Vol 11 No 7, July 2002
1009-1963/2002/11(07)/0678-06

Chinese Physics

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# A study of a main-road cellular automata traffic flow model*

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(Received 6 July 2001; revised manuscript received 11 March 2002)

A main-road cellular automata traffic flow model on two dimensions is presented based on the Biham–Middleton–Levine traffic model. Its evolution equations are given and the self-organization and organization cooperation phenomena in this model are also studied by using computer simulation.

**Keywords:** main-road cellular automata traffic flow model, self-organization and organization cooperation,computer simulation

**PACC:** 0550

## 1. Introduction

In recent years, much attention has been paid to traffic problems, and one of the main methods of studying these in physics is cellular automata (CA). This method has the advantage of being simple and easy to implement on computers and can be readily generalized to study different effects on roads.[1] Many models have been presented to simulate traffic flow both in one dimension and two dimensions based on CA. In the one-dimensional case, thorough studies of spontaneous and causable traffic jams have been made by many CA traffic flow models according to the attained radical figures.[1−10] Mean-field theories are always adopted to provide a basic understanding of the phenomena in dynamical systems.[2,5,6] Recently Wang *et al*[4] have studied the CA highway traffic flow model using a statistical mechanical approach. We have presented the relation between the correlation function and the creation, disappearance, braking probabilities of vehicles in a random traffic flow CA model based on their approach.[8] Moreover, Xue *et al*[9] proposed an improved single-lane traffic CA model by considering the relative motion of vehicles and the relation of deceleration probability to the density of vehicles, based upon the numerical simulation model.[9] Among all the two-dimensional models, the simplest model for city traffic is the so-called Biham–Middleton–Levine (BML) traffic model.[11] This model shows that a phase transition from a freely moving phase to a perfectly jammed phase occurs with increasing vehicle density. These two phases are the two results of self-organization. The evolution of the system is the competition of these two modes. Recently, many control strategies have been studied to keep the system in as freely moving a mode as possible. For example, Cuesta *et al*[12] proposed a model in which vehicles can change their directions on a square lattice. Nagatani *et al*[13] simulated the crossing status when a north-bound main road exists. On the other hand, Chen and co-workers[14,15] studied the effect of the braking probability. The evolution equations and the velocity expression of a two-dimensional CA traffic model in which the turning probability exists were presented and simulated by Lü *et al.*[16] In a word, the control of a traffic system is the cooperation between the self-organization and the organization of the system.[17] In the above-mentioned models, some are only for the study of traffic net on a pure square lattice, and some are for the study of traffic phenomena when only a certain factor exists in the road. In this paper, we present a main-road model in which a turning probability exists at the crossing and a braking probability works on the road and the evolution equations for the model. Then we obtain the fundamental diagrams of traffic flow by using computer simulation. They show the self-organization and organization behaviour of the system. Our aim is to obtain different effects of external factors to ameliorate the spontaneous and causable traffic jams through the

*Project supported by the National Natural Science Foundation of China (Grant No 10062001), and the Natural Science Foundation of Guangxi Zhuang Autonomous Region, China (Grant No 0007017) and the Foundation for University Key Teacher by the Ministry of Education of China.

study of the radical figures that come from the gradual application of the braking and turning probabilities in the models. It turns out that self-organization plays a main role in high density as well as in low density without the external organizational action, while the cooperation between the self-organization and the organization acts chiefly in low and middle densities with the existence of external factors.

# 2.Model

Our system consists of a north-bound (positive $y$-direction) main road and $k$ east-bound (positive $x$-direction) branch lanes which are equispaced mutually parallel (see Fig.1) with periodic boundary conditions. The length of the main road is denoted by $L_y$, and that of each branch lane is denoted by $L_x$. The velocity of each vehicle on the roads is an integer which is 0 or 1. Thus, we only consider the influence of the nearest neighbours, as shown in Fig.1. Each site of the main road (branch lanes) outside the crossings can be empty or occupied by only $\uparrow$ ($\rightarrow$). Each cell on the crossing can be in one of the three allowed states: either empty or occupied by $\rightarrow$ or $\uparrow$. Braking is permitted on both main road and branch lanes, and turnings are permitted at the crossing. The traffic lights, which are placed on each crossing, change synchronously. The system evolves according to the following rules:

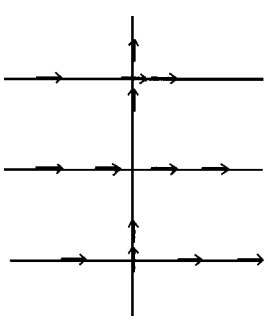


**Fig.1.**

### 2.1. Green lights time step (green lights on main road and red lights on branch lanes)

(1) The evolution of $\uparrow$ on the main road and $\rightarrow$ on the branch lanes except those $\rightarrow$ which are going to move into crossings: if a neighbouring site in the direction of motion of a vehicle is occupied, the vehicle cannot move; however, if it is empty, the vehicle moves to the site or brakes with probability $P_{by}$ (for $\uparrow$) and $P_{bx}$ (for $\rightarrow$).

(2) The evolution of $\rightarrow$ at the crossings: if the neighbouring site in the direction of motion of $\rightarrow$ is occupied, $\rightarrow$ cannot move forward, but if no vehicle exists on the left site of it, it can turn to the positive $y$-direction with probability $P_{txy}$ and move one site forward.

(3) The rule of synchronicity: in front of $\uparrow$, if there is $\rightarrow$ which can move away at this time step (that is, $\rightarrow$ can turn and move or it must move forward without braking if there is no vehicle on the next site), $\uparrow$ can move forward one site synchronously.

### 2.2. Red lights time step (red lights on main road and green lights on branch-lanes)

(1) The evolution of $\rightarrow$ on branch lanes and $\uparrow$ on the main road, except those $\uparrow$ which are going to move into the crossing, is the same as the green lights time step (1).

(2) The evolution of $\uparrow$ at the crossing: if the neighbouring site in the direction of motion of $\uparrow$ is occupied, it cannot move forward, but if no vehicle exists on the right site of it, it can turn to the positive $x$-direction with probability $P_{tyx}$ and move one site forward.

Compared with the BML model, this model makes more practical considerations:

(1) The model consists of a main road and several branch lanes.

(2) Traffic lights have effects only at the intersection of the main road and branch lanes: when the green light is on, the vehicles on branch lanes cannot move into crossings; when the red light is on, the vehicles on the main road cannot move into crossings; the vehicles which do not reach crossing can move regularly.

(3) When there are traffic jams, the vehicles at crossings can turn to other directions to keep moving as soon as possible, avoiding the long-time jamming that affects the interaction flow.

(4) The rule of synchronicity on the main road takes into account the interaction of two nearest neighbours. It shows fully the priority of the main road and guarantees the increase of velocity and flow to the highest degree.

(5) This model mimics the reaction of drivers in accidents by using the braking probability.

Defining $H_\alpha$=int$\left(\frac{L_x}{2}\right)+1$ and $R_{N_{\rm row}}=(N_{\rm row}-1)\times M_{\rm space}+1$ ($N_{\rm row}$=1,2, $\cdots$ $k$, $M_{\rm space}$ is the number of lattices between every two branch lanes adding 1) respectively as the main road (indicated by Rank) and branch lanes (indicated by Line). This model and its evolution rules can be expressed by a series of Boolean variables.

(i) $n(i,j,t)$ $(i=1,2,\cdots,L_y, j=H_\alpha)$, $e(i,j,t)$ $(i=R_{N_{\rm row}}, j=1,2,\cdots,L_x)$, $s(i,j,t)$ $(i=1,2,\cdots, L_y(i=R_{N_{\rm row}}, j=1,2,\cdots,L_x, i\neq R_{N_{\rm row}}, j=H_\alpha))$ indicate the state when the lattice $(i,j)$ is occupied by $\uparrow$ ($\uparrow$ exists only in the main road), $\rightarrow$ or a vehicle at the time of $t$ separately.

(ii) $\varphi_y(i,j,t)$ $(i=1,2,\cdots,L_y, j=H_\alpha)$ and $\varphi_x(i,j,t)$ $(i=R_{N_{\rm row}}, j=1,2,\cdots,L_x)$ indicate respectively the Boolean variables of the braking probabilities of $\uparrow$ and $\rightarrow$ when the lattice $(i,j)$ is occupied by $\uparrow$ or $\rightarrow$ at time $t$. The probabilities of $\varphi_y(i,j,t)$=1 and $\varphi_x(i,j,t)$=1 are $P_{by}$ and $P_{bx}$. Then the probabilities of $\varphi_y(i,j,t)$=0 and $\varphi_x(i,j,t)$=0 are 1–$P_{by}$, 1–$P_{bx}$.

(iii) $T_{yx}(i,j,t)$ $(i=R_{N_{\rm row}}, j=H_\alpha)$ and $T_{xy}(i,j,t)$ $(i=R_{N_{\rm row}}, j=H_\alpha)$ indicate the Boolean variables of the probabilities of $\uparrow$ turning to $\rightarrow$ and $\rightarrow$ turning to $\uparrow$ on the lattice $(i,j)$ at time $t$. The probabilities of $T_{yx}(i,j,t)$=1 and $T_{xy}(i,j,t)$=1 (turn) are $P_{tyx}$ and $P_{txy}$, while the probabilities of $T_{xy}(i,j,t)$=0 and $T_{xy}(i,j,t)$=0 (no turn) are 1–$P_{tyx}$ and 1–$P_{txy}$.

(iv) $\gamma(i,j)$ $(i=1,2,\text{- - -}\ L_y, j=H_\alpha)$ indicates the Boolean variable of the crossing (site of traffic lights). When the lattice $(i,\ j)$ is the crossing, $\gamma(i,j)$=1, otherwise $\gamma(i,j)$=0.

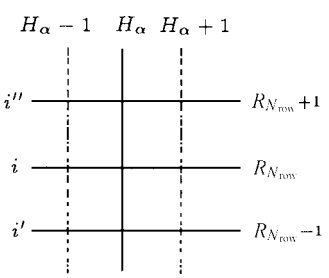



**Fig.2.**

The exclusion relations of the Boolean variables exist in the state of vehicle occupation:

$$e(i,j,t)\cdot n(i,j,t)=0,$$

$$s(i,j,t)=e(i,j,t)+n(i,j,t).$$

According to the definition of Boolean variables, we give the evolution equations for this model at any time as below.

(i) Green lights time step: $(t+1)$ mod 2=1.

Evolution of $\rightarrow$: $i=R_{N_{\rm row}}$

$$\begin{aligned}e(i,j,t+1)=&e(i,j,t)e(i,j+1,t)\\&+e(i,j,t)\overline{e}(i,j+1,t)\varphi_x(i,j,t)\\&+e(i,j-1,t)\overline{e}(i,j,t)\overline{\varphi}_x(i,j-1,t),\end{aligned}$$

where $j\neq H_\alpha-1, H_\alpha, H_\alpha+1$;

$$\begin{aligned}&e(i,H_\alpha-1,t+1)\\=&e(i,H_\alpha-2,t)\overline{e}(i,H_\alpha-1,t)\\&\times\overline{\varphi}_x(i,H_\alpha-2,t)+e(i,H_\alpha-1,t),\end{aligned}$$

$$\begin{aligned}&e(i,H_\alpha,t+1)\\=&e(i,H_\alpha,t)e(i,H_\alpha+1,t)s(i+1,H_\alpha,t)\\&+e(i,H_\alpha,t)e(i,H_\alpha+1,t)\\&\times\overline{s}(i+1,H_\alpha,t)\overline{T}_{xy}(i,H_\alpha,t)\\=&e(i,H_\alpha,t)e(i,H_\alpha+1,t)\overline{T}_{xy}(i,H_\alpha,t),\end{aligned}$$

$$\begin{aligned}&e(i,H_\alpha+1,t+1)\\=&e(i,H_\alpha,t)\overline{e}(i,H_\alpha+1,t)\\&+e(i,H_\alpha+1,t)e(i,H_\alpha+2,t)\\&+e(i,H_\alpha+1,t)\overline{e}(i,H_\alpha+2,t)\varphi_x(i,H_\alpha+1,t).\end{aligned}$$

Evolution of $\uparrow$: $j=H_\alpha$,

$$\begin{aligned}&n(i,j,t+1)\\=&n(i,j,t)\overline{n}(i,j,t)\varphi_y(i,j,t)\\&+n(i,j,t)n(i+1,j,t)\\&+n(i-1,j,t)\overline{n}(i,j,t)\overline{\varphi}_y(i-1,j,t),\end{aligned}$$

where $i\neq R_{N_{\rm row}}-1, R_{N_{\rm row}}, R_{N_{\rm row}}+1$;

$$\begin{aligned}&n(i,j,t+1)\\=&n(i-1,j,t)\overline{n}(i,j,t)\overline{\varphi}_y(i-1,j,t)\overline{\gamma}(i,j,t)\\&+n(i,j,t)e(i+1,j,t)e(i+1,j+1,t)\\&\times\overline{T}_{xy}(i+1,j,t)\overline{\gamma}(i,j,t)\\&+e(i,j,t)\overline{e}(i,j+1,t)n(i-1,j,t)\gamma(i,j,t)\\&+e(i,j,t)e(i,j+1,t)n(i-1,j,t)\overline{s}(i+1,j,t)\\&\times T_{xy}(i,j,t)\gamma(i,j,t)\\&+n(i-1,j,t)\overline{s}(i,j,t)\overline{\varphi}_y(i-1,j,t)\gamma(i,j,t)\\&+n(i,j,t)\overline{s}(i+1,j,t)\varphi_y(i,j,t),\end{aligned}$$

where $i=R_{N_{\rm row}}-1, R_{N_{\rm row}}, R_{N_{\rm row}}+1$.

(ii) Red lights time step: $(t+1)$ mod 2=0.

Evolution of $\rightarrow$: $i=R_{N_{\rm row}}$,

$$\begin{aligned}&e(i,j,t+1)\\=&e(i,j,t)e(i,j+1,t)\\&+e(i,j,t)\overline{e}(i,j+1,t)\varphi_x(i,j,t)\\&+e(i,j-1,t)\overline{e}(i,j,t)\overline{\varphi}_x(i,j-1,t),\end{aligned}$$

where $j\neq H_\alpha-1, H_\alpha, H_\alpha+1$;

$$\begin{aligned}&e(i,H_\alpha-1,t)\\=&e(i,H_\alpha-2,t)\overline{e}(i,H_\alpha-1,t)\\&\times\overline{\varphi}_x(i,H_\alpha-2,t)+e(i,H_\alpha-1,t)\\&\times\overline{s}(i,H_\alpha,t)\varphi_x(i,H_\alpha-1,t)\\&+e(i,H_\alpha-1,t)s(i,H_\alpha,t),\end{aligned}$$

$$
\begin{aligned}
&e(i, H_\alpha, t+1)\\
=&e(i, H_\alpha - 1, t)\overline{s}(i, H_\alpha - 1, t)\overline{\varphi}_x(i, H_\alpha - 1, t)\\
&+ e(i, H_\alpha, t)\overline{e}(i, H_\alpha + 1, t)\varphi_x(i, H_\alpha, t)\\
&+ e(i, H_\alpha, t)e(i, H_\alpha + 1, t),
\end{aligned}
$$

$$
\begin{aligned}
&e(i, H_\alpha + 1, t+1)\\
=&e(i, H_\alpha + 1, t)\overline{e}(i, H_\alpha + 2, t)\varphi_x(i, H_\alpha + 1, t)\\
&+ e(i, H_\alpha, t)\overline{e}(i, H_\alpha + 1, t)\overline{\varphi}_x(i, H_\alpha, t)\\
&+ e(i, H_\alpha + 1, t)e(i, H_\alpha + 2, t)\\
&+ n(i, H_\alpha, t)\overline{e}(i, H_\alpha + 1, t)\\
&\times s(i+1, H_\alpha, t)T_{yx}(i, H_\alpha, t).
\end{aligned}
$$

Evolution of $\uparrow$: $j = H_\alpha$,

$$
\begin{aligned}
n(i, j, t+1) =& n(i, j, t)n(i+1, j, t)\\
&+ n(i-1, j, t)\overline{n}(i, j, t)\overline{\varphi}_y(i-1, j, t)\\
&+ n(i, j, t)\overline{n}(i+1, j, t)\varphi_y(i, j, t),
\end{aligned}
$$

where $i \neq R_{N_{\rm row}} - 1, R_{N_{\rm row}}, R_{N_{\rm row}} + 1$;

$$
\begin{aligned}
&n(i, j, t)\\
=&n(i, j, t)s(i+1, j, t)\overline{T}_{yx}(i, j, t)\gamma(i, j)\\
&+ n(i, j, t)\gamma(i+1, j) + n(i, j, t)\\
&\times \overline{s}(i+1, j, t)\overline{\gamma}(i+1, j)\varphi_y(i, j, t)\\
&+ n(i-1, j, t)\overline{n}(i, j, t)\overline{\varphi}_y(i-1, j, t)\overline{\gamma}(i, j)\\
&+ n(i, j, t)s(i+1, j, t)\overline{\gamma}(i, j),
\end{aligned}
$$

where $i = R_{N_{\rm row}} - 1, R_{N_{\rm row}}, R_{N_{\rm row}} + 1$.

The periodic boundary conditions can be expressed as:

$$
\begin{aligned}
n(L_y + 1, H_\alpha, t) &= n(1, H_\alpha, t),\\
e(i, L_x + 1, t) &= e(i, 1, t).
\end{aligned}
$$

The rule of synchronicity is

$$
\begin{aligned}
&TS(i, j, t+1)\\
=&n(i-1, j, t)[e(i, j, t)e(i, j+1, t)\overline{s}(i+1, j, t)\\
&\times T_{xy}(i, j, t) + e(i, j, t)\overline{e}(i, j+1, t)].
\end{aligned}
$$

There are limitations in the Boolean variables of this model at the crossings ($i = R_{N_{\rm row}}$, $j = H_\alpha$):

$$
\begin{aligned}
TS(i, j, t)\overline{\varphi}_y(i-1, j, t) &= 1,\\
T_{xy}(i, j, t)\varphi_y(i, j, t) &= 0,\\
T_{yx}(i, j, t)\varphi_x(i, j, t) &= 0.
\end{aligned}
$$

The density, velocity and flow of the main road and branch lanes are given by

$$
\rho_y(t) = \frac{N_y(t)}{L_y}, \quad \rho_x(t) = \frac{N_x(t)}{L_x \times k},
$$

$$
\nu_y(t) = \frac{N_{ym}(t)}{N_y(t)}, \quad \nu_x(t) = \frac{N_{xm}(t)}{N_x(t)},
$$

$$
J_y(t) = \rho_y(t)\nu_y(t), \quad J_x(t) = \rho_x(t)\nu_x(t),
$$

where $N_y(t)$, $N_x(t)$, $N_{ym}(t)$ and $N_{xm}(t)$ denote the number of the vehicles and the movable vehicles in positive $y$- and $x$-directions at time $t$, respectively. The average velocity and flow of vehicles are calculated by the following equations

$$
\nu_{x,y} = \frac{1}{t_m - t_0}\sum_{t=t_0+1}^{t_m}\nu_{x,y}(t),
$$

$$
J_{x,y} = \frac{1}{t_m - t_0}\sum_{t=t_0+1}^{t_m}J_{x,y}(t),
$$

where $t_0$ is an idling run and $t_m$ is a maximum run.

## 3. Computer simulation and results

We can simulate the traits of this model with a computer. According to the changes of various parameters of the model, we study the influences on the average velocity and flow of the traffic. In the simulation, the period $T$ of the traffic lights is defined as the sum of the red light time steps ($Rs$) and green light time steps ($Gs$), that is $T = Gs + Rs$. In this model, we take $L_y$=800, $L_x$=100, $k$=4, $P_{bx} = P_{by} = P_b$, $Gs = Rs$=200, $t_0$=60000 and $t_m$=80000. To avoid the influence of the initial random distribution of vehicles, a hundred samples are considered to obtain the average value on each kind of evolution condition. The B-spline connection is also used to link every two points in each figure.

We present here the simulation result based on the parameters listed above. When the braking and turning probabilities do not exist, the characteristics of the main road shown in Fig.3 are similar to the one-dimensional DCA model. The system is in the state of free moving phase at green light time steps when the initial density is less than 0.5, but it becomes seriously jamming with increasing initial density; i.e. the first-order phase transition takes place on the main road. When the density is above 0.5 on the main road, without the support of the turning probability at the crossing, the jammed vehicles which are forbidden to pass because of the red light cannot be completely

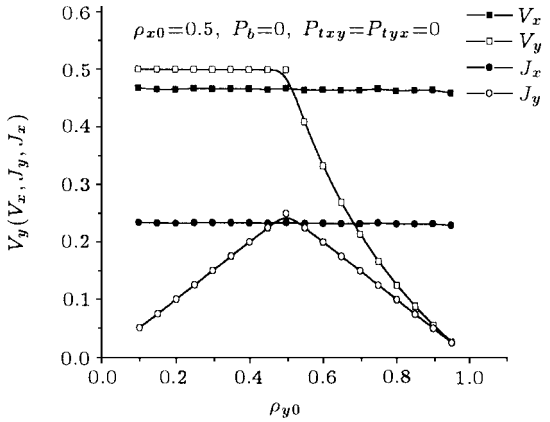



**Fig.3.** The average velocities and flows of the main road and branch lanes against the initial density of the main road for the case without braking and turning probabilities.

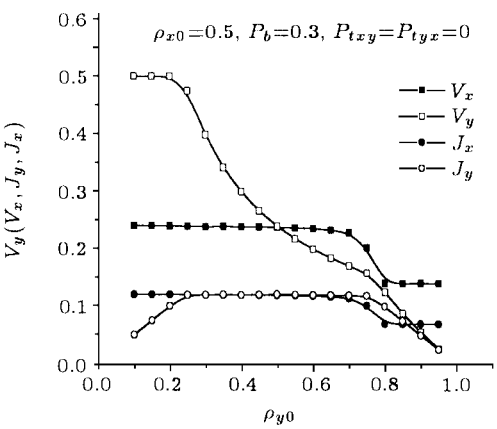



**Fig.4.** The average velocities and flows of the main road and branch lanes against the initial density of the main road for the case without turning probabilities but with braking probabilities.

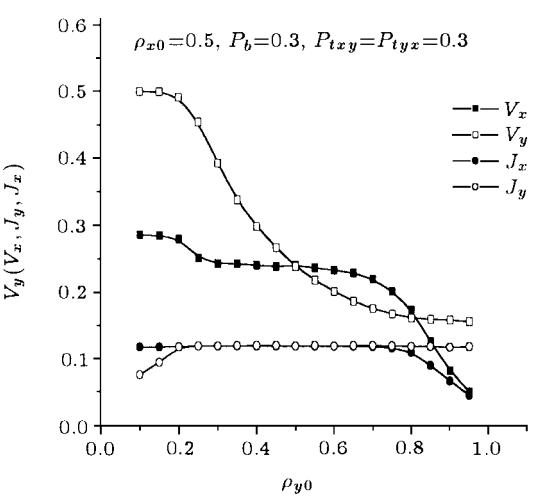



**Fig.5.** The average velocities and flows of the main road and branch lanes against the initial density of the main road for the case with braking probabilities and turning probabilities.

dispersed during the green light time step. So the speed and flow of the main road decreases and the seriousness of the traffic jam will become greater with the increase of the initial density. The formation of the steady status is the result of self-organization. When the braking probabilities appear during the course of driving, as shown in Fig.4, on the main road there appears to be an upper critical density, above which the velocity and flow on the main road are brought down more quickly and those of the branch lanes are also lessened. On the main road the phase transition point for the freely moving phase changing into jammed phase moves forward (near $\rho_{y0} \approx 0.25$), indicating that the existence of the braking probability leads to the same effect as that in the increase of the initial density, but the braking creates jams more easily. However, with the increase of the initial density, the speed on the main road decreases and a platform ($\rho_{y0} \approx 0.25$–$0.75$) appears, which indicates that the flow is not related to the density. If the density of the main road is greater than the upper critical density ($\rho_{y0} \approx 0.75$), due to the high density of vehicles, the effect of $P_b$ has been lessened greatly (not many vehicles can satisfy the rules of braking). This means that the entirely self-organizing behaviour exists in Fig.4 as in Fig.3. Above this critical density, the congestion on the main road is sure to bring about the slowing of speed and flow on the branch lanes. When the turning probability is added, as shown in Fig.5, we can see that the upper critical density on the main road disappears and the velocity and flow increase. However, the superiority of the main road leads to the rise of pressures in the high-density section of branch lanes, so the velocity and flow in the branch lanes fall gradually. The above phenomena embody that the self-organization works well in the low-density section while the pressure of the traffic jam can only be lessened by organization cooperation. The organization function of the turning and braking probabilities can also be seen in Figs.6 and 7. As in Fig.6, when the turning probability is not permitted on branch lanes, the higher the turning probability, the higher the velocity on the main road. But because turning exists only on the main road instead of the branch lanes, the density of the main road falls, and then the flow does not change. In contrast, since the vehicles on the branch lanes cannot turn, the velocity on the branch lanes decreases with the increase of the turning probability. The situation is alleviated after the vehicles on the branch lanes are permitted to turn, as in Fig.7.

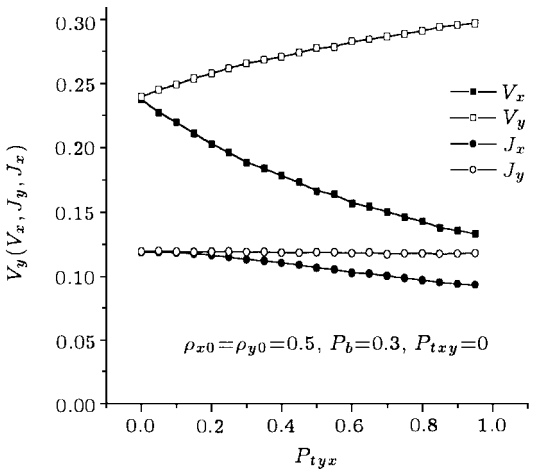



**Fig.6.** The average velocities and flows of the main road and branch lanes against the turning probabilities of the main road for the case without turning probabilities of the branch lanes but with braking probabilities.

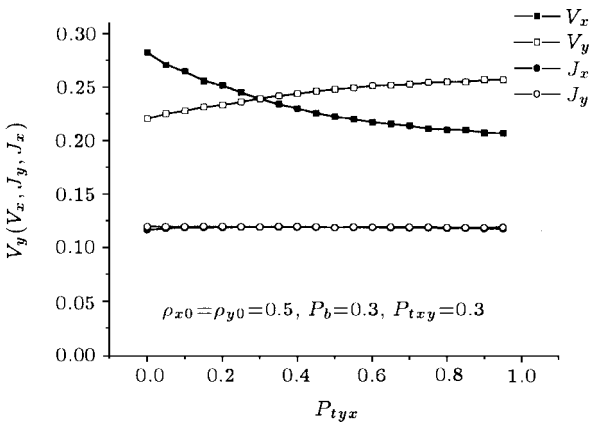



**Fig.7.** The average velocities and flows of the main road and branch lanes against the turning probabilities of the main road for the case with braking and turning probabilities of the branch lanes.

Meanwhile, the appearance of an intersection ($P_{tyx}$ =0.3) of the velocity curves shows the obvious competition of the turning probabilities between the main road and the branch lanes. When the density is higher in the main road, the high or low turning probability has less influence on the velocity and flow. Yet, the organization behaviour, like the turning probability, has a great effect as in Fig.8.

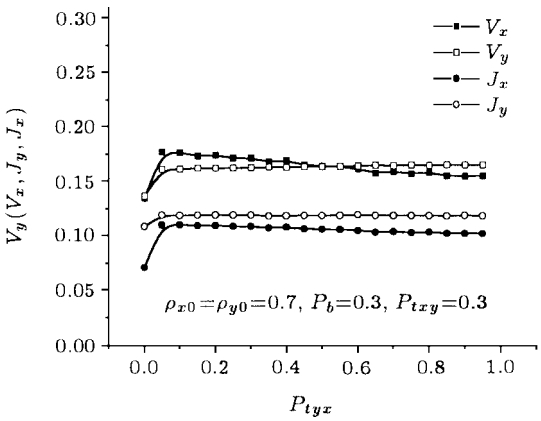



**Fig.8.** The average velocities and flows of the main road and branch lanes against the turning probabilities of the main road for the case with braking and turning probabilities of the branch lanes while the initial densities of the main road and the branch lanes increase.

## 4. Conclusion

This model has many complex phenomena to be researched, from which we can decide how to control the traffic. For example, we can study the influences of the number of branch lanes, the lengths of the main road and branch lanes, the difference of the braking probabilities between the main road and the branch lanes, the period of the traffic lights and the ratio of green lights to traffic lights, and so on. In general, we aim to find the best possible way to develop traffic flow as we can.

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