is much longer than the electron phase coherence length. Further calculations show

$$I_{n1} = 0, \qquad eV < 2\Delta; \tag{5a}$$

$$I_{n2} = \frac{\pi\Delta}{2eR_T}, \qquad eV = 2\Delta; \tag{5b}$$

$$I_{n3} = \frac{1}{eR_T}\left\{(2\Delta+eV)E(\alpha) - \frac{4\Delta(\Delta+eV)}{2\Delta+eV}K(\alpha)\right\}, \qquad eV > 2\Delta; \tag{5c}$$

where

$$R_T^{-1} = eAN_1(0)N_2(0), \tag{5d}$$

$$E(\alpha)=\int_0^1 \frac{(1-\alpha^2x^2)^{1/2}}{(1-x^2)^{1/2}}\mathrm{d}x,$$

$$K(\alpha)=\int_0^1 \frac{\mathrm{d}x}{(1-x^2)^{1/2}(1-\alpha^2x^2)^{1/2}},$$

$$\alpha=\frac{\mathrm{e}V-2\Delta}{\mathrm{e}V+2\Delta},\quad x=\frac{2E-\mathrm{e}V}{2\Delta-\mathrm{e}V}.$$

V is the external voltage applied to the capacitor junction. When $V\gg 2\Delta/\mathrm{e}$, the approximate solution to Eq.(5c) is

$$I_{n3}=R_{\mathrm{T}}^{-1}V. \tag{6}$$

This shows that the relationship between current and voltage is linear.

The displacement current through the junction I_{d} can be obtained through the external voltage:

$$I_{\mathrm{d}}=C\frac{\mathrm{d}V}{\mathrm{d}t}. \tag{7}$$

A junction, in fact, must comprise a complete circuit with external circuits. In mesoscopic LC circuits, because of the presence of the inductance L, there will be an oscillation effect between the capacitor and inductor. When the external voltage is V, the voltage exerted on C is about

$$\nu_{\mathrm{c}}(t)=V+[C^{-1}Q(0)-V]\cos(\omega t)+\omega^{-1}p(0)\sin(\omega t), \tag{8}$$

where $\omega^2=(LC)^{-1}$. Under the external voltage given by Eq.(8), the single electron tunnel current I_n is

$$I_n=R_{\mathrm{T}}^{-1}\nu_{\mathrm{c}}(t), \tag{9}$$

and the displacement current I_{d} is

$$I_{\mathrm{d}}=\omega[CV-Q(0)]\sin(\omega t)+Cp(0)\cos(\omega t). \tag{10}$$

4. Shapiro effect

Further experimental results indicate that steps appear in the characteristic curve. This means that the current through the tunnel junction consists not only of the two terms of current discussed above, but also the contribution of a third term. Of course, we may ascribe the third term to the interaction between electrons and phonons in normal metal. Both Coulomb repulsion between electrons and attraction between electrons and phonons exist in metal. Because the mesoscopic dimension is so small that the charge carrier preserves its phase memory, normal metal electrons form weak coupling through the proximity effect. So we believe, in the mesoscopic dimension scale, normal metal weak coupling has a certain probability to create Cooper pairs. That is to say, in the scale of mesoscopic phase coherence length, the phase coherence length of normal metal electrons enables the electrons to carry superconductive information, and weak coupling caused by normal metal electrons through the proximity effect forms a supercurrent. So, the quantum behaviour of normal electrons in a mesoscopic capacitor is comparable to an actual Josephson tunnel junction, and the current through the tunnel junction includes the single electron tunnel current I_n through resistance, the displacement current I_{d} through ideal capacitor, and the supercurrent I_{s} caused by the weak coupling.

While the mesoscopic capacitor takes the metal–insulator–metal model, the Hamiltonian of the tunnelling junction circuit can be written in the form[21,22]

$$H=H_1+H_2+H_{\mathrm{t}}+H_{\mathrm{em}}, \tag{11}$$

where H_1 and H_2 describe the electronic state of two electrodes, including intra-electrode interactions due to the electron–impurity and electron–electron scatterings as well as kinetic terms. We neglect the interactions for the moment;

$$H_1+H_2=\sum_{j=1}^{2}\sum_{k,\sigma}\varepsilon_j(k)a_{k,\sigma}^{(j)+}a_{k,\sigma}^{(j)},$$

$a_{k,\sigma}^{(j)+}$ ($a_{k,\sigma}^{(j)}$) refers to the creation (annihilation) operator of electrons with wave vector k, energy $\varepsilon_j(k)$ and spin σ in the electrode. The electromagnetic energy of the entire circuit is

$$H_{\mathrm{em}}=\frac{1}{2C}Q^2+\frac{1}{2L}\Phi^2-QV,$$

where V is the voltage of external source, Q is the electrical charge of capacitor, while its conjugation variable is Φ. The Hamiltonian that describes the transfer of electrons by tunnelling through junctions is

$$H_{\mathrm{t}}=\sum_{k,k',\sigma}(T_{kk'}a_{k\sigma}^{+}a_{k'\sigma}+T_{k'k}a_{k'\sigma}^{+}a_{k\sigma}).$$

We suppose wavefunctions of the electron pairs across mesoscopic junction are ψ_1 and ψ_2, the phase difference is $\Delta\varphi=\varphi_2-\varphi_1$, and ε_1 and ε_2 are the eigenvalue energies of Hamiltonians H_1 and H_2, respectively. The wavefunctions satisfy the following equations

$$\mathrm{i}\hbar\frac{\partial\psi_1}{\partial t}=\varepsilon_1\psi_1+\lambda\psi_2, \tag{12a}$$

$$\mathrm{i}\hbar\frac{\partial\psi_2}{\partial t} = \varepsilon_2\psi_2 + \lambda\psi_1, \tag{12b}$$

where λ is a coupling coefficient. Because the coupling is weak, λ is small. Letting $\varepsilon_2 - \varepsilon_1 = 2\mathrm{e}V$, where V is the external voltage across the junction, we obtain

$$I_\mathrm{s} = I_\mathrm{c}\sin\Delta\varphi, \tag{13a}$$

$$\frac{\mathrm{d}}{\mathrm{d}t}\Delta\varphi = \frac{2\mathrm{e}V}{\hbar}. \tag{13b}$$

I_c is the critical current, which is related to electron density of states and the coupling coefficient, and φ is the phase difference of the electron wavefunctions on the two sides, which varies with time. In the equilibrium state, neglecting the influence of the magnetic field created by the current itself, the relationship between φ and the tunnel voltage is

$$\varphi = \int\frac{2\mathrm{e}}{\hbar}V\mathrm{d}t + \varphi_0,$$

where φ_0 is an integral constant. Here we need to point out that, for the weak coupling of normal metal, φ_0 is very small. So, when the external voltage $V = 0$, the supercurrent is approximately equal to zero; when $V \neq 0$,

$$I_\mathrm{s} = I_\mathrm{c}\sin\left(\frac{2\mathrm{e}}{\hbar}Vt + \varphi_0\right). \tag{14}$$

Obviously, the mean value of the supercurrent over time is zero.

We suppose that the effect of the external source is directly added to the ac component of voltage, from Eq.(8). In a mesoscopic LC circuit, because of the inductance L, there is an electromagnetic oscillation between the capacitor and the inductor, which can be considered as an alternating voltage exerted on the tunnelling junction. So, with the external source power voltage V, the total voltage across the junction is approximately

$$\nu_\mathrm{c}(t) = V + [C^{-1}Q(0) - V]\cos(\omega t) + \omega^{-1}p(0)\sin(\omega t).$$

The time-dependent junction supercurrent then has the form

$$I_\mathrm{s} = I_\mathrm{c}\sin\left[\left(\frac{2\mathrm{e}}{\hbar}Vt\right) + \frac{2\mathrm{e}V}{\hbar\omega}\sin(\omega t + \theta) + \varphi_0\right],$$

where

$$V_1 = \sqrt{[C^{-1}Q(0) - V]^2 + \omega^{-2}p^2(0)},$$

$$\tan\theta = \omega p^{-1}(0)[C^{-1}Q(0) - V],$$

and the mean value of the supercurrent is

$$I_\mathrm{s} = I_\mathrm{c}\sum_{m=-\infty}^{\infty}(-1)^n J_n\left(\frac{2\mathrm{e}V_1}{\hbar\omega}\right) \times\sin\left[\left(\frac{2\mathrm{e}}{\hbar}V - n\omega\right)t - n\theta + \varphi_0\right], \tag{15}$$

where J_n is the n-order Bessel function of the first kind. Generally speaking, $(2\mathrm{e}\hbar^{-1}V - n\omega)$ will not be zero, and each item in Eq.(15) refers to the alternating current, whose mean value over time is zero. If $(2\mathrm{e}\hbar^{-1}V - n\omega)$ is equal to zero, we obtain

$$V = \mathrm{e}^{-1}\hbar n\omega, \qquad (n = 0, \pm1, \pm2, \cdots). \tag{16}$$

Thus, in Eq.(15), except for the number n item, which is time independent, the mean values over time of the other items are all equal to zero, and then the dc component supercurrent appears. When the external voltage V satisfies Eq.(16), the mean value over time of Eq.(15) is

$$I_\mathrm{s} = (-1)^n I_\mathrm{c} J_n\left(\frac{\mathrm{e}V_1}{\hbar\omega}\right)\sin(\varphi_0 - n\theta), \qquad (n = 0, 1, 2, \cdots). \tag{17}$$

It is shown that the supercurrent created by the weak coupling of the left and right electrodes is not equal to zero. From the current–voltage characteristic curve of the junction, we find that the direct current has weak peaks at points where $V = \mathrm{e}^{-1}\hbar n\omega$, which is caused by the charge quantization. According to the current continuity theorem, current through the tunnelling junction should be equal to the external circuit current. So, the current through the mesoscopic LC circuit should be

$$I = I_n + I_\mathrm{d} + I_\mathrm{s}. \tag{18}$$

Obviously, the experimentally measured current is the mean value. When taking Eq.(8) into account, when $V \gg 2\Delta/\mathrm{e}$, the current through the tunnelling junction approximately equals

$$\langle I\rangle = R_\mathrm{t}^{-1}(V - V_\mathrm{t}) + (-1)^n I_\mathrm{c} J_n\left(\frac{\mathrm{e}V_1}{\hbar\omega}\right)\sin(\varphi_0 - n\theta). \tag{19}$$

Here $V_\mathrm{t} = \mathrm{e}/C$ is the original threshold voltage. So, as to the influence of the external circuit, if we vary the external voltage V, in the current–voltage characteristic curve of the mesoscopic LC circuit, we can observe current steps instead of the dc weak peaks at points where $V = \mathrm{e}^{-1}\hbar n\omega$.

5. Conclusion

In a mesoscopic capacitor, there is a Coulomb gap because of the influences caused by the Coulomb force and interaction between electrons and phonons. In addition, as the mesoscopic dimension is comparable to the scale in which the charge carriers maintain their phase memory, normal metal electrons can produce weak coupling through the proximity effect. So, the phase coherence of normal electrons in the scale of the mesoscopic phase coherence length enables the electrons to generate current, and the current Shapiro phenomena appears in the current–voltage curve of the mesoscopic LC circuit.

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