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Hawking radiation of Weyl neutrinos in a rectilinearly non-uniformly accelerating Kinnersley black hole*

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The quantum thermal effect of Weyl neutrinos in a rectilinearly non-uniformly accelerating Kinnersley black hole is investigated using the generalized tortoise coordinate transformation. The equations that determine the location, the Hawking temperature of the event horizon and the thermal radiation spectrum of neutrinos are derived. Our results show that the location and the temperature of the event horizon depend not only on the time but also on the angle.

Keywords: Hawking radiation, Weyl neutrino, non-stationary Kinnersley black hole, generalized tortoise coordinate transformation

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1. Introduction

The discovery of Hawking radiation[1] has not only resolved some contradictions in black hole thermodynamics but it has also revealed a profound intrinsic relation between quantum mechanics, thermodynamics and general relativity. The investigation of the thermal properties of various black holes is an important subject of black hole physics.[2] Owing to the existence of various astrophysical processes, such as evaporation and accretion,[3] a black hole in the universe must change with time. Thus, a deeper investigation of the Hawking radiation of an evaporating black hole is very significant to understanding dark celestial bodies such as black holes. In recent years, research on the Hawking radiation of non-stationary black holes has attracted much more attention, but most studies have concentrated on the quantum thermal effect of scalar fields. The investigation of the Hawking radiation of fermions in non-static black holes is merely limited to those in spherically symmetric ones.[2] Recently, by making use of the generalized tortoise transformation proposed by Zhao,[2] Wu and Cai[4] have successfully dealt with the Hawking radiation of Dirac particles in a non-stationary Kerr black hole.[5] They simultaneously treated the first-order and second-order Dirac equations and derived both the event horizon equation and the Hawking temperature which coincide with previous results.[6] A new quantum effect observed in the thermal radiation spectrum of fermions probably originates from the coupling of the spin of particles with the angular momentum of black holes. It is absent from the Bosonic radiation spectrum of scalar particles.[6]

In this paper, we use the method developed in Ref.[4] to cope with the Hawking radiation of massless fermions, namely Weyl neutrinos in a rectilinearly non-uniformly accelerating Kinnersley black hole.[7] The corresponding quantum thermal effect of scalar particles in the same space–time has already been studied in Ref.[8].

2. The Weyl equation in the background space-time

The metric of a rectilinearly non-uniformly accelerating Kinnersley black hole that we take into account can be rewritten in the advanced Eddington–Finkelstein coordinate system as[7]

$$\mathrm{d}s^2 = 2\mathrm{d}v(G\mathrm{d}v - \mathrm{d}r - r^2 f\mathrm{d}\theta) - r^2(\mathrm{d}\theta^2 + \sin^2\theta\mathrm{d}\varphi^2), \tag{1}$$

where $2G = 1 - 2M/r - 2ar\cos\theta - r^2 f^2$, $f = -a\sin\theta$, the parameter $a = a(v)$ being the magnitude of acceleration, and the mass $M(v)$ of the hole is a function of time v.

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We choose such a complex null-tetrad $\{\boldsymbol{l},\boldsymbol{n},\boldsymbol{m},\overline{\boldsymbol{m}}\}$ that satisfies the orthogonal conditions $\boldsymbol{l}\cdot\boldsymbol{n}=-\boldsymbol{m}\cdot\overline{\boldsymbol{m}}=1$. Thus the covariant one-forms can be written as

$$\begin{aligned}\boldsymbol{l}&=\mathrm{d}v\,,\qquad \boldsymbol{n}=G\mathrm{d}v-\mathrm{d}r-r^2f\mathrm{d}\theta\,,\\ \boldsymbol{m}&=\frac{-r}{\sqrt{2}}\left(\mathrm{d}\theta+\mathrm{i}\sin\theta\mathrm{d}\varphi\right),\\ \overline{\boldsymbol{m}}&=\frac{-r}{\sqrt{2}}\left(\mathrm{d}\theta-\mathrm{i}\sin\theta\mathrm{d}\varphi\right),\end{aligned}\tag{2}$$

and their corresponding directional derivatives are

$$\begin{aligned}D&=-\partial_r\,,\qquad \Delta=\partial_v+G\partial_r,\\ \delta&=\frac{1}{\sqrt{2}r}\left(-r^2f\partial_r+\partial_\theta+\frac{\mathrm{i}}{\sin\theta}\partial_\varphi\right),\\ \overline{\delta}&=\frac{1}{\sqrt{2}r}\left(-r^2f\partial_r+\partial_\theta-\frac{\mathrm{i}}{\sin\theta}\partial_\varphi\right).\end{aligned}\tag{3}$$

It is not difficult to compute the non-vanishing Newman–Penrose (NP)[9] spin coefficients as follows (we denote $G_{,r}=\mathrm{d}G/\mathrm{d}r$):

$$\begin{aligned}&\rho=1/r\,,\quad \mu=G/r\,,\quad \gamma=-G_{,r}/2,\\ &\tau=-\tilde{\pi}=f/\sqrt{2}\,,\quad \beta=\cot\theta/(2\sqrt{2}\,r),\\ &\alpha=-\cot\theta/(2\sqrt{2}\,r)+f/\sqrt{2},\\ &\nu=\left[(2rG-r^2G_{,r})f+r^2f_{,v}+G_{,\theta}\right]/(\sqrt{2}\,r).\end{aligned}\tag{4}$$

Inserting the related spin coefficients and directional derivatives into the spinor form of the Weyl neutrino equation[5,10] in the NP formalism

$$\begin{aligned}&(D+\epsilon-\rho)\eta_1-(\overline{\delta}+\tilde{\pi}-\alpha)\eta_0=0,\\ &(\Delta+\mu-\gamma)\eta_0-(\delta+\beta-\tau)\eta_1=0,\end{aligned}\tag{5}$$

we obtain

$$\begin{aligned}&\mathcal{D}_1\eta_1+\frac{1}{\sqrt{2}\,r}\left(\mathcal{L}-r^2f\mathcal{D}_2\right)\eta_0=0,\\ &\left(\partial_v+G\mathcal{D}_1+G_{,r}/2\right)\eta_0-\frac{1}{\sqrt{2}\,r}\left(\mathcal{L}^\dagger-r^2f\mathcal{D}_1\right)\eta_1=0,\end{aligned}\tag{6}$$

in which we have defined operators

$$\begin{aligned}\mathcal{D}_n&=\partial_r+n/r,\\ \mathcal{L}&=\partial_\theta+\frac{1}{2}\cot\theta-\frac{\mathrm{i}}{\sin\theta}\partial_\varphi,\\ \mathcal{L}^\dagger&=\partial_\theta+\frac{1}{2}\cot\theta+\frac{\mathrm{i}}{\sin\theta}\partial_\varphi\,.\end{aligned}$$

With further substitutions $\chi_1=\sqrt{2}\,r\eta_1$, $\chi_0=\eta_0$, into Eq.(6), we can reduce them to the form

$$\begin{aligned}&\mathcal{D}_0\chi_1+\left(\mathcal{L}-r^2f\mathcal{D}_2\right)\chi_0=0,\\ &r^2\left(2\partial_v+2G\mathcal{D}_1+G_{,r}\right)\chi_0-\left(\mathcal{L}^\dagger-r^2f\mathcal{D}_0\right)\chi_1=0.\end{aligned}\tag{7}$$

3. Event horizon equation

Though Eq.(7) cannot be decoupled with respect to θ, however to deal with the problem of Hawking radiation, one should be concerned about its asymptotic behaviours near the horizon only. Based upon the symmetry of the space-time under consideration, we introduce a generalized tortoise coordinate transformation as follows:

$$\begin{aligned}r_*&=r+\frac{1}{2\kappa}\ln[r-r_{\rm h}(v,\theta)],\\ v_*&=v-v_0\,,\quad \theta_*=\theta-\theta_0.\end{aligned}\tag{8}$$

Namely

$$\begin{aligned}\mathrm{d}r_*&=\mathrm{d}r+\frac{1}{2\kappa(r-r_{\rm h})}\left(\mathrm{d}r-\dot{r}_{\rm h}\mathrm{d}v-r'_{\rm h}\mathrm{d}\theta\right),\\ \mathrm{d}v_*&=\mathrm{d}v\,,\quad \mathrm{d}\theta_*=\mathrm{d}\theta,\end{aligned}$$

where $r_{\rm h}=r_{\rm h}(v,\theta)$ is the location of the event horizon, and κ is an adjustable parameter to be determined. The parameters v_0 and θ_0 are arbitrary constants, all of which remain unchanged under the tortoise coordinate transformation. $\dot{r}_{\rm h}=\partial_v r_{\rm h}$ and $r'_{\rm h}=\partial_\theta r_{\rm h}$ can be viewed as parameters depicting the evolution of the event horizon. All induced relations among the first-order and second-order derivatives from transformation (8) can be found in Ref.[2] and are not listed here.

Now we apply transformation (8) to Eq.(7) and take the $r\to r_{\rm h}(v_0,\theta_0)$, $v\to v_0$ and $\theta\to\theta_0$ limits. Then we can obtain its limiting form near the event horizon as

$$\begin{aligned}&\frac{\partial}{\partial r_*}\chi_1-\left(r'_{\rm h}+r_{\rm h}^2f\right)\frac{\partial}{\partial r_*}\chi_0=0,\\ &\left(r'_{\rm h}+r_{\rm h}^2f\right)\frac{\partial}{\partial r_*}\chi_1+2r_{\rm h}^2\left(G-\dot{r}_{\rm h}\right)\frac{\partial}{\partial r_*}\chi_0=0.\end{aligned}\tag{9}$$

Obviously, the condition that Eq.(9) has a non-trivial solution for χ_0 and χ_1 is that its determinant vanishes, which gives the following equation to determine the location of the event horizon:

$$2G-2\dot{r}_{\rm h}+\left(r_{\rm h}f+\frac{r'_{\rm h}}{r_{\rm h}}\right)^2=0\,.\tag{10}$$

It is easy to see that this result is consistent with that inferred from the null-surface condition: $g^{ij}\partial_iF\partial_jF=$

0. Making a similar prescription as above to the null-surface equation

$$(2G + r^2 f^2)(\partial_r F)^2 + 2\partial_v F \partial_r F - 2f\partial_\theta F \partial_r F + \frac{1}{r^2}(\partial_\theta F)^2 + \frac{1}{r^2 \sin^2\theta}(\partial_\varphi F)^2 = 0\,, \tag{11}$$

we can arrive at an equation

$$\left(2G + r_{\rm h}^2 f^2 - 2\dot{r}_{\rm h} + 2f r'_{\rm h} + \frac{r'^2_{\rm h}}{r_{\rm h}^2}\right)\left(\frac{\partial}{\partial r_*}F\right)^2 = 0,$$

which results in the same equation (10) that determines the event horizon after we put the coefficient in front of the derivative $\left(\frac{\partial}{\partial r_*}F\right)^2$ to zero. Equation (10) shows that $r_{\rm h}$ depends not only on v but also on θ. This means that the location of the event horizon and the shape of the black hole change with time.

4. Hawking temperature

To investigate the Hawking radiation of Weyl neutrinos, we are now in a position to consider the second-order form of the Weyl equation. After a direct calculation of the second-order Weyl equations

$$[r^2\mathcal{D}_2(2\partial_v + 2G\mathcal{D}_1 + G_{,r}) + (\mathcal{L}^\dagger - r^2 f\mathcal{D}_2)(\mathcal{L} - r^2 f\mathcal{D}_2)]\chi_0 = 0\,, \tag{12}$$

and

$$\begin{aligned}&[r^2(2\partial_v + 2G\mathcal{D}_1 + G_{,r})\mathcal{D}_0 + (\mathcal{L} - r^2 f\mathcal{D}_0)(\mathcal{L}^\dagger - r^2 f\mathcal{D}_0)]\chi_1\\ =&2r^2\Big\{[(2rG - r^2 G_{,r})f + r^2 f_{,v} + G_{,\theta}]\partial_r - \left(rG_{,r} - 3G + r^2 G_{,rr}/2\right)f + 2rf_{,v} + G_{,\theta}/r + G_{,r\theta}/2\Big\}\chi_0\,,\end{aligned} \tag{13}$$

we obtain their explicit forms as

$$\begin{aligned}&\Big[(2r^2G + r^4 f^2)\partial_r^2 + 2r^2\partial_{vr}^2 + \partial_\theta^2 + \frac{1}{\sin^2\theta}\partial_\varphi^2 - 2r^2 f\partial_{r\theta}^2 + \frac{{\rm i}\cos\theta}{\sin^2\theta}\partial_\varphi\\ &+ (\cot\theta - 4rf)\partial_\theta + 4r\partial_v + (3r^2 G_{,r} + 6rG + 6r^3 f^2 - 2r^2 f\cot\theta)\partial_r\\ &+ r^2 G_{,rr} + 4rG_{,r} + 2G + 6r^2 f^2 - 4rf\cot\theta - \frac{1}{4\sin^2\theta} - \frac{1}{4}\Big]\chi_0 = 0\,,\end{aligned} \tag{14}$$

and

$$\begin{aligned}&\Big[(2r^2G + r^4 f^2)\partial_r^2 + 2r^2\partial_{vr}^2 + \partial_\theta^2 + \frac{1}{\sin^2\theta}\partial_\varphi^2 - 2r^2 f\partial_{r\theta}^2 - \frac{{\rm i}\cos\theta}{\sin^2\theta}\partial_\varphi\\ &+ \cot\theta\partial_\theta + (r^2 G_{,r} + 2rG + 2r^3 f^2 - 2r^2 f\cot\theta)\partial_r - \frac{1}{4\sin^2\theta} - \frac{1}{4}\Big]\chi_1\\ =&2r^2\Big\{[(2rG - r^2 G_{,r})f + r^2 f_{,v} + G_{,\theta}]\partial_r - \left(rG_{,r} - 3G + r^2 G_{,rr}/2\right)f + 2rf_{,v} + G_{,\theta}/r + G_{,r\theta}/2\Big\}\chi_0\,.\end{aligned} \tag{15}$$

The application of a similar procedure as in the last section to Eqs.(14) and (15) leads to

$$\begin{aligned}&\left[\frac{A}{2\kappa} + 2r_{\rm h}^2(2G - \dot{r}_{\rm h}) + 2r_{\rm h}^4 f^2 + 2f r_{\rm h}^2 r'_{\rm h}\right]\frac{\partial^2}{\partial r_*^2}\chi_0 - 2\left(f r_{\rm h}^2 + r'_{\rm h}\right)\frac{\partial^2}{\partial r_*\partial\theta_*}\chi_0\\ &+ 2r_{\rm h}^2\frac{\partial^2}{\partial r_*\partial v_*}\chi_0 + [-A + 3r_{\rm h}^2 G_{,r} + r_{\rm h}(6G - 4\dot{r}_{\rm h}) + 6r_{\rm h}^3 f^2 - 2r_{\rm h}^2 f\cot\theta_0\\ &+ (4fr_{\rm h} - \cot\theta_0)r'_{\rm h} - r''_{\rm h}]\frac{\partial}{\partial r_*}\chi_0 = 0\,,\end{aligned} \tag{16}$$

and

$$\begin{aligned}&\left[\frac{A}{2\kappa} + 2r_{\rm h}^2(2G - \dot{r}_{\rm h}) + 2r_{\rm h}^4 f^2 + 2f r_{\rm h}^2 r'_{\rm h}\right]\frac{\partial^2}{\partial r_*^2}\chi_1 - 2\left(f r_{\rm h}^2 + r'_{\rm h}\right)\frac{\partial^2}{\partial r_*\partial\theta_*}\chi_1\\ &+ 2r_{\rm h}^2\frac{\partial^2}{\partial r_*\partial v_*}\chi_1 + [-A + r_{\rm h}^2 G_{,r} + 2r_{\rm h}G + 2r_{\rm h}^3 f^2 - 2r_{\rm h}^2 f\cot\theta_0 - \cot\theta_0 r'_{\rm h} - r''_{\rm h}]\frac{\partial}{\partial r_*}\chi_1\\ =&2r_{\rm h}^2[(2r_{\rm h}G - r_{\rm h}^2 G_{,r})f + r_{\rm h}^2 f_{,v} + G_{,\theta}]\frac{\partial}{\partial r_*}\chi_0\\ =&-\frac{r'_{\rm h} + r_{\rm h}^2 f}{G - \dot{r}_{\rm h}}[(2r_{\rm h}G - r_{\rm h}^2 G_{,r})f + r_{\rm h}^2 f_{,v} + G_{,\theta}]\frac{\partial}{\partial r_*}\chi_1\,,\end{aligned} \tag{17}$$

in which the coefficient A is an infinite form of $\frac{0}{0}$-type. Its value can be computed by means of L'Hôspital rule as

$$\begin{aligned}A =&\lim_{r\to r_{\rm h}(v_0,\theta_0)}\frac{2r^2(G - \dot{r}_{\rm h}) + r^4 f^2 + 2f r^2 r'_{\rm h} + r'^2_{\rm h}}{r - r_{\rm h}}\\ =&2r_{\rm h}^2 G_{,r} + 4r_{\rm h}(G - \dot{r}_{\rm h}) + 4r_{\rm h}^3 f^2 + 4f r_{\rm h} r'_{\rm h}\\ =&2r_{\rm h}^2 G_{,r} + 2r_{\rm h}^3 f^2 - 2r'^2_{\rm h}/r_{\rm h}\,.\end{aligned}$$

Now we adjust the parameter κ in Eqs.(16) and (17) such that the ratio of the coefficients before $\frac{\partial^2}{\partial r_*^2}$

and $\dfrac{\partial^2}{\partial r_* \partial v_*}$ is equal to $1:2$, namely

$$\frac{A}{2\kappa} + 2r_{\rm h}^2(2G - \dot{r}_{\rm h}) + 2r_{\rm h}^4 f^2 + 2f r_{\rm h}^2 r'_{\rm h} \equiv r_{\rm h}^2 \,,$$

and we obtain the surface gravity of the event horizon

$$\kappa = \frac{r_{\rm h}^2 G_{,r} + r_{\rm h}^3 f^2 - {r'_{\rm h}}^2/r_{\rm h}}{r_{\rm h}^2(1-2G) - r_{\rm h}^4 f^2 + {r'_{\rm h}}^2} \,. \tag{18}$$

Dividing both sides of Eqs.(16) and (17) by $r_{\rm h}^2$ after substituting the coefficient A into them, we can reduce both of these to a standard wave equation near the event horizon

$$\begin{aligned} &\frac{\partial^2}{\partial r_*^2}\Psi + 2\frac{\partial^2}{\partial r_* \partial v_*}\Psi \\ &- 2B\frac{\partial^2}{\partial r_* \partial \theta_*}\Psi + 2C\frac{\partial}{\partial r_*}\Psi = 0 \,, \end{aligned} \tag{19}$$

where $B = f + r'_{\rm h}/r_{\rm h}^2$. For $\Psi = \chi_0$ we have

$$\begin{aligned} 2C =& \frac{2G}{r_{\rm h}} + G_{,r} + 2r_{\rm h} f^2 \\ &- \left(2f r_{\rm h} + \frac{r'_{\rm h}}{r_{\rm h}^2}\right)\cot\theta_0 - \frac{r''_{\rm h}}{r_{\rm h}^2} \,, \end{aligned}$$

while for $\Psi = \chi_1$ we have

$$\begin{aligned} 2C =& \frac{2G}{r_{\rm h}} - G_{,r} - \left(2f + \frac{r'_{\rm h}}{r_{\rm h}^2}\right)\cot\theta_0 \\ &+ \frac{2{r'_{\rm h}}^2}{r_{\rm h}^3} - \frac{r''_{\rm h}}{r_{\rm h}^2} + \frac{f + r'_{\rm h}/r_{\rm h}^2}{G - \dot{r}_{\rm h}} \\ &\times \left[(2r_{\rm h}G - r_{\rm h}^2 G_{,r})f + r_{\rm h}^2 f_{,v} + G_{,\theta}\right] . \end{aligned}$$

From the above prescriptions, we can see that it is with the aid of the relations (9) derived in the preceding section that we can recast the coupled equation (17) into a second-order wave equation of one component. This demonstrates that our disposition of the first-order and second-order equations is self-consistent and meets with the needs on physical grounds.

5. Thermal radiation spectrum

Now that all coefficients in Eq.(19) can be regarded as finite real constants, we can separate variables as

$$\Psi = R(r_*)\Theta(\theta_*)\mathrm{e}^{\mathrm{i}(m\varphi - \omega v_*)}, \tag{20}$$

and with this ansatz substituted into equation (19), we obtain the following equation

$$\Theta' = \lambda\Theta \,, \quad R'' = 2(\mathrm{i}\omega - C_0)R' \,,$$

with the solutions

$$\Theta = \mathrm{e}^{\lambda\theta_*} \,, \quad R = C_1\mathrm{e}^{2(\mathrm{i}\omega - C_0)r_*} + C_2 \,, \tag{21}$$

where λ is a real constant introduced in the separation variables, $C_0 = C - \lambda B$.

The incoming wave and the outgoing wave, respectively, are

$$\begin{aligned} \Psi_{\rm in} \sim& \mathrm{e}^{\mathrm{i}(m\varphi - \omega v_*) + \lambda\theta_*} \,, \\ \Psi_{\rm out} \sim& \mathrm{e}^{\mathrm{i}(m\varphi - \omega v_*) + \lambda\theta_*}\mathrm{e}^{\mathrm{i}\omega - C_0)r_*} \end{aligned}$$

and

$$= \Psi_{\rm in}\mathrm{e}^{2(\mathrm{i}\omega - C_0)r_*} \,, \quad (r > r_{\rm h}) \,. \tag{22}$$

Near the event horizon, we have $r_* \sim \dfrac{1}{2\kappa}\ln(r - r_{\rm h})$. The incoming wave $\Psi_{\rm in}$ is regular at the event horizon, but the outgoing wave $\Psi_{\rm out}(r > r_{\rm h})$ is irregular at the event horizon $r = r_{\rm h}$. It can be analytically extended from the outside of the hole into the inside of the hole through the lower complex r-plane

$$(r - r_{\rm h}) \to (r_{\rm h} - r)\mathrm{e}^{-\mathrm{i}\pi}$$

to

$$\begin{aligned} \widetilde{\Psi}_{\rm out} =& \mathrm{e}^{\mathrm{i}(m\varphi - \omega v_*) + \lambda\theta_*}\mathrm{e}^{2(\mathrm{i}\omega - C_0)r_*}\mathrm{e}^{\pi(\omega + \mathrm{i}C_0)/\kappa} \\ =& \Psi_{\rm out}\mathrm{e}^{\pi(\omega + \mathrm{i}C_0)/\kappa} \,, \quad (r < r_{\rm h}) \,. \end{aligned} \tag{23}$$

According to the method suggested by Damour and Ruffini, and Sannan,[11] the relative scattering probability of the outgoing wave at the horizon is

$$\left|\frac{\Psi_{\rm out}}{\widetilde{\Psi}_{\rm out}}\right|^2 = \mathrm{e}^{-2\pi\omega/\kappa} \,, \tag{24}$$

and the thermal radiation Fermionic spectrum of Weyl neutrinos from the event horizon of the hole is given by

$$\langle \mathcal{N}(\omega) \rangle = \frac{1}{\mathrm{e}^{\omega/T_{\rm H}} + 1} \,, \tag{25}$$

with the obvious expression of the Hawking temperature

$$T_{\rm H} = \frac{\kappa}{2\pi} = \frac{1}{4\pi r_{\rm h}} \cdot \frac{M r_{\rm h} - r_{\rm h}^3 a\cos\theta_0 - {r'_{\rm h}}^2}{M r_{\rm h} + r_{\rm h}^3 a\cos\theta_0 + {r'_{\rm h}}^2/2} \,. \tag{26}$$

It follows that the temperature depends not only on the time, but also on the angle θ. The temperature is in agreement with the corresponding result presented in Ref.[8].

6. Conclusions

Equations (10) and (26) give the location and the temperature of the event horizon, which depend not only on the advanced time v but also on the polar angle θ. These are just the same results as those obtained in the discussion of the thermal radiation

of scalar fields.[8] Equation (25) shows the Hawking radiation spectrum of Weyl neutrinos in a rectilinearly non-uniformly accelerating Kin nersley black hole.

This study and those of Refs.[4] and [12]–[14] show that the method of generalized coordinate transformation is a powerful tool to investigate black hole radiation. It was initially proposed by Damour and Ruffini[11] to deal with the quantum thermal effect of scalar fields in a static (Schwarzschild) black hole and in a stationary (Kerr) black hole. Later, Sannan[11] developed this method and made it suitable to deal not only with the thermal radiation of scalar fields but also with that of fermions. Zhao[2] further extended this transformation in the static and stationary cases to that in the non-static and non-stationary cases and has made several discussions about the quantum thermal effects of scalar fields in various black holes and that of Dirac particles in some non-static and spherically symmetric black holes. But this method meets with great difficulties when it is applied to the Hawking radiation of Dirac particles in a most general space-time, such as a non-static and non-spherically symmetric black hole or a non-stationary axisymmetric black hole. The reason for this is that the Dirac equation cannot be completely separable for all variables in the most general space-time. Now this difficulty is overcome when we simultaneously treat the first-order and second-order equations, and use the relations between the first-order derivatives to remove the crossing terms in the second-order equations. This procedure enables us to reduce each second-order equation of one component to a simple separable standard wave equation near the event horizon after we have made a generalized tortoise coordinate transformation.

In conclusion, the work in this paper and in Refs.[4] and [12]–[14] has made the method of generalized tortoise transformation more perfect. It has been indicated that this theory becomes a fairly integrated system. The method we have developed is applicable not only to discussing the Hawking radiation of Weyl neutrinos in a non-static and non-spherically symmetric Kinnersley black hole with arbitrary acceleration and that of electrons in a non-stationary axisymmetric black hole, but also, in principle, to studying the quantum thermal effect of any space-time with a non-degenerate event horizon.[14] Also it is easily extended to discuss the Hawking effect of fields with an arbitrary spin.[12,13]

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