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The isolated critical value phenomenon in local–global riddling bifurcation*

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A chaotic synchronized system of two coupled skew tent maps is discussed in this paper. The locally and globally riddled basins of the chaotic synchronized attractor are studied. It is found that there is a novel phenomenon in the local–global riddling bifurcation of the attractive basin of the chaotic synchronized attractor in some specific coupling intervals. The coupling parameter corresponding to the locally riddled basin has a single value which is embedded in the coupling parameter interval corresponding to the globally riddled basin, just like a breakpoint. Also, there is no relation between this phenomenon and the form of the chaotic synchronized attractor. This phenomenon is found analytically. We also try to explain it in a physical sense. It may be that the chaotic synchronized attractor is in the critical state, as it is infinitely close to the boundary of its attractive basin. We conjecture that this isolated critical value phenomenon will be common in a system with a chaotic attractor in the critical state, in spite of the system being discrete or differential.

Keywords: skew tent map, chaotic synchronized system, riddled basins
PACC: 0545

1. Introduction

The control of chaos has commanded increasing interest in recent years,[1–7] especially chaos synchronization, due to the extensive applications.[8–15] Since chaotic synchronized systems have many important applications, numerous investigators have studied them together with some new striking phenomena, such as riddled basins and riddling bifurcations.[16–22]

Consider two identical chaotic maps $x_{j+1} = f(x_j)$ and $y_{j+1} = f(y_j)$, each of which has an asymptotically stable chaotic attractor. With one-to-one coupling, a two-dimensional map $F_\varepsilon(x, y)$ can be obtained:

$$\begin{cases} x_{j+1} = f(x_j) + \varepsilon(y_j - x_j), \\ y_{j+1} = f(y_j) + \varepsilon(x_j - y_j). \end{cases} \tag{1}$$

The chaotic synchronized attractor A is restricted to the one-dimensional invariant sub-space $x{=}y$ when the coupled system exhibits chaotic synchronization.

Many authors have studied the model and found that the attractive basin of the chaotic synchronized attractor is riddled for some coupling parameter $\varepsilon \in R$.[18–21] They have used critical curves[23,24] to construct the absorbing area of A. If the boundaries of the absorbing area are in contact with the boundaries of the attractive basin of A or some new attractors are born inside the absorbing area, the local–global riddling bifurcation will occur and the locally riddled basin of A will be globally riddled.[17,18]

In this paper a coupled system of two skew tent maps is studied. The system can exhibit chaotic synchronization and the attractive basin of the chaotic synchronized attractor shows locally and globally riddled properties in some specific coupling parameter intervals or values. In order to look for the critical values of the coupling parameter for the transition from locally to globally riddled basins, we find that the method of absorbing area cannot be applied for some specific coupling parameter interval because A is infinitely close to its attractive basin. Inside this specific coupling parameter interval, we analytically find that the coupling parameter value corresponding to the locally riddled basin is an isolated value. Kapitaniak and Maistrenko[18] have pointed out other isolated value phenomena of $p_c = p_{cc}$, where p_c expresses the critical value of riddling bifurcation[25] and p_{cc} expresses the critical value of local–global riddling bifurcation. They explained that there are new attractors born inside the absorbing area of the chaotic attractor so the attractive basin of the chaotic attractor directly becomes a globally riddled basin. But the novel phenomenon we have found is not the same as the known results. Firstly, due to the special property of the chaotic attractor we have discussed, it is an attractor

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infinitely close to its attractive basin for some specific coupling parameter. We cannot find the absorbing area of this chaotic attractor here because there is no neighbourhood to the absorbing area, which has such a strong trapping ability that points in the neighborhood are all mapped inside the area. Secondly, the coupling parameter value corresponding to the locally riddled basin is an isolated value and is embedded in the coupling parameter interval corresponding to the globally riddled basin. For this case, there are no new attractors born here.

This phenomenon is found analytically. We also try to explain it in a physical sense. We think that this novel phenomenon may take place because the chaotic attractor is infinitely close to its attractive basin. We conjecture that this isolated value phenomenon will be common in a system with a chaotic attractor in the critical state if we call the state of the chaotic attractor infinitely close to its attractive basin the critical state of the chaotic attractor, in spite of the system being discrete or differential.

2. Model

The base map is a usual skew tent map

$$f(x): x_{j+1} = \begin{cases} 2hx_j, & 0 \le x_j \le 1/2, \\ 2h(1-x_j), & 1/2 < x_j \le 1. \end{cases} \left(\frac{1}{2} < h \le 1\right) \quad (2)$$

For every $h \in (\frac{1}{2}, 1]$, there will be a chaotic attractor lying in $[2h(1-h), h]$. The numerical calculations indicate that the chaotic attractor is piecewise when $\frac{1}{2} < h < 0.71$ but it will be full of $[2h(1-h), h]$ when $0.71 \le h \le 1$ (0.71 is an approximate value). Cursory descriptions are given in Figs.1 and 2.

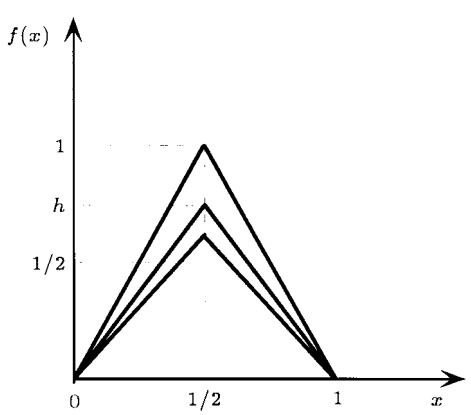


Fig.1. The skew tent map expressed by map (2).

Considering the coupled system $F_\varepsilon(x, y)$ of the basic map $f(x)$, the chaotic synchronized attractor A lies in the invariant subspace $x = y$ if $F_\varepsilon(x, y)$ exhibits chaotic synchronization. A is piecewise in the line segment $\{2h(1-h) \le x = y \le h\}$ when $\frac{1}{2} < h < 0.71$ but A will be full of this line segment when $0.71 \le h \le 1$. So the attractive basin of A will be considered not only in the area [0,1]×[0,1] but in the whole plane (x, y).

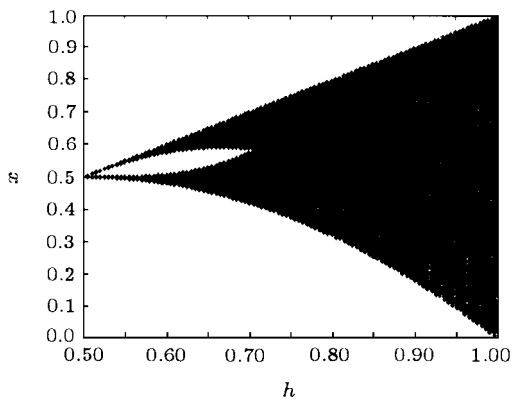


Fig.2. Bifurcation diagram of the chaotic attractor of map (2).

3. The riddled basins of coupled system $F_\varepsilon(x, y)$

It has been proven that if the transversal Lyapunov exponent of A is negative, the set attracted to A can be found.[26] It has also been proven that if some lower period orbits or fixed points embedded in A are transversally repelled, then the repelled set near A can be found.[25] So we can say the attractive basin of A is a riddled basin. In the following we look for the coupling parameter values of the riddled basin of A (refer to Refs.[19,26] for a rigorous analytical proof).

The Jacobian matrix of the coupled system $F_\varepsilon(x, y)$ in the invariant sub-space is[19,22]

$$DJ(x,y)|_{x=y} = \begin{bmatrix} u-\varepsilon & \varepsilon \\ \varepsilon & u-\varepsilon \end{bmatrix}, \quad (3)$$

where $u = \begin{cases} 2h, & 0 \le x \le 1/2, \\ -2h, & 1/2 < x \le 1. \end{cases}$

The eigenvalue corresponding to the motion in the sub-space is u and the eigenvalue corresponding to the motion transversal to the sub-space is $u - 2\varepsilon$.

Since A has absolutely continuous invariant probability in the sub-space and the invariant probability density in the sub-space is $\rho(x, y)|_{x=y}$=1, the transversal Lyapunov exponent of A can be computed explicitly:

$$\lambda_\perp = \int_{2h(1-h)}^{h} \ln|u-2\varepsilon|\rho(x)\mathrm{d}x$$
$$= \int_{2h(1-h)}^{1/2} \ln|2h-2\varepsilon|\mathrm{d}x + \int_{1/2}^{h} \ln|-2h-2\varepsilon|\mathrm{d}x$$
$$= \ln|2h-2\varepsilon|^{\frac{1}{2}-2h(1-h)}|2h+2\varepsilon|^{h-\frac{1}{2}}. \quad (4)$$

If $\lambda_\perp < 0$, the set attracted to A can be found.[24]

It is difficult to solve the inequality $\ln|2h-2\varepsilon|^{\frac{1}{2}-2h(1-h)}|2h+2\varepsilon|^{h-\frac{1}{2}}<0$. But we note that $\varepsilon_c = \pm h$ always satisfies the inequality. So the coupling parameter interval satisfying the inequality can be written as $r_1(\varepsilon)\bigcup r_2(\varepsilon)$, where $r_1(\varepsilon)$:$\{h-\Delta_1<\varepsilon<h+\Delta_2\}$, $r_2(\varepsilon)$: $\{-h-\Delta_3<\varepsilon<-h+\Delta_4\}$, Δ_1, Δ_2, Δ_3, Δ_4 can be validated by $\lambda_\perp$=0. Figure 3 shows a sketch of $\lambda_\perp < 0$ in the parameter phase (h, ε).

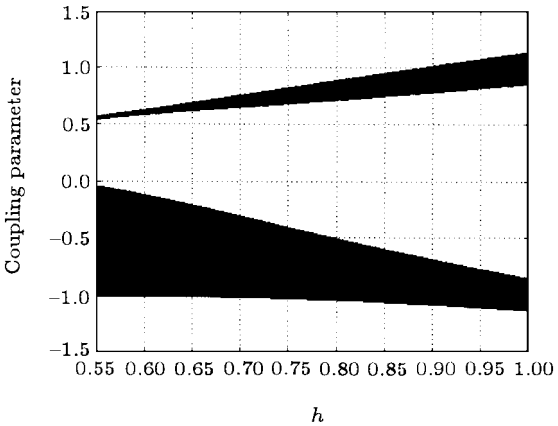


Fig.3. The coupling parameter interval of ε corresponding to different values of h when $F_\varepsilon(x, y)$ exhibits chaotic synchronization.

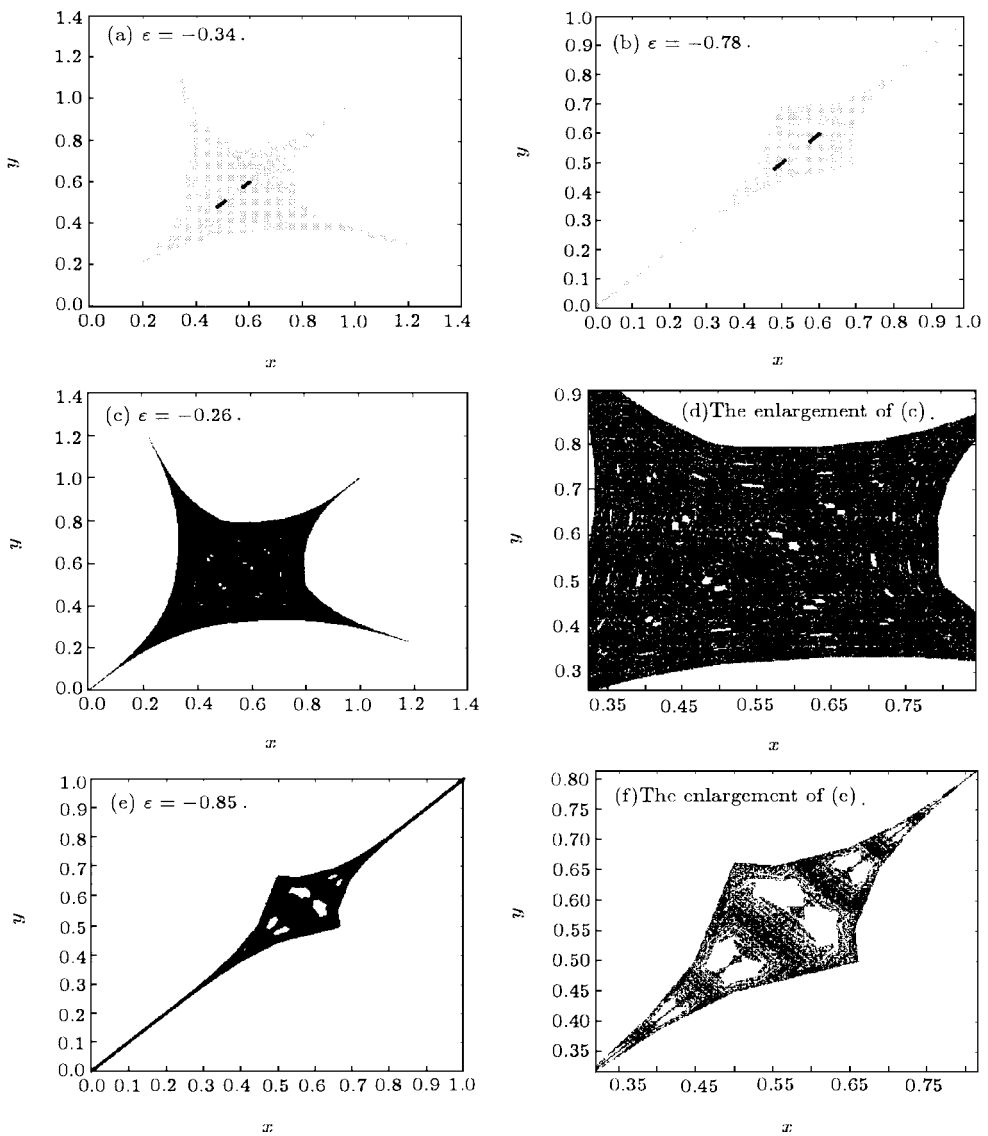


Fig.4. The attractive basin of A under different values of ε when h=0.6, where: (a) and (b) are locally riddled basins, in which the black lines denote A; (c), (d), (e) and (f) are globally riddled basins.

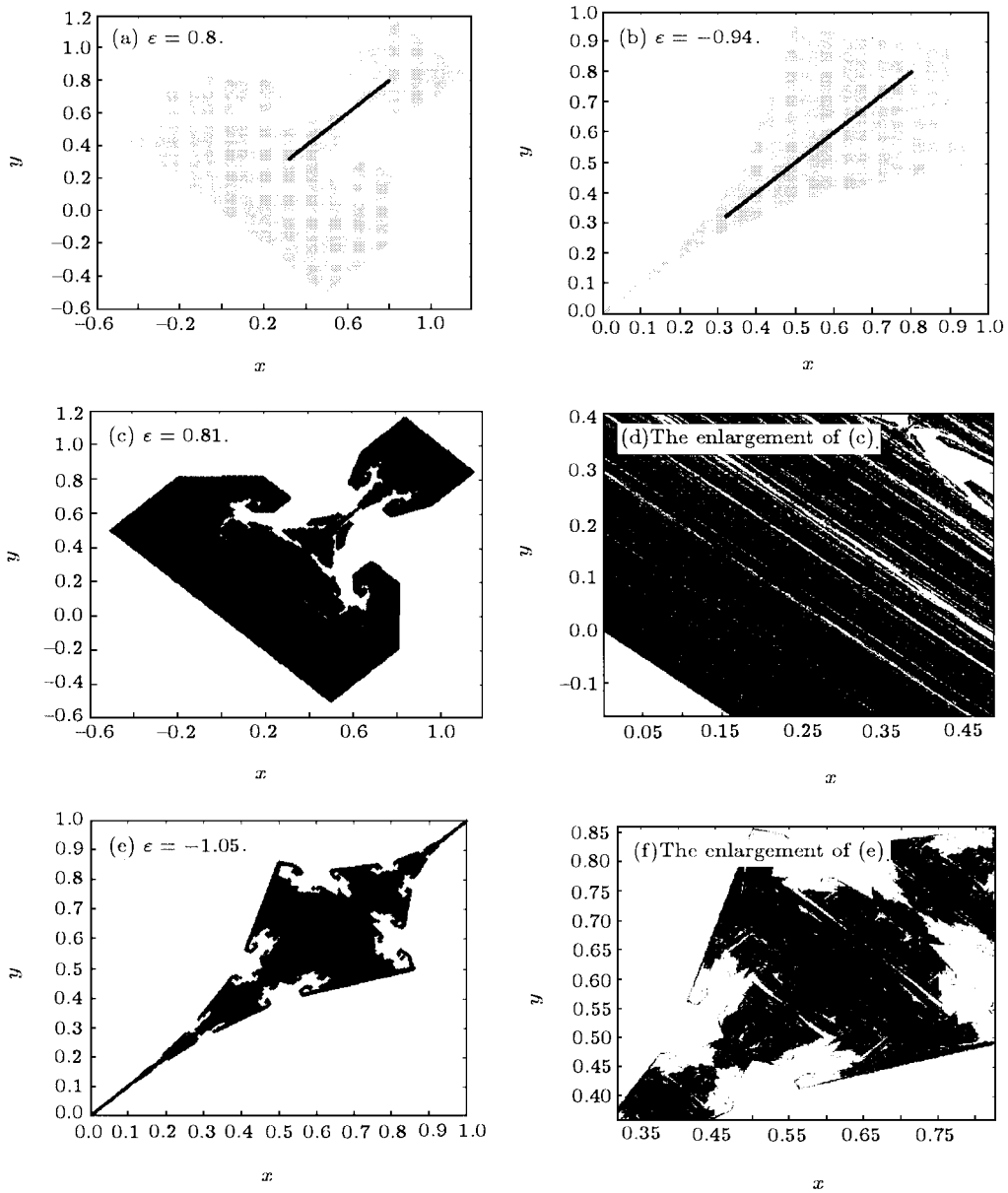


Fig.5. The attractive basin of A under different values of ε when $h = 0.8$, where: (a) and (b) are locally riddled basins, in which the black line denotes A; (c),(d),(e) and (f) are globally riddled basins.

When $\frac{1}{2} < h < 0.71$ A is piecewise in the line $\{2h(1-h) \le x = y \le h\}$. The numerical calculations hint to us that the attractive basin of A can be riddled for some ε.

When $0.71 \le h < 1$ we can easily find that a fixed point $C\left(\frac{2h}{2h+1}, \frac{2h}{2h+1}\right)$ is transversally unstable for every $\varepsilon \in r_1(\varepsilon)$ (the transversal eigenvalue is $-2h - 2\varepsilon$), which is embedded in A. So we can say the attractive basin of A is riddled for every $\varepsilon \in r_1(\varepsilon)$. But for every $\varepsilon \in r_2(\varepsilon)$, the fixed point C is transversally stable so we consider the higher-period orbits. If the period-2 orbits embedded in A are transversally unstable the minimal intervals of ε corresponding to the riddled basins are $-h - \frac{1}{2} < \varepsilon < -\sqrt{h^2 + \frac{1}{4}}$ and $-\sqrt{h^2 - \frac{1}{4}} < \varepsilon < -h + \frac{1}{2}$.

When $h = 1.0$, $F_\varepsilon(x, y)$ is a coupled system of two standard tent maps. Tan *et al*[19] point out that the attractive basin of the chaotic synchronized attractor is a riddled basin once a coupled system of two standard tent maps exhibits chaotic synchronization.

If we call the basin of attraction of A being riddled a state of A, according to the analysis above, we think that for every $\varepsilon \in r_1(\varepsilon)$ when $0.71 \le h \le 1$ and $\varepsilon \in r_2(\varepsilon)$ when h=1.0, there is only one state for A; the attractive basin of A must be riddled. But there

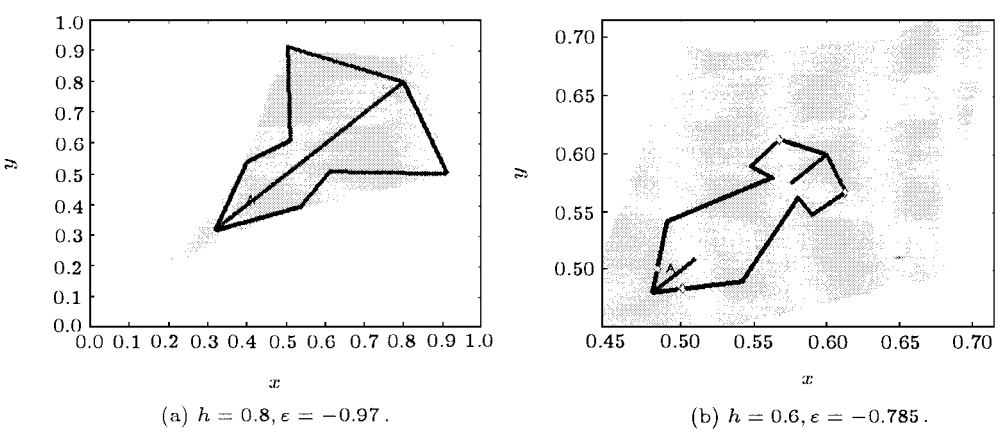
will be two states for A: the absolutely stable state (there is no riddled basin) and the riddled basin state if $\varepsilon \in r_1(\varepsilon) \bigcup r_2(\varepsilon)$ when $\frac{1}{2} < h < 0.71$ and $\varepsilon \in r_2(\varepsilon)$ when $0.71 \leq h < 1$.

It is seen from Fig.4 (h=0.6) and Fig.5 (h=0.8) that the riddled basins of A exhibit locally and globally riddled properties. Also, the fractal structure can be seen only when the basin is globally riddled. We find that there are two cases for the globally riddled basin. One case is that the repelled orbits go to a finite attractive set; the other case is that the repelled orbits go to infinitely.

4. The regular phenomenon in the local–global riddling bifurcation

Many authors have studied the transition from locally to globally riddled basins of systems. They have used the method of critical curves[23,24] to obtain the boundary of the absorbing area. The absorbing area, which contains the one-dimensional chaotic attractor A completely, is an invariant set. It has strong trapping ability so that a neighbourhood of the area exists, whose points are all mapped inside the area. It has been shown that only the invariant and minimal absorbing area is useful to characterize the transition from locally to globally riddled basins.[16] When the boundary of the absorbing area is in contact with the boundary of the attractive basin of A or some new attractors are born inside the absorbing area, the locally riddled basin will turn out to be a globally riddled basin.[17,18] It is clear that A must be contained in its attractive basin. If A is infinitely close to the boundary of its attractive basin, we say A is in critical state, so the absorbing area cannot be used here.

We note that there is only one two-piece chaotic synchronized attractor in $F_\varepsilon(x, y)$ when $\frac{1}{2} < h < 0.71$ and the attractor is contained in its attractive basin completely if $\varepsilon \in r_1(\varepsilon) \bigcup r_2(\varepsilon)$. When $0.71 \leq h < 1$ there is a non-piecewise chaotic synchronized attractor A in $F_\varepsilon(x, y)$. At this time A is contained in its attractive basin completely for $\varepsilon \in r_2(\varepsilon)$. But A is infinitely close to the boundary of its attractive basin for $\varepsilon \in r_1(\varepsilon)$ (the fixed point $C\left(\frac{2h}{2h+1}, \frac{2h}{2h+1}\right)$ embedded in A is in the boundary of the attractive basin of A). When h=1.0, A is also infinitely close to the boundary of its attractive basin for $\varepsilon \in r_1(\varepsilon) \bigcup r_2(\varepsilon)$. The chaotic synchronized attractor $\{0 < x = y < 1\}$ ought to be a closed set[27] but we can easily find that the point (0,0) is a fixed point and the point (1,1) is the previous map of (0,0). Thus, the chaotic synchronized attractor is only close to (0,0) and (1,1) infinitely. Consequently, the chaotic synchronized attractor A is in a critical state for $\varepsilon \in r_1(\varepsilon)$ when $0.71 \leq h < 1$ and $\varepsilon \in r_1(\varepsilon) \bigcup r_2(\varepsilon)$ when h=1.0. According to the definition of the absorbing area, we can use the method of the absorbing area to analyse the local–global riddling bifurcation only when A is contained by its attractive basin completely (see Fig.6). So we ought to look for other methods to analyse the local–global riddling bifurcation when A is in a critical state.



(a) $h = 0.8, \varepsilon = -0.97$. (b) $h = 0.6, \varepsilon = -0.785$.

Fig.6. The absorbing area of A when A is enclosed by its attractive basin completely (the region enclosed by black lines is the absorbing area): (a) describes the contact between boundaries of the absorbing area and the attractive basin; (b) describes the birth of new attractors inside the absorbing area.

5. The isolated critical value phenomenon in the local–global riddling bifurcation

Since A is a critical attractor for $\varepsilon \in r_1(\varepsilon)$ when $0.71 \le h < 1$ and $\varepsilon \in r_1(\varepsilon) \bigcup r_2(\varepsilon)$ when h=1.0, the absorbing area of A is not valid. We find that there is no new attractor born in the attractive basin of A and the repelled orbits go to infinity ultimately. So we look for the coupling parameter intervals corresponding to the globally riddled basin under the hypothesis that the repelled orbits go to infinity ultimately.

Property: A is close to the boundary of its attractive basin for $\varepsilon \in \{\varepsilon \in r_1(\varepsilon)$ and $0.71 \le h < 1\} \bigcup \{\varepsilon \in r_1(\varepsilon) \bigcup r_2(\varepsilon)$ and h=1.0$\}$. If the repelled orbits go to infinity ultimately when the attractive basin of A is globally riddled, the coupling parameter interval corresponding to the globally riddled basin is the whole interval $\varepsilon \in \{\varepsilon \in r_1(\varepsilon)$ and $0.71 \le h < 1\} \bigcup \{\varepsilon \in r_1(\varepsilon) \bigcup r_2(\varepsilon)$ and h=1.0$\}$ except $\varepsilon_c = h$ when $0.71 \le h < 1$ and $\varepsilon_c = \pm h$ when $h = 1.0$, where ε_c corresponds to the locally riddled state.

Proof: Given a neighbourhood $S_E(A)$ of $A(0 < E \ll 1)$, consider a two-dimensional set $\Theta = \{(w, v) :$ (w, v) in $S_E(A)$ and $L^+(w, v) \not\subset A\}$, where $L^+(w, v)$ is the ω-set of point (w, v). The attractive basin of A will be globally riddled if the Lebegue measure of Θ is positive.

For the coupled system $F_\varepsilon(x, y)$: $\begin{cases} x_{j+1} = f(x_j) + \varepsilon(y_j - x_j) \\ y_{j+1} = f(y_j) + \varepsilon(x_j - y_j) \end{cases}$ of the base map of (2), using the coordinate translation $\begin{cases} w_j = \dfrac{\sqrt{2}}{2}(x_j + y_j) \\ v_j = \dfrac{\sqrt{2}}{2}(y_j - x_j) \end{cases}$ and $\begin{cases} x_j = \dfrac{\sqrt{2}}{2}(w_j - v_j) \\ y_j = \dfrac{\sqrt{2}}{2}(w_j + v_j) \end{cases}$, the coupled system can be rewritten as:

for (I)

$$x \le \frac{1}{2} \text{ and } y \le \frac{1}{2}, \begin{cases} w_{j+1} = 2hw_j, \\ v_{j+1} = (2h - 2\varepsilon)v_j; \end{cases} \tag{5}$$

for (II)

$$x \le \frac{1}{2} \text{ and } y > \frac{1}{2}, \begin{cases} w_{j+1} = \sqrt{2}h - 2hv_j, \\ v_{j+1} = \sqrt{2}h - 2hw_j - 2\varepsilon v_j; \end{cases} \tag{6}$$

for(III)

$$x > \frac{1}{2} \text{ and } y \le \frac{1}{2}, \begin{cases} w_{j+1} = \sqrt{2}h + 2hv_j, \\ v_{j+1} = -\sqrt{2}h + 2hw_j - 2\varepsilon v_j; \end{cases} \tag{7}$$

for (IV)

$$x > \frac{1}{2} \text{ and } y > \frac{1}{2}, \begin{cases} w_{j+1} = 2\sqrt{2}h - 2hw_j, \\ v_{j+1} = (-2h - 2\varepsilon)v_j. \end{cases} \tag{8}$$

Here, w denotes the dynamics in the invariable subspace and v describes the evolution transversal to the invariant sub-space.

If the repelled orbit originally at $(w_0, v_0) \in S_E(A)$ stays in $S_E(A)$, it mainly evolves in regions (I) and (IV) (we note that the area of (II) and (III) can be ignored because of $\dfrac{E^2}{\sqrt{2}E} = \dfrac{E}{\sqrt{2}} \ll 1$). At this time the evolutions of v and w are unrelated. If it leaves away from $S_E(A)$, the evolution will be determined totally by the four equations (I), (II), (III) and (IV). Thus a global analysis is needed.

For $\varepsilon \in \{\varepsilon \in r_1(\varepsilon)$ and $0.71 \le h < 1\} \bigcup \{\varepsilon \in r_1(\varepsilon) \bigcup r_2(\varepsilon)$ and $h = 1.0\}$ it is clear that $|v_j(w_0, v_0)| \equiv 0$ from the equations in (I) and (II) if $\varepsilon = \varepsilon_c = h$ when $0.71 \le h < 1$ and $\varepsilon = \varepsilon_c = \pm h$ when $h = 1.0$. So the repelled orbits will stay in $S_E(A)$ forever. So the attractive basin of A is locally riddled.

If $\varepsilon \ne \varepsilon_c$, there may be two possibilities for the riddled basin of A. One is to remain locally riddled and the other is to become globally riddled. If the repelled orbit goes to infinity ultimately in the case of the globally riddled basin, the following procedure will help us to look for the coupling parameter intervals corresponding to the globally riddled basin.

There are only two possibilities for the repelled orbits which go to infinity: (i) $|w_j| \to \infty$, $|v_j| \to 0$ if $j \to \infty$; (ii) $|w_j| \to \infty$, $|v_j| \to \infty$ if $j \to \infty$. So there must exist at least one stable node in infinity in these four regions. We carry out a global analysis to determine whether these infinity points exist.

According to the properties of the skew tent map, the orbit evolving in region (IV) will go to region (I) ultimately, so the global analysis for the equation of (IV) has no meaning. Moreover, the equations of (II) and (III) are symmetric about coordinate w. So we only make the global analysis for the evolution in (I) and (III). Once we find at least a stable node in the infinity, the above proposition will be proven.

For $\varepsilon \in \{\varepsilon \in r_1(\varepsilon)$ and $0.71 \le h < 1\} \bigcup \{\varepsilon \in r_1(\varepsilon) \bigcup r_2(\varepsilon)$ and $h = 1.0\}$ and $\varepsilon \ne h$ when $0.71 \le h < 1$ and $\varepsilon \ne \pm h$ when h=1.0, there are

$$|2h - 2\varepsilon| < 1, \quad |-2h - 2\varepsilon| > 1. \tag{9}$$

It is clear that $|w_j| \to \infty$, $|v_j| \to 0$ when $j \to \infty$ from region (I), so there must be a stable node in infinity in region (I). The above proposition is proven.

6. Discussion

For the coupled skew tent map above, it has a chaotic synchronized attractor A only and the attractive basin of A exhibits locally and globally riddled properties. The method of the absorbing area can be used to study the local–global riddling bifurcation when A is contained by its attractive basin completely. But when A is very close to the boundary of the attractive basin itself in some certain coupling parameter interval, the absorbing area cannot be applied to the system. Since the repelled orbits go to infinity ultimately for every $\varepsilon \in \{\varepsilon \in r_1(\varepsilon)$ and $0.71 \leq h < 1\} \bigcup \{\varepsilon \in r_1(\varepsilon) \bigcup r_2(\varepsilon)$ and $h = 1.0\}$ we analytically find that the coupling parameter value corresponding to the locally riddled basin is embedded in the coupling parameter interval corresponding to the globally riddled basin, like an isolated point. Also, there is no relationship between this phenomenon and the form of the chaotic attractor. This phenomenon is not the same as the known results.

We also try to explain this novel phenomenon in a physical sense. It may be that the chaotic synchronized attractor is in a critical state, close to the boundary of its attractive basin infinitely. If the attractive basin of the chaotic attractor A is riddled, once the coupling parameter value is apart from the isolated value, there may be more segments of the chaotic attractor A close to the attractive basin infinitely. So the ability of A to attract will diminish. The locally riddled basin will be globally riddled easily here. The isolated critical value phenomenon may occur.

Fig.7 was obtained by calculating the riddled factor in Ref.[19], where the basic map of the coupled system is the standard tent map, h=1.0. It is clear that the coupling parameters corresponding to locally riddled basins are embedded in the coupling parameter interval corresponding to globally riddled basins, $\varepsilon = \pm 1.0$. However, at that time, we had not yet recognized this phenomenon.

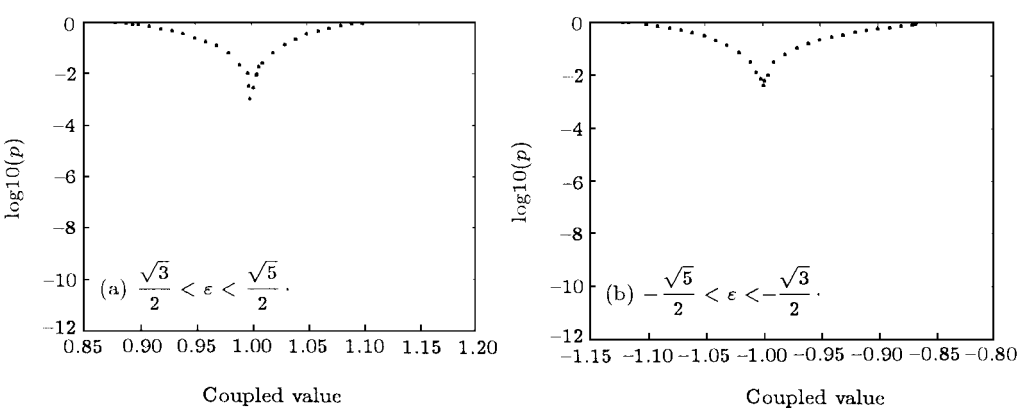


Fig.7. The point-graph of $\log10(p) \sim \varepsilon$ of the coupled standard tent maps, where p is the riddled factor of the riddled basin of the chaotic synchronized attractor.[19]

We have also studied another coupled system in other papers, whose basic map is another skew tent map

$$x_{j+1} = \begin{cases} \dfrac{x_j}{a} & 0 \leq x_j \leq a \\ \dfrac{1 - x_j}{1 - a} & a < x_j \leq 1 \end{cases} \quad (0 < a < 1). \qquad (10)$$

The same isolated critical value phenomenon can be found analytically in the local–global riddling bifurcation, where the chaotic synchronized attractor is in a critical state (the chaotic synchronized attractor is close to the boundary of its attractive basin infinitely).

Though the systems we have discussed are piecewise linear maps, we think that it is not an artefact phenomenon of piecewise linear maps. This isolated critical value phenomenon in the local–global riddling bifurcation may be general if the chaotic attractor in the system is in a critical state.

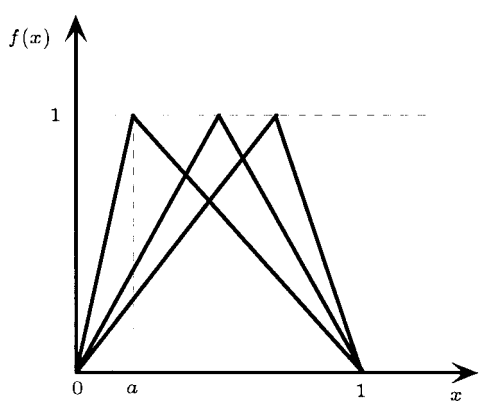


Fig.8. Another skew tent map expressed by Eq.(10).

References

[1] Luo X S and Fang J Q 2000 *Chin. Phys.* **9** 333

[2] Gu C M and Shen K 1999 *Acta Phys. Sin.* (Overseas Edition) **8** 733

[3] Peng Z W and Zhong T X 2000 *Chin. Phys.* **9** 244

[4] Li Z and Han C Z 2001 *Chin. Phys.* **10** 494

[5] Hong L and Xu J X 2001 *Int. J. Bifurcation Chaos* **11** 723

[6] Cui F S, Chew C H, Xu J X and Cai Y L 1997 *Nonlinear Dyn.* **12** 251

[7] Gong P L, Xu J X and Hu S J 2002 *Chaos Solitons Fractals* **13** 885

[8] Zhu S Q *et al* 2000 *Chin. Phys.* **9** 337

[9] Liu F, Shan X M *et al* 2001 *Chin. Phys.* **10** 606

[10] Huang X G and Xu J X 2001 *Chin. Phys.* **10** 1113

[11] Huang X G, Xu J X *et al* 2001 *Int. J. Bifurcation Chaos* **11** 561

[12] Wang J L and Chen G Z 1999 *Acta Phys. Sin.* **48** 1605 (in Chinese)

[13] Zheng Z G, Hu G *et al* 2000 *Acta Phys. Sin.* **49** 2320 (in Chinese)

[14] Guan X P, Tang Y G *et al* 2001 *Acta Phys. Sin.* **50** 2112 (in Chinese)

[15] Kuang J Y, Deng K *et al* 2001 *Acta Phys. Sin.* **50** 1856 (in Chinese)

[16] Biaschi G-I and Gardini L 1998 *Phys. Rev.* E **58** 5710

[17] Kapitaniak T, Maistrenko Y *et al* 1998 *Phys. Rev.* E **57** 6253

[18] Kapitaniak T and Maistrenko Y 1999 *Physica* D **126** 18

[19] Tan N, Chen Y H *et al* 2000 *Acta Phys. Sin.* **49** 1215 (in Chinese)

[20] Maistrenko Y L, Maistrenko V L *et al* 1998 *Phys. Rev. Lett.* **80** 1638

[21] Maistrenko Y L, Maistrenko V L *et al* 1998 *Phys. Rev.* E **57** 2713

[22] Hasler M and Maistrenko Y L 1997 *IEEE. Trans. Circuits Syst.* I **44** 856

[23] Frouzakis C E, Gardini L *et al* 1997 *Int. J. Bifurcation Chaos* **7** 1167

[24] Mira C, Rauzy C *et al* 1996 *Int. J. Bifurcation Chaos* **6** 2299

[25] Lai Y C, Grebogi C *et al* 1996 *Phys. Rev. Lett.* **77** 55

[26] Alexander J C, Yorker J A *et al* 1992 *Int. J. Bifurcation Chaos* **2** 795

[27] Ott E 1993 *Chaos in Dynamical Systems* (London: Cambridge University Press)