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Mass distribution in n-polymer stochastic aggregation and large-mass behaviour*

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The exact solutions of the rate equations of the n-polymer stochastic aggregation involving two types of clusters, active and passive for the kernel $\prod_{k=1}^{n} s_{i_k}(s_{i_k}=i_k)$ and $\sum_{k=1}^{n} s_{i_k}(s_{i_k}=i_k)$, are obtained. The large-mass behaviours of the final mass distribution of the active and passive clusters have scaling-like forms, although the models exhibit different properties. Respectively, they have different decay exponents $\gamma=\dfrac{2n+1}{2(n-1)}$ and $\gamma=q+\dfrac{2n+1}{2(n-1)}$ for $\prod_{k=1}^{n} s_{i_k}(s_{i_k}=i_k)$ and $\gamma=\dfrac{3}{2(n-1)}$ and $\gamma=q+\dfrac{3}{2(n-1)}$ for $\sum_{k=1}^{n} s_{i_k}(s_{i_k}=i_k)$, which include exponents of two-polymer stochastic aggregation. We also find that gelation is suppressed for kernel $\prod_{k=1}^{n} s_{i_k}(s_{i_k}=i_k)$ which is different from the deterministic aggregation.

Keywords: stochastic aggregation, rate kernel, gelation, mass distribution
PACC: 0550, 0520, 6470

1. Introduction

Aggregation, coagulation or agglomeration are kinetic processes in which a number of basic units (or clusters) stick irreversibly together to build clusters. Most approaches assume that the merging of two clusters of masses i and j into a cluster of mass $i+j$ occurs with a rate kernel $\boldsymbol{K}(i,j)$. The clustering process evolves according to the Smoluchovski aggregation equation[1]

$$\frac{\mathrm{d}A_k}{\mathrm{d}t}=\frac{1}{2}\sum_{i+j=k}K(i,j)A_iA_j-A_k\sum_{j}K(j,k)A_j. \quad (1)$$

This equation has been widely used to describe the kinetics of aggregation in many fields in physical chemistry, biology, astronomy, etc.[2,3] However, the irreversible aggregation process, in practice, is very complex and the coagulation process may involve two-polymer, three-polymer or n-polymer interactions to stick together to form clusters. The aggregation process of a single n-polymer collision has been investigated via the generalized Smoluchovski equation.[5−9] The practical aggregation process involves 2-polymer, 3-polymer and n-polymer interactions and different numbers of polymers collide together to develop large clusters. The kinetic equation concerning the interactions of different numbers of polymers is the joint aggregation kinetic equation.[6,9] The multi-component aggregation process has been investigated and described by the multi-component Smoluchovski equation.[11] By the above-mentioned investigations, we can further understand the kinetic behaviours of those aggregations, especially the critical phenomena of gelation and scaling properties.[7] The practical aggregation process of the interaction between clusters has stochastic dynamic behaviour. Even if two clusters have a similar mass and size, the character of a cluster may be altered every time it undergoes stochastic aggregation. Recently, Krapivsky Ben-Naim[12] have proposed the rate equation to describe stochastic aggregation in which active clusters may become passive after merging events. The master equations governing the time evolution of the stochastic aggregation are

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$$\frac{\mathrm{d}A_k}{\mathrm{d}t} = \frac{1}{2}p\sum_{i+k=k} K(i,j)A_iA_j - A_k\sum_{j\geq 1} K(k,j)A_j,$$

$$\frac{\mathrm{d}P_k}{\mathrm{d}t} = \frac{1}{2}q\sum_{i+j=k} K(i,j)A_iA_j. \qquad (2)$$

The equation explains that the newborn cluster remains active with probability p or becomes passive with probability q $(q+p=1)$ and the passive cluster does not further aggregate. In this equation $K(i,j)$ is the rate at which clusters of mass i,j aggregate and $A_k(P_k)$ denotes an active (passive) cluster of mass k. The master equation obeys the mono-disperse initial conditions

$$A_k(0) = \delta_{k,1}, \qquad P_k(0) = 0. \qquad (3)$$

The overall mass is conserved

$$\sum_{k\geq 1} k[A_k(t) + P_k(t)] = 1. \qquad (4)$$

Some kinetic equations of the deterministic aggregation, i.e., the Smoluchovski equation with the rate kernel $K(i,j) = s_is_j$ $(s_k = k)$ or the generalized Smoluchovski equation with the rate kernel $\prod_{k=1}^{n} s_{i_k}(s_{i_k} = i_k)$, are solvable models. In the time evolution of the aggregation, the sol–gel transition will occur, but the kinetic behaviour of two-polymer differs from that of n-polymer. Moreover, they have different gel points.[4,5,8] The stochastic kinetic equations with the rate kernels $K(i,j) = s_is_j$ $(s_k = k)$ and $k(i,j) = s_i + s_j$ $(s_k = k)$ are exactly solved. Gelation is suppressed for the rate kernel $k(i,j) = s_is_j$ $(s_k = k)$ and the large-mass and long-time behaviour of the mass distribution of active and passive clusters have scaling-like forms, which is different from the deterministic aggregation.[12] Based on the master equation (2), we have proposed the single n-polymer stochastic aggregation and exactly solved the stochastic kinetic equation with the rate kernel $K(i_1, i_2, \cdots, i_n)$=const. We also found similar scaling-like forms with different scaling exponents.[10]

In this paper, we generalize the master equation (2) to the single n-polymer stochastic aggregation with the rate kernels $K(i_1, i_2, \cdots, i_n) = \prod_{k=1}^{n} s_{i_k}(s_{i_k} = i_k)$ and $K(i_1, i_2, \cdots, i_n) = \sum_{k=1}^{n} s_{i_k}(s_{i_k} = i_k)$ and exactly solve the stochastic kinetic equation to investigate the properties of the mass distribution.

2. Exact solution of the kinetic equation with rate kernel $\prod_{k=1}^{n} s_{i_k}(s_{i_k} = i_k)$

Yu and Hu[5] exactly solved the generalized Smoluchovski equation with the rate kernel $K(i,j,k) = s_is_js_k$ $(s_k = k)$ in the deterministic aggregation. They found critical phenomena of the sol–gel transition and defined the gel point and scaling relation with scaling exponents. We generalize the stochastic kinetic equation of the two-polymer to the single n-polymer stochastic aggregation. The stochastic generalized Smoluchovski equation reads

$$\frac{\mathrm{d}A_m(t)}{\mathrm{d}t} = \frac{p}{n!}\sum_{i_1+i_2+\cdots+i_n=m}\prod_{k=1}^{n} s_{i_k}A_{i_k}(t) - \frac{s_mA_m(t)}{(n-1)!}\sum_{i_1,i_2,\cdots,i_{n-1}}\prod_{k=1}^{n} s_{i_k}A_{i_k}(t),$$

$$\frac{\mathrm{d}P_m(t)}{\mathrm{d}t} = \frac{q}{n!}\sum_{i_1+i_2+\cdots+i_n=m}\prod_{k=1}^{n} s_{i_k}A_{i_k}(t). \qquad (5)$$

The mono-disperse initial condition is

$$A_m(0) = \delta_{m,1}, \quad P_m(0) = 0,$$

where $s_m = m$. By introducing the following generating function

$$g(z(t),t) = \sum_{m\geq 1} m\mathrm{e}^{mz(t)}A_m(t)$$

Eq.(5) can be reduced to

$$\frac{\partial g(z,t)}{\partial t} = \frac{1}{(n-1)!}[pg^{n-1}(z,t) - M^{n-1}(t)]\frac{\partial g(z,t)}{\partial z}. \qquad (6)$$

By rewriting partial derivatives as Jacobians

$$\frac{\partial g}{\partial t} = \frac{\partial(g,z)}{\partial(t,z)}, \quad \frac{\partial g}{\partial z} = \frac{\partial(g,t)}{\partial(z,t)},$$

and substituting these Jacobians into Eq.(6) and using the cyclic-relation $\frac{\partial z}{\partial t} = \frac{\partial(z,g)}{\partial(t,g)}$, we have

$$\frac{\partial z}{\partial t} = \frac{1}{(n-1)!}(-pg^{n-1} + M^{n-1}), \qquad (7)$$

where $M(t) = \sum_{m\geq 1} mA_m(t)$. Thus, for the mono-disperse initial conditions, we obtain

$$z(g,t) = \frac{1}{(n-1)!}\left[\int_0^t \mathrm{d}t' M^{n-1}(t') - pg^{n-1}t + (n-1)!\ln g\right]. \qquad (8)$$

Letting z=0 and $g(0,t) = M(t)$, Eq.(8) gives

$$z(M(t),t) = \frac{1}{(n-1)!}\Big[\int_0^t \mathrm{d}t' M^{n-1}(t') - pM(t)^{n-1}t + (n-1)!\ln M(t)\Big]. \quad (8b)$$

Then differentiating Eq.(8b) leads to

$$qM^n = (n-1)[pM^{n-1}t - (n-2)!]\frac{\mathrm{d}M}{\mathrm{d}t}. \quad (9)$$

Hence, Eq.(9) can be solved as

$$t = (n-2)!\Big[M^{-(n-1)} - M^{\frac{(n-1)p}{q}}\Big]. \quad (10)$$

Summing the first equation of Eq.(5) leads to

$$M_0 = \frac{M}{n}\Big(n-1+q+(n-1)pM^{\frac{n-1}{q}}\Big), \quad (11)$$

where $M_0 = \sum_{m\geq 1} A_m(t)$. Thus, the mass densities and number of active clusters decay similarly in the long-time limit

$$M \sim t^{-\frac{1}{n-1}}, \quad M_0 \sim \frac{(n-1)+q}{n}t^{-\frac{1}{n-1}}. \quad (12)$$

To obtain the mass distribution of active clusters, Eq.(8) can be rewritten as

$$\exp\Big[z - \frac{1}{(n-1)!}\int_0^t \mathrm{d}t' M^{n-1}(t')\Big] = g\mathrm{e}^{-\frac{p}{(n-2)!}tg^{n-1}}. \quad (13)$$

Let $u = \frac{1}{(n-2)!}tg^{n-1}$,

$$u\mathrm{e}^{-u} = \frac{pt}{(n-2)!}\exp\Big[(n-1)z - \frac{1}{(n-2)!}\int_0^t \mathrm{d}t' M^{n-1}(t')\Big]. \quad (14)$$

The solution for the equation $u\mathrm{e}^{-u} = w$ can be expressed in terms of the series $u = \sum_{k\geq 1}\frac{k^{k-1}}{k!}w^k$. Applying this to Eq.(14) yields:

$$g^{n-1} = (n-2)!\sum_{m\geq 1}\frac{\Big[\frac{mpt}{(n-1)!}\Big]^{m-1}}{m!}\mathrm{e}^{(n-1)mz} \times \exp\Big[-\frac{m}{(n-2)!}\int_0^t \mathrm{d}t' M^{n-1}(t')\Big]. \quad (15)$$

Therefore, the mass distribution of active clusters is given by

$$A_m(t) = \Big\{\frac{m^{m-n}}{m!}\Big[\frac{pt}{(n-2)!}\Big]^{m-1}\Big\}^{\frac{1}{n-1}} \times \exp\Big[-\frac{m}{(n-1)!}\int_0^t \mathrm{d}t' M^{n-1}(t')\Big], \quad (16)$$

where $m = (n-1)q'+1, q' = 0,1,2\cdots, q' \in Z$, as $m \neq (n-1)q'+1, A_m(t) = 0$.

Using Eq.(10), the mass distribution of active clusters can be again expressed as the following in terms of the variable $\nu = M^{\frac{n-1}{q}}$:

$$A_m(\nu) = \Big[\frac{\nu^q m^{m-n}[p(1-\nu)]^{m-1}}{m!}\Big]^{\frac{1}{n-1}}\mathrm{e}^{-\frac{mp}{n-1}(1-\nu)}. \quad (17)$$

In the large-mass limit $m \to \infty$, using the Stirling approximate formula $n! \approx n^n\mathrm{e}^{-n}\sqrt{2\pi n}$, the leading behaviour of this mass distribution is given by

$$A_m(\nu) \cong \Big[\frac{\nu^q}{\sqrt{2\pi p^2(1-\nu)^2}}\Big]^{\frac{1}{n-1}} m^{-\frac{2n+1}{2(n-1)}}\mathrm{e}^{-\frac{m}{n-1}\Lambda(\nu,q)}, \quad (18)$$

where $\Lambda(\nu,q) = q\nu - [q+\ln(1-q)] - [\nu+\ln(1-\nu)]$. As $m \ll \Lambda^{-1}(\nu,q)$,

$$A_m(\nu) \sim m^{-\frac{2n+1}{2(n-1)}}t^{-\frac{1}{n-1}}. \quad (19)$$

Using Eq.(16), the evolution equation for $P_m(t)$ can be simplified to

$$\frac{\mathrm{d}P_m(t)}{\mathrm{d}t} = \frac{q}{p}\Big[\frac{\mathrm{d}A_m(t)}{\mathrm{d}t} + \frac{mA_m(t)}{(n-1)!}M^{n-1}\Big]. \quad (20)$$

Again, we present the mass distribution of passive clusters in terms of $\nu = M^{\frac{n-1}{q}}$:

$$P_m(\nu) = \frac{q}{p}\Big[A_m(\nu) + \int_\nu^1 \frac{q+p\nu'}{(n-1)\nu'}mA_m(\nu')\mathrm{d}\nu'\Big]. \quad (21)$$

Substituting Eq.(17) into Eq.(21) gives

$$P_m(\nu) = \frac{q}{p}\Big\{A_m(\nu) + \int_\nu^1 \frac{q+px}{(n-1)}x^{-1+\frac{q}{n-1}} \times \Big\{\frac{m^{m-n}}{m!}[p(1-x)]^{m-1}\Big\}^{\frac{1}{n-1}}\mathrm{e}^{-\frac{mp(1-x)}{(n-1)}}\mathrm{d}x\Big\}. \quad (22)$$

Introducing the variable $y = mx$, keeping only the leading terms and performing the integration by simplification yields the mass distribution of passive clus-

ters in the large-mass limit $m \to \infty$

$$P_m(\infty) \cong \frac{\left(\frac{q}{(n-1)}\right)^{p+1} \Gamma\left(\frac{q}{n-1}\right)}{[\sqrt{2\pi p^2}]^{\frac{1}{n-1}}} \times m^{-q\frac{2n+1}{2(n-1)}} \mathrm{e}^{-\frac{m}{n-1}\Lambda(q)}, \quad (23)$$

where $\Lambda(q) = -[q + \ln(1-q)]$. In the limit $t \to \infty$ and $m \to \infty$, the scaling variable $\xi = \frac{mq\nu}{n-1}$ remains finite, and the mass distributions of active and passive clusters can be respectively recast into scaling-like forms:

$$A_m(t) \sim \mathrm{e}^{-\frac{m}{n-1}\Lambda(q)} m^{-\frac{2n+1}{2(n-1)}} t^{-\frac{1}{n-1}} F(\xi),$$

$$P_m(t) \sim P_m(\infty) \frac{\Gamma\left(\frac{q}{n-1}, \xi\right)}{\Gamma\left(\frac{q}{n-1}\right)}, \quad F(\xi) = \mathrm{e}^{-\xi}. \quad (24)$$

We directly use the generating function to derive the mass distribution of the active and passive clusters in the procedure of n-polymer stochastic aggregation. The asymptotic behaviours of the large-mass are similar to that of the two-polymer,[12] but scaling exponents and magnitude are different. The power law $P_m \sim m^{-\gamma}$ with $\gamma = q + \frac{2n+1}{2(n-1)}$ and $A_m \sim m^{-\gamma}$ with $\gamma = \frac{2n+1}{2(n-1)}$ is found in the limit $m \to \infty$. It is also found that gelation is suppressed for the rate kernel $\prod_{k=1}^{n} s_{i_k} (s_{i_k} = i_k)$, which is different from the deterministic aggregation. The gel phase is inhibited because of the form of the passive clusters.

3. Exact solution of the kinetic equation with rate kernel $\sum_{k=1}^{n} s_{i_k} (s_{i_k} = i_k)$

The deterministic aggregation kinetic equation with the rate kernel $\sum_{k=1}^{n} s_{i_k} (s_{i_k} = i_k)$ has been exactly solved and the long-time asymptotic behaviour of the mass distribution has been discussed.[9] The stochastic generalized Smoluchovski equation with the same rate kernel reads

$$\frac{\mathrm{d}A_m(t)}{\mathrm{d}t} = \frac{1}{n!} p \sum_{i_1+i_2+\cdots+i_n=m} \left(\sum_{k=1}^{n} s_{i_k}\right) \prod_{k=1}^{n} A_{i_k} - \frac{A_m}{(n-1)!} \sum_{i_1,i_2,\cdots,i_n} \left(\sum_{k=1}^{n} s_{i_k} + S_m\right) \prod_{k=1}^{n} A_{i_k},$$

$$\frac{\mathrm{d}P_m}{\mathrm{d}t} = \frac{1}{n!} q \sum_{i_1+i_2+\cdots+i_n=m} \left(\sum_{k=1}^{n} s_{i_k}\right) \prod_{k=1}^{n} A_{i_k}. \quad (25)$$

The mono-disperse initial condition is

$$A_m(0) = \delta_{m,1}, \quad P_m(0) = 0. \quad (26)$$

The overall mass of the system is

$$\sum_{m\geq 1} m[A_m(t) + P_m(t)] = 1. \quad (27)$$

Considering $S_m = m$, the first of Eqs.(25) is rewritten as

$$\frac{\mathrm{d}A_m}{\mathrm{d}t} = \frac{p}{n!} m \sum_{i_1+i_2+\cdots+i_n=m} \prod_{k=1}^{n} A_{i_k} - \frac{A_m}{(n-1)!}(mM_0^{n-1} + (n-1)MM_0^{n-1}). \quad (28)$$

Performing the following transformation,

$$\mathcal{A}_m(t) = A_m(t) \exp\left(\frac{m}{(n-1)!} \int_0^t \mathrm{d}t' M_0^{n-1} + (n-1) \int_0^t \mathrm{d}t' M M_0^{n-2}\right), \quad (29)$$

Eq.(28) can be simplified to

$$\frac{\mathrm{d}\mathcal{A}_m}{\mathrm{d}\tau} = \frac{1}{n} m \sum_{i_1+i_2+\cdots+i_n=m} \prod_{k}^{n} \mathcal{A}_k, \quad (30)$$

where

$$\tau = \frac{p}{(n-1)!} \int_0^t \mathrm{d}t'' \exp\left(-\frac{n-1}{(n-2)!} \int_0^t \mathrm{d}t' M M_0^{n-2}\right).$$

Introducing the generating function $g(z(t), t)$,

$$g(z, \tau) = \sum_{m\geq 1} \mathrm{e}^{mz} \mathcal{A}_m(\tau), \quad (31)$$

we obtain the following equation

$$\frac{\partial g(z,\tau)}{\partial \tau} = g^{n-1}(z,\tau) \frac{\partial g(z,\tau)}{\partial z}. \quad (32)$$

Rewriting partial derivatives as Jacobians and using the cyclic relation $\frac{\partial z}{\partial t} = \frac{\partial(z,g)}{\partial(t,g)}$ and initial condition Eq.(26), we derive

$$z = -g^{n-1}\tau + \ln g. \quad (33)$$

From Eq.(28),we give

$$\frac{\mathrm{d}M_0}{\mathrm{d}t} = -\frac{(n-1)+q}{(n-1)!} M M_0^{n-1}. \quad (34)$$

From Eq.(29), we perform the transform

$$\sum_{m\geq 1}\mathcal{A}_m\exp\Big(-\frac{m}{(n-1)!}\int_0^t \mathrm{d}t' M_0^{n-1}\Big)$$
$$=\sum_{m\geq 1}A_m\exp\Big(-\frac{1}{(n-2)!}\int_0^t \mathrm{d}t' M M_0^{n-2}\Big). \quad (35)$$

Introducing the variable $z=-\frac{1}{(n-1)!}\int_0^t \mathrm{d}t' M_0^{n-1}$ and using Eqs.(34) and (31), thus we obtain

$$\begin{aligned} g(z,\tau)&=\sum_{m\geq 1}\mathrm{e}^{mz}\mathcal{A}_m(\tau)\\ &=\sum_{m\geq 1}A_m\exp\Big(\frac{1}{(n-2)!}\int_0^t \mathrm{d}t' M M_0^{n-2}\Big)\\ &=M_0^{\frac{q}{(n-1)+q}}. \end{aligned} \quad (36)$$

Differentiating Eq.(33) to M_0, we can solve

$$\tau=\frac{p}{(n-1)}\Big(1-M_0^{\frac{1}{(n-1)+q}}\Big)M_0^{-\frac{(n-1)q}{(n-1)+q}}. \quad (37)$$

From Eq.(33), we perform the transform

$$g^{n-1}\mathrm{e}^{-(n-1)g^{(n-1)}\tau}=\mathrm{e}^{(n-1)z}. \quad (38)$$

Eq.(38) can be expressed as the following in terms of the variable

$$u=(n-1)g^{(n-1)}\tau, \quad (39)$$
$$u\mathrm{e}^{-u}=(n-1)\tau\mathrm{e}^{(n-1)z}. \quad (40)$$

Using a similar solving method, we obtain

$$\mathcal{A}_m(\tau)=\Big\{\frac{[(n-1)m]^{m-1}}{m!}\Big\}^{\frac{1}{n-1}}\tau^{\frac{m-1}{n-1}}, \quad (41)$$

where $m=(n-1)q'+1$, $q'=1,2,\cdots$. $q'\in Z$, as $m\neq(n-1)q'+1, A_m(t)=0$. Letting $\nu=M_0^{\frac{1}{(n-1)+q}}$ and applying Eqs.(29) and (37) to Eq.(41), the mass distribution of active clusters is given by

$$A_m(\nu)=\Big\{\frac{[(p(1-\nu)]^{m-1}m^{m-1}}{m!}\Big\}^{\frac{1}{n-1}}\nu^{\frac{n-1+q}{n-1}}\mathrm{e}^{-\frac{mp(1-\nu)}{n-1}}. \quad (42)$$

In the large-mass limit $m\to\infty$, the leading behaviour of this mass distribution is obtained

$$A_m(\nu)\cong\frac{\nu^{1+\frac{q}{n-1}}}{[\sqrt{2\pi p^2(1-\nu)^2}]^{\frac{1}{n-1}}}m^{-\frac{3}{2(n-1)}}\mathrm{e}^{-\frac{m}{n-1}\Lambda(\nu,q)}. \quad (43)$$

Similar to the method of solving the rate kernel $\prod_{k=1}^{n}s_{i_k}(s_{i_k}=i_k)$, we present the mass distribution of passive clusters in terms of $\nu=M^{\frac{n-1}{q}}$:

$$\begin{aligned} P_m(\nu)=&\frac{q}{p}\Big\{A_m(\nu)+m\int_\nu^1 \mathrm{d}x x^{-1+\frac{q}{n-1}}\\ &\times\Big\{\frac{m^{m-n}[p(1-x)]^{m-1}}{m!}\Big\}^{\frac{1}{n-1}}\\ &\times\mathrm{e}^{-\frac{mp(1-x)}{n-1}}\mathrm{d}x\Big\}. \end{aligned} \quad (44)$$

Also, we introduce the variable $y=mx$ and keep only the leading terms. Performing the integration by rearrangement yields the mass distribution of passive clusters in the large-mass limit $m\to\infty$:

$$\begin{aligned} P_m(\infty)\cong&\Big(\frac{q}{n-1}\Big)^{(n-1)(p+1)}\Gamma\Big(1+\frac{q}{n-1}\Big)\\ &\times[2\pi p^2]^{-\frac{1}{2(n-1)}}m^{-q-\frac{3}{2(n-1)}}\mathrm{e}^{-\frac{m}{n-1}\Lambda(q)}. \end{aligned} \quad (45)$$

Similarly, in the limit case of $t\to\infty$, $m\to\infty$, the scaling variable $\xi=\frac{mq\nu}{n-1}$ remains finite. The mass distributions of active and passive clusters respectively have the following scaling-like universal forms

$$A_m(t)\sim\mathrm{e}^{-\frac{m}{n-1}\Lambda(q)}m^{-\frac{3}{2(n-1)}}t^{-\frac{1}{n-1}}F(\xi),$$

$$P_m(t)\sim P_m(\infty)\frac{\Gamma\Big(1+\frac{q}{n-1},\xi\Big)}{\Gamma\Big(1+\frac{q}{n-1}\Big)},\quad F(\xi)=\mathrm{e}^{-\xi}. \quad (46)$$

The asymptotic behaviours of the mass distributions of active and passive clusters respectively are similar to the rate kernel $\prod_{k=1}^{n}s_{i_k}(s_{i_k}=i_k)$ under the first approximation, but the scaling exponents are different. This shows that the clustering processes of active and passive clusters with the rate kernels $\prod_{k=1}^{n}s_{i_k}(s_{i_k}=i_k)$ and $\sum_{k=1}^{n}s_{i_k}(s_{i_k}=i_k)$ in n-polymer stochastic aggregation evolve in the same form in the limit $t\to\infty$, $m\to\infty$.

4. Conclusions

In this paper, we have investigated the single n-polymer stochastic aggregation. We have solved the analytical expression of the mass distributions of active and passive clusters for rate kernels $\prod_{k=1}^{n} s_{i_k}(s_{i_k} = i_k)$ and $\sum_{k=1}^{n} s_{i_k}(s_{i_k} = i_k)$ and we have discussed their asymptotic behaviours. Although these results indicate that the asymptotic behaviours of the large-mass are similar to those of the two-polymer, the scaling exponents and magnitudes which describe speed rate in the aggregation process are different. The scaling-like forms of active and passive clusters respectively are similar for rate kernels $\prod_{k=1}^{n} s_{i_k}(s_{i_k} = i_k)$ and $\sum_{k=1}^{n} s_{i_k}(s_{i_k} = i_k)$ under the first approximation, except for the difference of their scaling exponents. As n=2, we also can derive the result of the two-polymer stochastic aggregation, which is in agreement with Krapivsky *et al.* [12] We again confirm that the critical phenomena of sol–gel transition cannot occur because the gel phase is inhibited, which is apparently different from the n-polymer deterministic aggregation.

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