Vol 11 No 7, July 2002
1009-1963/2002/11(07)/0666-04

Chinese Physics

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Adaptive synchronization of Rossler and Chen chaotic systems*

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(Received 14 December 2001; revised manuscript received 4 February 2002)

A novel adaptive synchronization method is proposed for two identical Rossler and Chen systems with uncertain parameters. Based on Lyapunov stability theory, we derive an adaptive controller without the knowledge of the system parameters, which can make the states of two identical Rossler and Chen systems globally asymptotically synchronized. Especially, when some unknown uncertain parameters are positive, we can make the controller more simple and, besides, the controller is independent of those positive uncertain parameters. All results are proved using a well-known Lyapunov stability theorem. Numerical simulations are given to validate the proposed synchronization approach.

Keywords: chaotic systems, chaos control, adaptive synchronization, Rossler system, Chen attractor
PACC: 0545

1. Introduction

Chaos synchronization has been studied extensively in recent years. Since chaos synchronization is an important problem in nonlinear science, much effort has been devoted to investigating it and numerous achievements[1−15] have been made. For example, the nonlinear observer theory method,[6] the neural network method,[5] the impulse control method,[4] the Lyapunov function method[15] and others are presented for synchronization of chaotic systems. More recently, two identical Rossler and Chen systems have been synchronized by using active control.[11] However, these results[11] on chaos synchronization are under the following hypotheses: all the parameters of drive and response systems are precisely known, and the controller can be constructed using those known parameters. To our knowledge, some system parameters cannot be exactly known in advance. The synchronization will be destroyed by these uncertainties. Consequently, adaptive synchronization may play an essential role in chaos synchronization.

In this paper, we try to derive an adaptive controller based on Lyapunov stability theory which can make both Rossler and Chen systems synchronized in the presence of unknown system parameters. Especially, when some unknown uncertain parameters are positive, we can make the controller more simple and, besides, the controller is independent of those positive uncertain parameters.

2. Adaptive synchronization of two identical Rossler systems

The Rossler system[17] is described by the following ordinary differential equations

$$\dot{x} = -y - z, \quad \dot{y} = x + ay,$$
$$\dot{z} = b + z(x - c), \tag{1}$$

where x, y and z are state variables and a, b and c are three unknown uncertain parameters. We assume that we have two Rossler systems where the drive system with the subscript 1 drives the response system, having identical equations denoted by the subscript 2. However, the initial condition of the drive system is different from that of the response system. Therefore, the drive and response systems are defined as follows

$$\dot{x}_1 = -y_1 - z_1,$$
$$\dot{y}_1 = x_1 + ay_1,$$
$$\dot{z}_1 = b + z_1(x_1 - c), \tag{2}$$

and

$$\dot{x}_2 = -y_2 - z_2 + u_1,$$
$$\dot{y}_2 = x_2 + ay_2 + u_2,$$
$$\dot{z}_2 = b + z_2(x_2 - c) + u_3, \tag{3}$$

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where $u = [u_1, u_2, u_3]^T$ is the controller introduced in Eq.(3). The controller is to be determined for the purpose of synchronizing the two identical Rossler systems with the same but unknown parameters a, b and c in spite of the differences in initial conditions.

Subtracting Eq.(2) from (3), yields the error dynamical system between Eqs.(2) and (3)

$$\dot{e}_1 = -e_2 - e_3 + u_1,$$

$$\dot{e}_2 = e_1 + ae_2 + u_2,$$

$$\dot{e}_3 = e_1e_2 + e_1z_1 + e_3x_1 - ce_3 + u_3, \tag{4}$$

where $e_1 = x_2 - x_1$, $e_2 = y_2 - y_1$, $e_3 = z_2 - z_3$.
The goal of control is to find a controller $u = [u_1, u_2, u_3]^T$ and a parameter estimation update law for Eq.(4) such that the states of response system (3) and the states of drive system (2) are globally synchronized asymptotically, i.e.

$$\lim_{t\to\infty} ||e(t)|| = 0, \quad \forall a, b, c \in R,$$

where $e(t) = [e_1, e_2, e_3]^T$.

Letting the unknown uncertain parameter $c > 0$, we can choose a Lyapunov function for Eq.(4)

$$V(e, \tilde{a}) = \frac{1}{2}e^Te + \frac{1}{2}\tilde{a}^2, \tag{5}$$

where $\tilde{a} = a - \hat{a}$, $\hat{a}$ is the estimated value of the unknown parameter a, and we require that its derivative along the solutions of Eq.(4) satisfies

$$\frac{\mathrm{d}V(e, \tilde{a})}{\mathrm{d}t} \le -W(e), \tag{6}$$

where $W(e) = e_1^2 + e_2^2 + ce_3^2$. We therefore need to find a controller u and a parameter estimation update law $\dot{\hat{a}}$ to guarantee that for all $e \in R^n$ the inequality (6) holds. There are many possible choices for the controller u and the parameter estimation update law $\dot{\hat{a}}$. We choose

$$u_1 = -e_1, \quad u_2 = -(\hat{a} + 1)e_2,$$

$$u_3 = -e_1e_3 - x_1e_3 - z_1e_1 + e_1, \quad \dot{\hat{a}} = e_2^2. \tag{7}$$

With this choice, we have the time derivative of $V(e, \tilde{a})$ along the solutions of Eq.(4)

$$\begin{aligned}\frac{\mathrm{d}V(e, \tilde{a})}{\mathrm{d}t} =& e^T\dot{e} + \tilde{a}\dot{\tilde{a}}\\ =& e_1(-e_2 - e_3 + u_1) + e_2(e_1 + ae_2 + u_2)\\ &+ e_3(e_1e_3 + e_1z_1 + e_3x_1\\ &- ce_3 + u_3) + \tilde{a}(-\dot{\hat{a}})\\ =& -e_1^2 - e_2^2 - ce_3^2.\end{aligned} \tag{8}$$

This leads to

$$\lim_{t\to\infty} ||e(t)|| = 0, \quad \forall a, b \in R, c > 0.$$

Hence, the synchronization of two Rossler systems is achieved under the controller and a parameter estimation update law Eq.(7).

It is clear that the controller in Eq.(7) is independent of the unknown uncertain parameter c if $c > 0$.

If the condition of unknown uncertain parameter $c > 0$ is cancelled, we take a Lyapunov function for Eq.(4)

$$V(e, \tilde{a}, \tilde{c}) = \frac{1}{2}e^Te + \frac{1}{2}(\tilde{a}^2 + \tilde{c}^2), \tag{9}$$

where $\tilde{a} = a - \hat{a}$, $\tilde{c} = c - \hat{c}$, and $\hat{a}$ and $\hat{c}$ are estimated values of the unknown parameters a and c, respectively.
We choose a controller and a parameter estimation update law $\dot{\hat{a}}$ and $\dot{\hat{c}}$ as follows

$$u_1 = -e_1, \quad u_2 = -(\hat{a} + 1)e_2,$$

$$u_3 = -e_1e_3 - x_1e_3 - z_1e_1 + e_1 - e_3 + \hat{c}e_3,$$

$$\dot{\hat{a}} = e_2^2, \quad \dot{\hat{c}} = -e_3^2. \tag{10}$$

With this choice, we have the time derivative of $V(e, \tilde{a}, \tilde{c})$ along the solutions of Eq.(4)

$$\frac{\mathrm{d}V(e, \tilde{a}, \tilde{c})}{\mathrm{d}t} = -e_1^2 - e_2^2 - e_3^2, \tag{11}$$

where $W(e) = e_1^2 + e_2^2 + e_3^2$. This translates to

$$\lim_{t\to\infty} ||e(t)|| = 0, \quad \forall a, b, c \in R.$$

Therefore, the synchronization of two Rossler systems is achieved under the controller and a parameter estimation update law of Eq.(10).

Numerical results

In this sub-section, we show the numerical simulations to demonstrate the effectiveness of the proposed synchronization approach when unknown uncertain parameters a, b and $c > 0$ exist in Eqs.(2) and (3). It has been proven that the Rossler system (1) presents chaos when a=b=0.2, c=5.7.

The true values of "unknown" parameters are chosen as a=b=0.2 and c=5.7 in simulation. The initial values of the drive system states, the initial values of the response system states and the initial values of estimates for "unknown" parameters a, are taken as $x_1(0) = y_1(0) = z_1(0) = 6$, $x_2(0) = y_2(0) = z_2(0)$=10

and $\hat{a}(0)$=0.1, respectively.

The time response of states determined by drive system Eq.(2) and the time response of states which are determined by response system Eq.(3) with $u_1 = u_2 = u_3$=0 (t <50s) and the controller $u_1 = -e_1, u_2 = -(\hat{a}+1)e_2$, $u_3 = -e_1e_3 - x_1e_3 - z_1e_1 + e_1$ with $\dot{\hat{a}} = e_2^2$ at t=50s are shown in Fig.1.

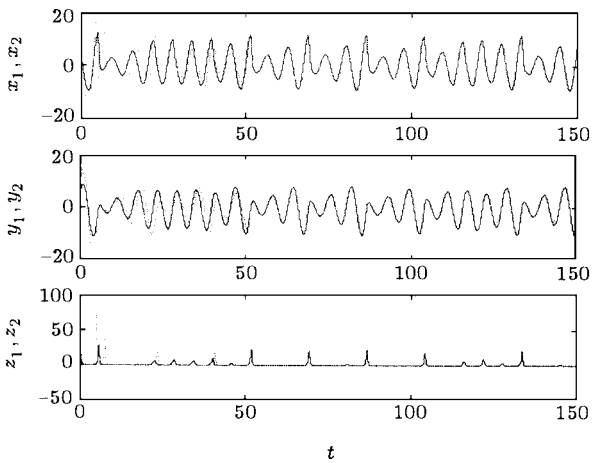


Fig.1. The time response of states for the drive system (x_1, y_1, z_1 –) and the response system (x_1, y_2, z_2 - - -), when the controller is deactivated (t <50s) and the controller with $\dot{\hat{a}} = e_2^2$ is activated to the response system at t=50s.

3. Adaptive synchronization of two identical Chen systems

Recently, on studying anticontrol of chaos, Chen[16] introduced a new chaotic attractor (Chen attractor), which is described by the following nonlinear differential equations

$$\dot{x} = a(y - x),$$

$$\dot{y} = (c - a)x - xz + cy, \quad \dot{z} = xy - bz, \tag{12}$$

where x, y and z are state variables and a, b and c are three unknown uncertain parameters. Our purpose is to achieve synchronization of Eq.(12) based on Lyapunov stability theory. For the Chen chaotic attractor, the drive and response systems are defined as follows

$$\dot{x}_1 = a(y_1 - x_1),$$

$$\dot{y}_1 = (c - a)x_1 - x_1z_1 + cy_1, \quad \dot{z}_1 = x_1y_1 - bz_1, \tag{13}$$

and

$$\dot{x}_2 = a(y_2 - x_2) + u_1,$$

$$\dot{y}_2 = (c - a)x_2 - x_2z_2 + cy_2 + u_2,$$

$$\dot{z}_2 = x_2y_2 - bz_2 + u_3, \tag{14}$$

where $u = [u_1, u_2, u_3]^T$ is the controller introduced in Eq.(14). The controller is to be determined for the purpose of synchronizing the two identical Chen chaotic attractors with the same but unknown parameters a, b and c in spite of the differences in initial conditions.

Then, the error dynamical system between Eqs.(13) and (14) can be expressed by

$$\dot{e}_1 = a(e_2 - e_1) + u_1,$$

$$\dot{e}_2 = (c - a)e_1 + ce_2 - e_1e_3 - x_1e_3 - z_1e_1 + u_2,$$

$$\dot{e}_3 = e_1e_2 + x_1e_2 + y_1e_1 - be_3 + u_3, \tag{15}$$

where $e_1 = x_2 - x_1, e_2 = y_2 - y_1, e_3 = z_2 - z_3$.

If the unknown uncertain parameters $a > 0$ and $b > 0$, we consider a Lyapunov function candidate as

$$V(e, \tilde{c}) = \frac{1}{2}e^Te + \frac{1}{2}\tilde{c}^2, \tag{16}$$

where $\tilde{c} = c - \hat{c}$, $\hat{c}$ is the estimated value of the unknown parameter c.

We choose a controller and a parameter estimation update law $\dot{\hat{c}}$ as follows

$$u_1 = 0, \quad u_2 = -\hat{c}(e_1 + e_2) + z_1e_1 - e_2,$$

$$u_3 = -y_1e_1, \quad \dot{\hat{c}} = e_2(e_1 + e_2). \tag{17}$$

With this choice, the derivative of Eq.(16) is

$$\frac{\mathrm{d}V(e, \tilde{c})}{\mathrm{d}t} = -ae_1^2 - e_2^2 - be_3^2, \tag{18}$$

where $W(e) = ae_1^2 + e_2^2 + be_3^2$. This translates to

$$\lim_{t\to\infty} ||e(t)|| = 0, \quad \forall c \in R, a, b > 0.$$

Thus, the synchronization of two Chen systems is achieved under the controller and a parameter estimation update law Eq.(17).

It is clear that the controller in Eq.(17) is independent of the unknown uncertain parameters a and b, if $a > 0$ and $b > 0$.

When the condition of unknown uncertain parameters $a > 0$ and $b > 0$ is cancelled, we can construct a Lyapunov function for Eq.(15)

$$V(e, \tilde{a}, \tilde{b}, \tilde{c}) = \frac{1}{2}e^Te + \frac{1}{2}(\tilde{a}^2 + \tilde{b}^2 + \tilde{c}^2), \tag{19}$$

where $\tilde{a} = a - \hat{a}$, $\tilde{b} = b - \hat{b}$, $\tilde{c} = c - \hat{c}$, $\hat{a}$, $\hat{b}$ and $\hat{c}$ are the estimated values of the unknown parameters a, b and c, respectively.

We choose a controller and a parameter estimation update law as follows

$$u_1 = \hat{a}e_1 - e_1,$$

$$u_2 = -\hat{c}(e_1 + e_2) + z_1 e_1 - e_2,$$

$$u_3 = -y_1 e_1 + \hat{b}e_3 - e_3,$$

$$\dot{\hat{a}} = -e_1^2, \quad \dot{\hat{b}} = -e_3^2, \quad \dot{\hat{c}} = e^2(e_1 + e_2). \tag{20}$$

From Eqs.(19), (15) and (20), we obtain the derivative of Eq.(19)

$$\frac{\mathrm{d}V(e, \tilde{a}, \tilde{b}, \tilde{c})}{\mathrm{d}t} = -e_1^2 - e_2^2 - be_3^2. \tag{21}$$

It is concluded that the synchronization of two Chen systems is achieved under the controller and a parameter estimation update law Eq.(20), i.e.

$$\lim_{t\to\infty} ||e(t)|| = 0, \quad \forall a, b, c \in R.$$

Numerical results

Now, we give numerical simulations to show the effectiveness of the proposed synchronization approach in the presence of unknown uncertain system parameters $a > 0$, $b > 0$ and c. It has been proven that the Chen system (12) has chaos behaviour when a=35, b=3 and c=28.

The true values of "unknown" parameters are selected as a=35, b=3 and c=28 in simulation. The initial values of the drive system states, the initial values of the response system states and the initial estimated values for "unknown" parameters c are taken as $x_1(0) = 2.5$, $y_1(0) = 3$, $z_1(0) = 7$, $x_2(0) = 40.5$, $y_2(0) = 20$, $z_2(0) = 50$ and $\hat{c}(0) = 0.5$, respectively. The time response of states determined by the drive system Eq.(13) and the time response of states which are determined by the response system Eq.(14) with $u_1 = u_2 = u_3 = 0$ (t <10s) and the controller $u_1 = 0$, $u_2 = -\hat{c}(e_1 + e_2) + z_1 e_1 - e_2$, $u_3 = -y_1 e_1$, with $\dot{\hat{c}} = e_2(e_1 + e_2)$ at t=10s can be seen in Fig.2.

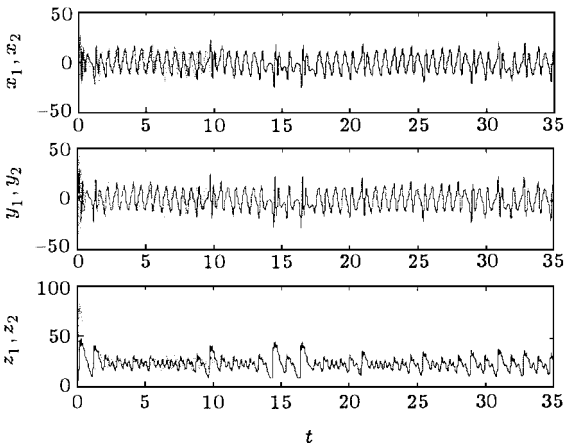


Fig.2. The time response of states for the drive system (x_1, y_1, z_1 –) and the response system (x_2, y_2, z_2 - - -), when the controller is deactivated (t <10s) and the controller with $\dot{\hat{c}} = e_2(e_1 + e_2)$ is activated to the response system at t=10s.

4. Conclusions

A novel adaptive synchronization method is proposed for two identical Rossler and Chen systems with uncertain parameters. Based on Lyapunov stability theory, we derive an adaptive controller without the knowledge of the system parameters, which can make the states of two identical Rossler and Chen systems globally asymptotically synchronized. Especially, when some unknown uncertain parameters are positive, we can make the controller more simple and, besides, the controller is independent of those positive uncertain parameters. All results are proven using a well-known Lyapunov stability theorem. Numerical simulations are given to validate the proposed synchronization approach.

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