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Generation of motional cat states for a trapped ion in a cavity*

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Beyond the Lamb–Dicke limit, we propose a simple scheme for generating Schrödinger cat states of the motion of a trapped two-level ion interacting with a quantized light field in a single-mode cavity.

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Over the past few years, there has been much interest in the problem of generating quantum states consisting of superpositions of macroscopically distinguishable quantum states, known as Schrödinger cat (SC) states.[1] Due to the interference between the coherent components such superposition states may exhibit various nonclassical properties, such as squeezing and sub-Poissonian statistics. There have been many theoretical schemes proposed for the generation of such cat states of the radiation field[2−4] and the centre-of-mass (CM) vibrational motion of ions (atoms).[5−7] Experimentally, the motional SC states have been observed.[8] As far as we know, almost all schemes for generating the motional SC states are proposed under the condition that the trapped ion system is driven by a classical light field.

On the other hand, in the experiment where only a single ion is trapped,[8,9] the interaction of the ion with the radiation field, namely, absorbing or radiating a photon, could be a quantized process. Recently, Buzek *et al*[10] and Luo *et al*[11] have proposed a fully quantum-mechanical model in which the trapped two-level ion interacts with a quantized cavity mode of the light field in a high-finesse resonator. In Ref.[10], Buzek *et al* have studied the dynamics of the fully quantized system in the Lamb–Dicke limit (LDL) as the motional state of the trapped ion is initially in a coherent state. They have shown that the SC states of the vibrational motion are produced at one-half of the revival time $T_{\rm r}$. It is worth noting, however, that the SC states of the motion of the trapped ion appear only at a fixed point of time. As time goes on, the SC states disappear. In this paper, we propose another simple scheme for generating the motional SC states of the trapped ion system in this single-mode cavity. Unlike the method used in Ref.[10], we can obtain the time-independent SC states of the motion of the trapped ion by detecting the number of photons in the cavity or measuring the electronic state of the ion after a short interaction time. Furthermore, our scheme is not restricted to the LDL.

We consider the system with a single two-level trapped ion placed inside a cavity that supports a single-mode quantized radiation field with frequency ω_a. For a standing wave travelling in the x-direction, the Hamiltonian of the system is given by ($\hbar = 1$)[11]

$$H = \omega_a a^+ a + \nu b^+ b + \frac{1}{2}\omega_0 \sigma_z + \frac{1}{2}\Omega(a^+ + a)(\sigma_- + \sigma_+)\sin(\eta\hat{x} + \phi), \quad (1)$$

where η is the Lamb–Dicke parameter, $\hat{x} = b^+ + b$ denotes a dimensionless position operator for the CM of the ion with b^+ and b respectively being the creation and annihilation operators of the CM vibrational motion mode, and ϕ accounts for the relative position of the CM of the ion to the standing wave. ν denotes the vibrational frequency and ω_0 is the transition frequency between the internal electronic states $|e\rangle$ and $|g\rangle$. ν is generally supposed to be much greater than

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the atomic decay rate, called the strong confinement limit (SCL), for neglecting the effect of the atomic decay. a^+ and a are the creation and annihilation operators of the single mode field, respectively. The coupling parameter Ω is directly proportional to the ion–radiation interaction strength. σ_+, σ_- and σ_z are the familiar pseudo-spin operators defined by $\sigma_+ = |e\rangle\langle g|$, $\sigma_- = |g\rangle\langle e|$ and $\sigma_z = |e\rangle\langle e| - |g\rangle\langle g|$, respectively.

Transforming H with the unitary operator defined by

$$U = \exp(-\mathrm{i}\omega_a a^+ a t)\exp\left(-\frac{1}{2}\mathrm{i}\omega_0\sigma_z t\right), \tag{2}$$

in the interaction picture, we have

$$\begin{aligned} H_\mathrm{i} =& \nu b^+ b + \frac{1}{2}\Omega\sin[\eta(b^+ + b) + \phi] \\ & \times (a^+\sigma_- \mathrm{e}^{\mathrm{i}(\omega_a-\omega_0)t} \\ & + a^+\sigma_+ \mathrm{e}^{\mathrm{i}(\omega_a+\omega_0)t} + \mathrm{H.c.}). \end{aligned} \tag{3}$$

Taking the resonant case $\omega_0 = \omega_a$ and the rotating-wave approximation, we can obtain

$$H_\mathrm{i} = \nu b^+ b + \frac{1}{2}\Omega\sin[\eta(b^+ + b) + \phi](a^+\sigma_- + a\sigma_+). \tag{4}$$

We now assume $|\Psi(0)\rangle = |e\rangle|0\rangle_a|0\rangle_b$, i.e. the ion is initially in the excited electronic state $|e\rangle$, and the single-mode and the vibrational motion field are both in the vacuum state. After an interaction time τ, the state of the system is given by

$$|\Psi(\tau)\rangle_\mathrm{i} = \exp(-\mathrm{i}H_\mathrm{i}\tau)|e\rangle|0\rangle_a|0\rangle_b. \tag{5}$$

As the laser field is treated as the quantum light field, the ion–radiation interaction strength Ω may be small, after a short interaction time τ, we have $\Omega\tau \ll 1$. Then $|\Psi(\tau)\rangle_\mathrm{i}$ can be approximated by

$$\begin{aligned} |\Psi(\tau)\rangle_\mathrm{i} \simeq& |e\rangle|0\rangle_a|0\rangle_b - \mathrm{i}H_\mathrm{i}\tau|e\rangle|0\rangle_a|0\rangle_b \\ =& |e\rangle|0\rangle_a|0\rangle_b - \mathrm{i}\frac{1}{2}\Omega\tau\sin[\eta(b^+ + b) \\ & + \phi]|g\rangle|1\rangle_a|0\rangle_b. \end{aligned} \tag{6}$$

We now detect the number of photons in the cavity. If we find one photon in the cavity,[a)] the vibrational motion is given by

$$\begin{aligned} |\Psi(\tau)\rangle_{\mathrm{i},b} =& C(\mathrm{e}^{\mathrm{i}\eta(b^+ + b)} - \mathrm{e}^{-\mathrm{i}2\phi}\mathrm{e}^{-\mathrm{i}\eta(b^+ + b)})|0\rangle_b \\ =& C(|\mathrm{i}\eta\rangle_b - \mathrm{e}^{-\mathrm{i}2\phi}|-\mathrm{i}\eta\rangle_b), \end{aligned} \tag{7}$$

where C is a normalization factor, $|\mathrm{i}\eta\rangle_b$ is the coherent state

$$|\mathrm{i}\eta\rangle_b = \mathrm{e}^{\mathrm{i}\eta(b^+ + b)}|0\rangle_b, \tag{8}$$

and the state $|\Psi(\tau)\rangle_{\mathrm{i},b}$ is a superposition of two coherent states of the ion motion in a single-mode cavity separated in phase by π. The relative phase 2ϕ between the two coherent components is controllable by adjusting the relative position of the CM of the ion to the standing wave. For the special case $\phi = 0$ (i.e. the ion is centred at a node of the standing wave), the vibrational motional state is

$$|\Psi(\tau)\rangle_{\mathrm{i},b} = C(|\mathrm{i}\eta\rangle_b - |-\mathrm{i}\eta\rangle_b), \tag{9}$$

it is an odd coherent state. If the ion is centred at an antinode of the standing wave ($\phi = \pi/2$), the motional state

$$|\Psi(\tau)\rangle_{\mathrm{i},b} = C(|\mathrm{i}\eta\rangle_b + |-\mathrm{i}\eta\rangle_b), \tag{10}$$

it is an even coherent state.

In summary, we have proposed a simple scheme for the generation of the motional SC states of the trapped ion interacting with a single-mode radiation field under the SCL condition. Although there is only one half chance for obtaining the motional SC states of the trapped ion in our scheme, it is convenient for the experiment realization. Firstly, our scheme is not restricted to the LDL, so the trapped ions are less tightly confined and more rapidly laser-cooled. Secondly, when the laser field is treated as the quantum light field and the ion–radiation interaction strength Ω is small, the work condition in the present scheme $\Omega\tau \ll 1$ can be satisfied easily. Finally, the different (odd and even) coherent states can be prepared by controlling carefully the relative position of the ion in the standing wave.

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